Wind Turbine Structural Dynamics with the Finite Element Method

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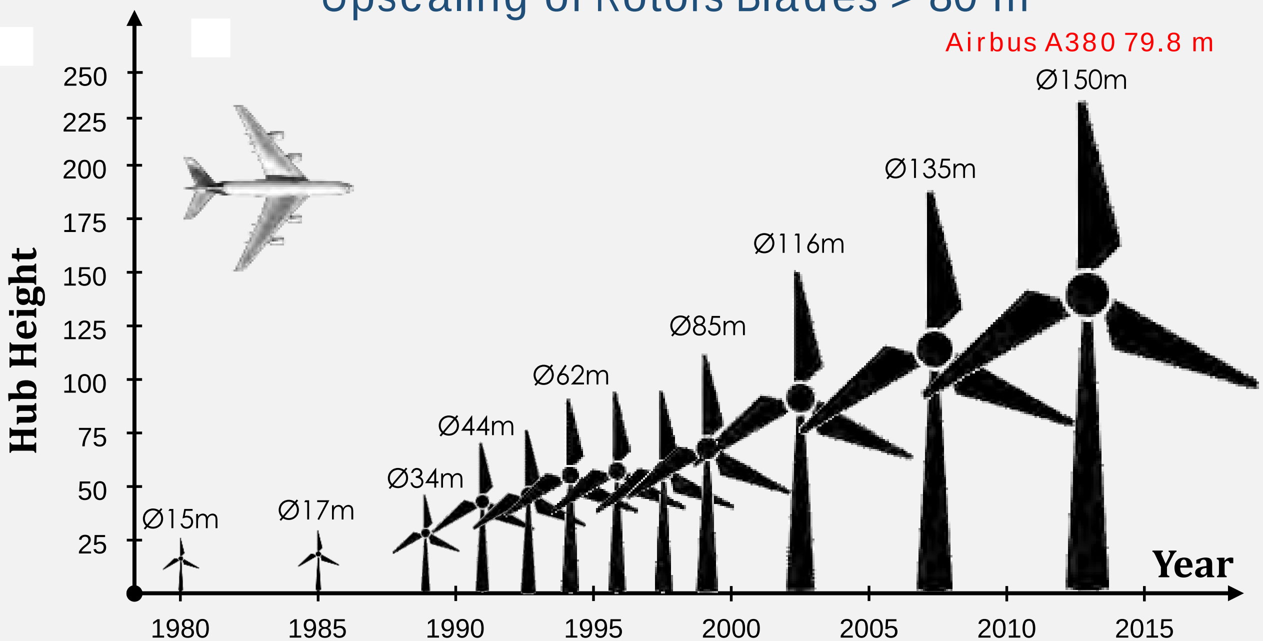
Preliminaries

Trends and Challenges

- Reduce cost per kWh
- Increase Power Generation Efficiency
- Reach Higher Altitudes

Upscaling of Rotors Blades > 80 m



Challenges for Large Rotor Blades:

- Increased steady & unsteady aerodynamic loads
- Increased Weight
- Increased blade flexibility
- Increased Fatigue Stresses
- Nonlinear blade structural behavior

Aerodynamic Load Control:

- Passive
  - o Bending-Twist Coupling (BTC)
  - o Damping
  - o Stall
- Active
  - o Pitch
  - o Flaps
  - o Local Morphing

Significance of Blade/Rotor Structural Dynamics

Blade Structural Dynamics

$$[M]\ddot{u}(t)+[C]\dot{u}(t)+[K]u(t)=F(t)$$

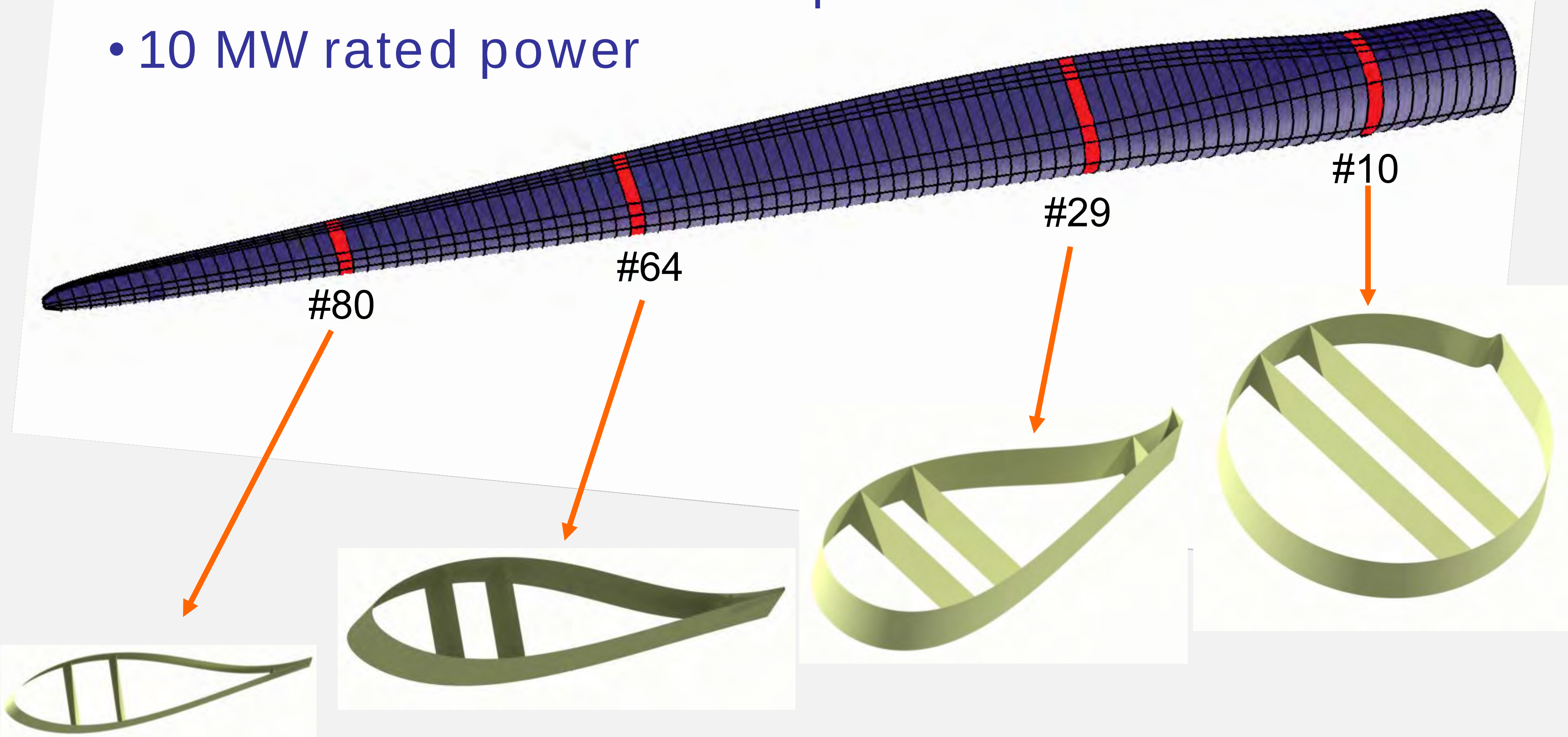
Blade Unsteady Aerodynamics

Blade Aeroelastic Model

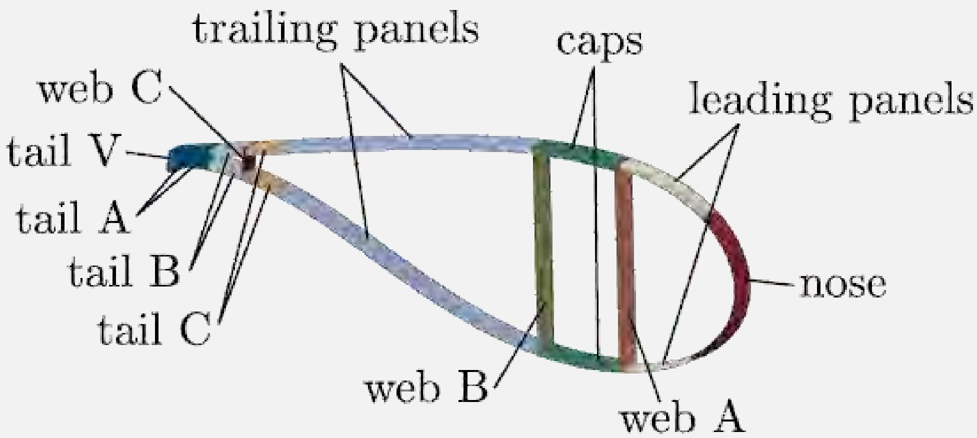
- Unsteady Aerodynamic Loads on the blade F(t) → Fatigue Loads
- Aeroelastic stability
- Time trajectory or vibration response of the blade u(t)
- Dynamic stresses → Fatigue life of the blade
- Can encompass Load Alleviation Mechanisms

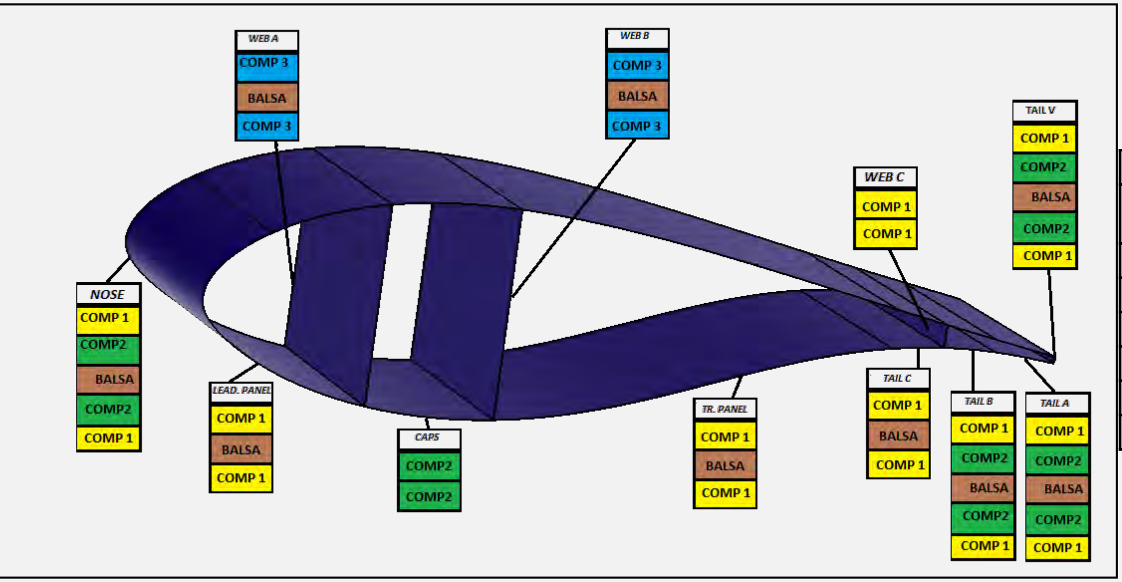
Structural Complexity of WT Blades

- 89 m composite blade
- 9.6 rpm (0.16Hz) rated speed
- 10000 kNm rated rotor torque
- 10 MW rated power



Cross-Section Configuration & Materials

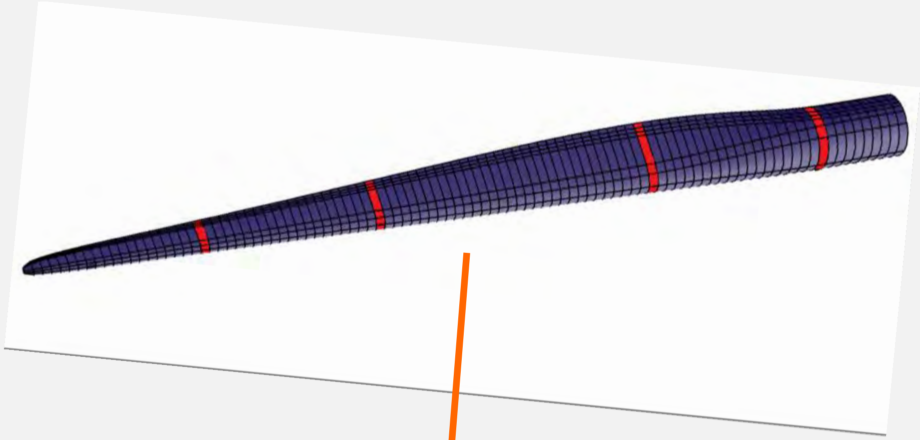


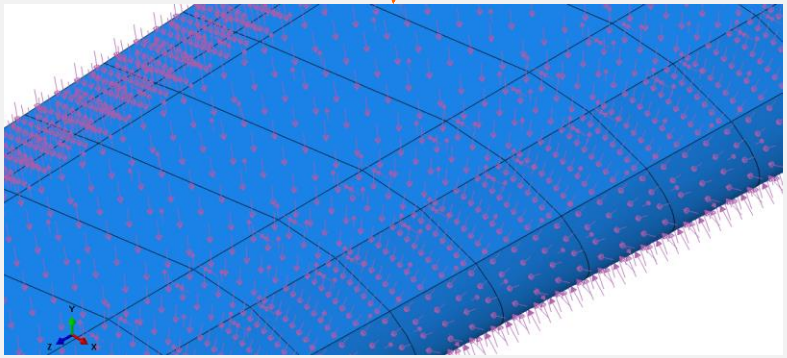


Multidirectional Ply	***Uniax***	***Biax***	***Triax***	***Balsa***	***Units***
Fiber Volume Fraction V_f	*0.55*	*0.5*	*0.5*	-	
Mass Density ρ	*1915.5*	*1845.0*	*1845.0*	*110*	Kg/m^3
Young's Modulus E_1	*41.63*	*13.92*	*21.79*	*0.05*	GPa
Young's Modulus E_2	*14.93*	*13.92*	*14.67*	*0.05*	GPa
Poisson's ratio v_{12}	*0.241*	*0.533*	*0.478*	*0.5*	-
Shear Modulus G_{12}	*5.047*	*11.50*	*9.413*	*0.01667*	GPa
Shear Modulus G_{13}	*5.04698*	*4.53864*	*4.53864*	*0.15*	GPa

Glass/Epoxy [0], [0/90], [0/45/-45]
Carbon/Epoxy [0]

Detailed FEA Structural Models

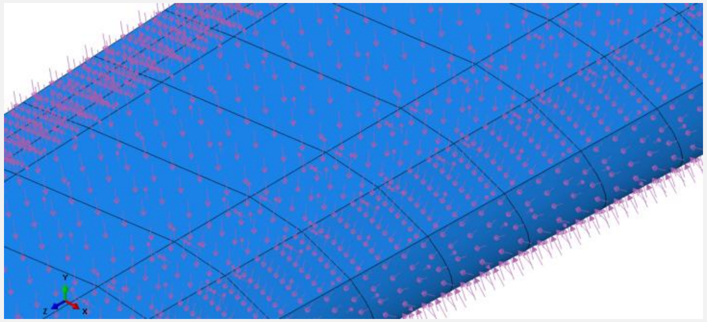




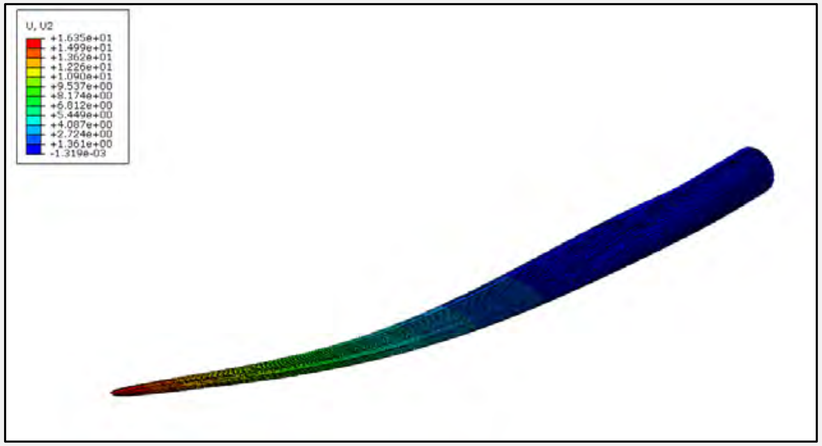
- Involve 3D surface or volume elements
- 3D Laminated Composite Shell elements (1 element per skin thickness)
- Solid elements with anisotropic continuum capabilities (1 element per composite layer)
- Result in Large Size discretizations
- Mainly used for
  - Static Analysis
  - Stress calculations
  - Strength and Fatigue Life predictions
  - Buckling Analysis

Static & Stress FEA Results

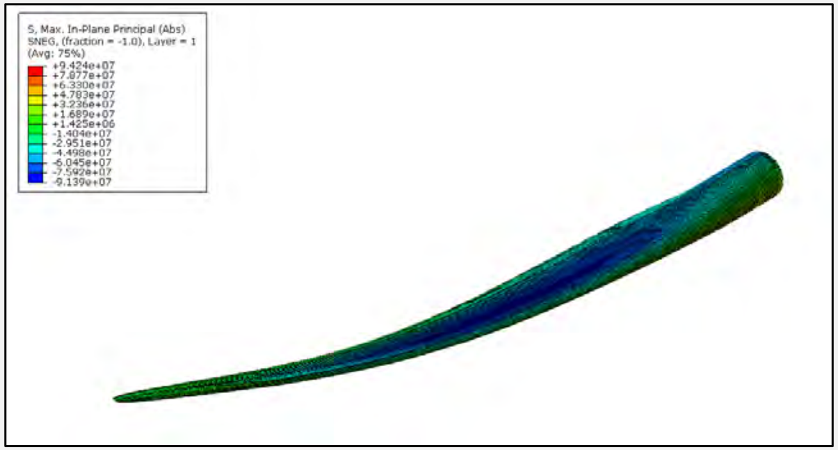
Aerodynamic pressure distribution
(close up)



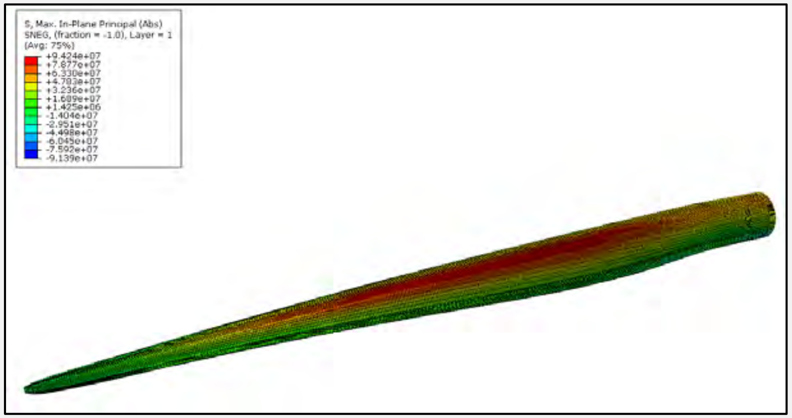
Displacement distribution



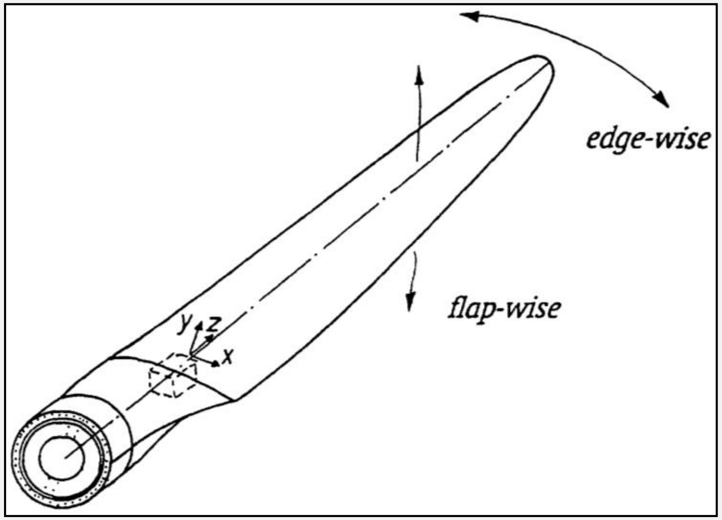
Stress distribution-Upper Surface



Stress distribution-Bottom Surface

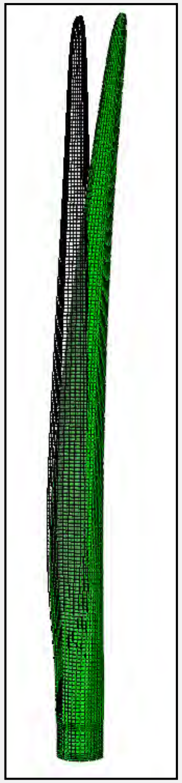


Modal Analysis

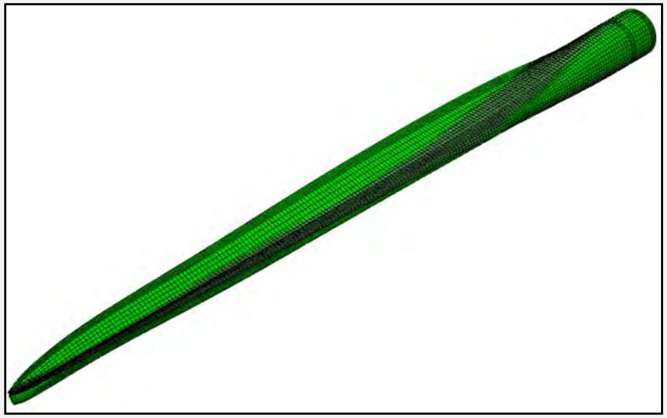


Mode Shape	Eigen Frequencies(Hz)
Flapwise	0.668
Edgewise	0.936
Torsional	6.236

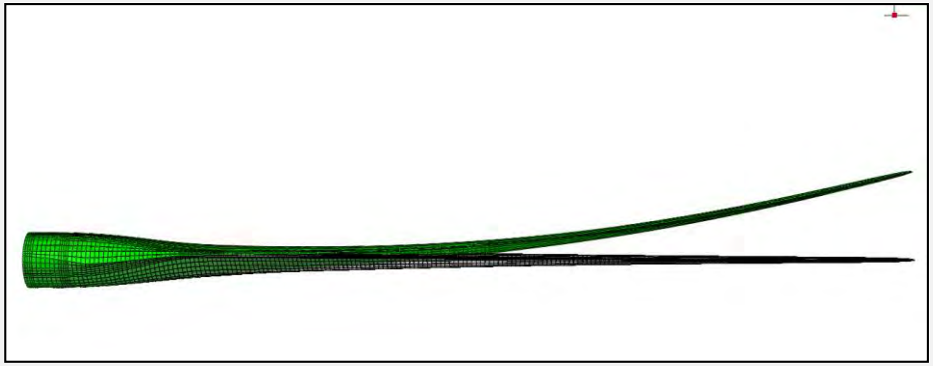
Edgewise



Torsional

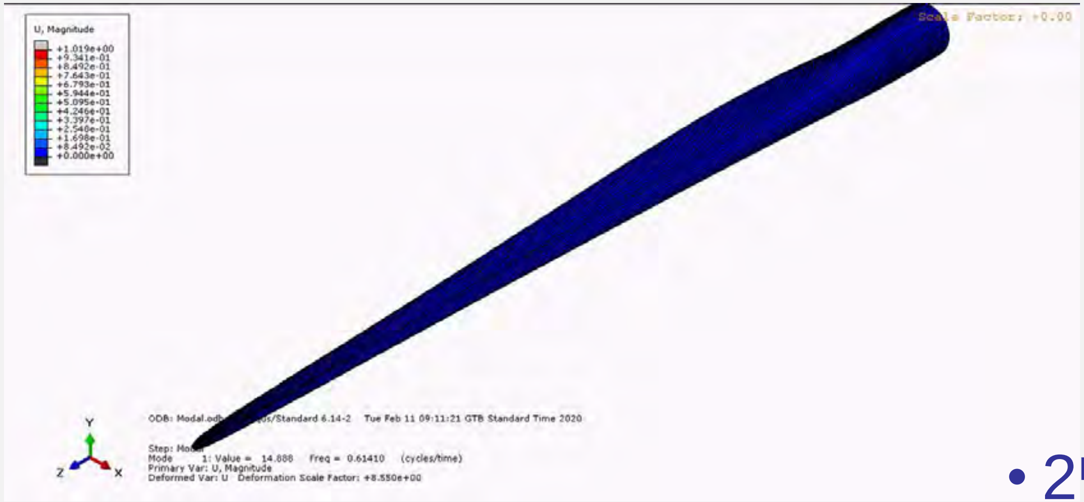


Flapwise



Flapwise Modes

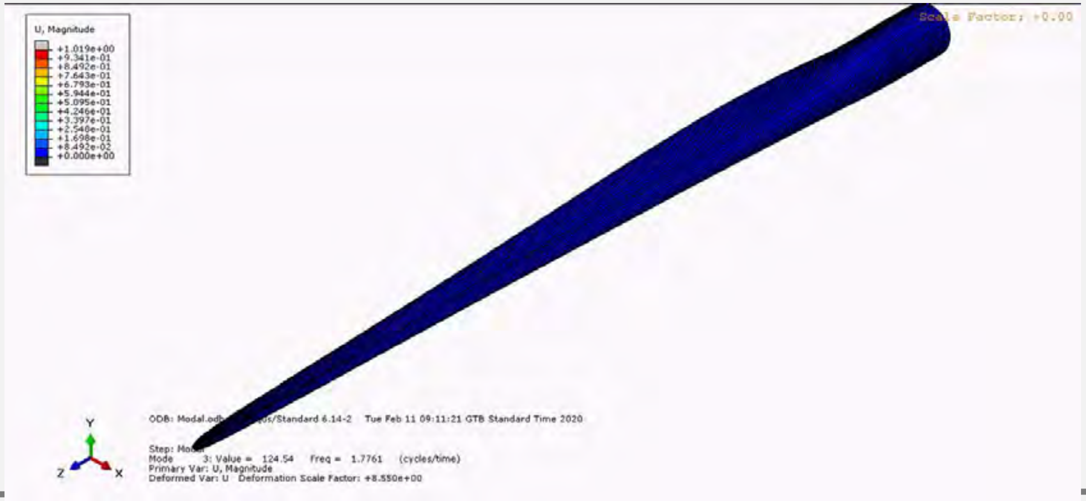
- 1st Flapwise @0.614 Hz



To avoid Resonance:
Modal (Resonance) frequencies
>
rotor rotation harmonics:

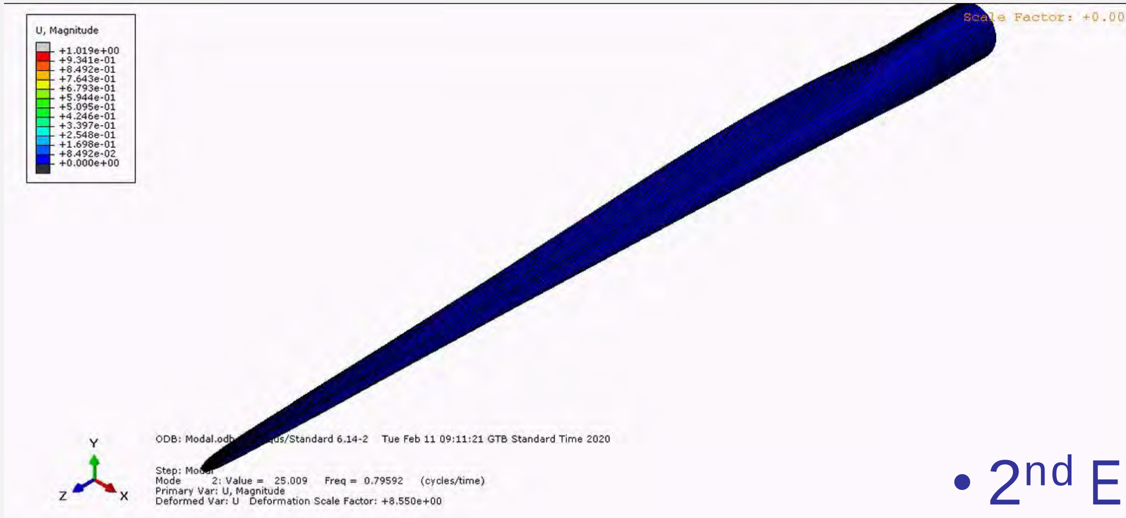
- 1 per rev, 2 per rev, **3 per rev**, …

- 2nd Flapwise @1.776 Hz

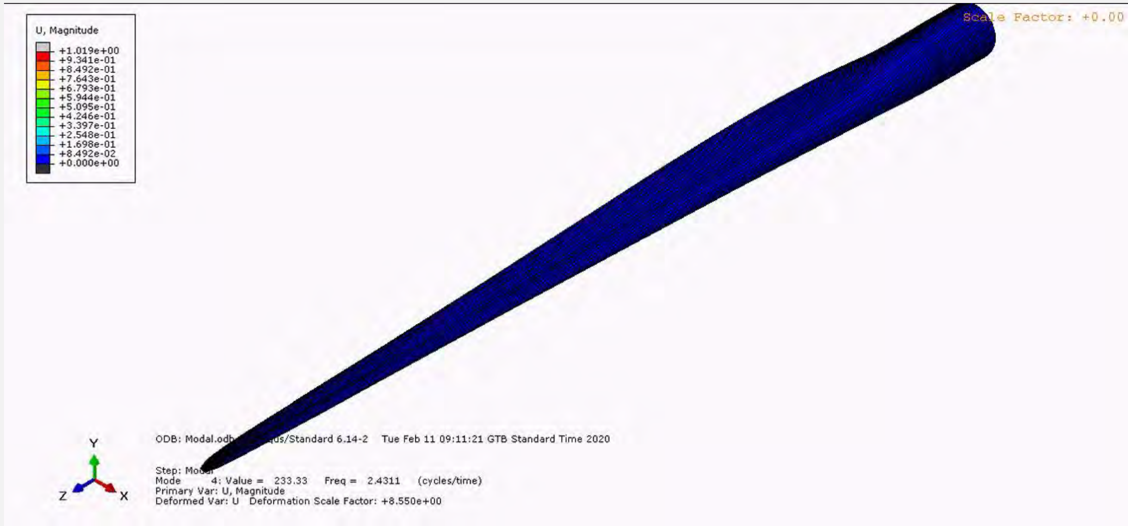


Edgewise Eigenmodes

- 1st Edgewise @0.795 Hz

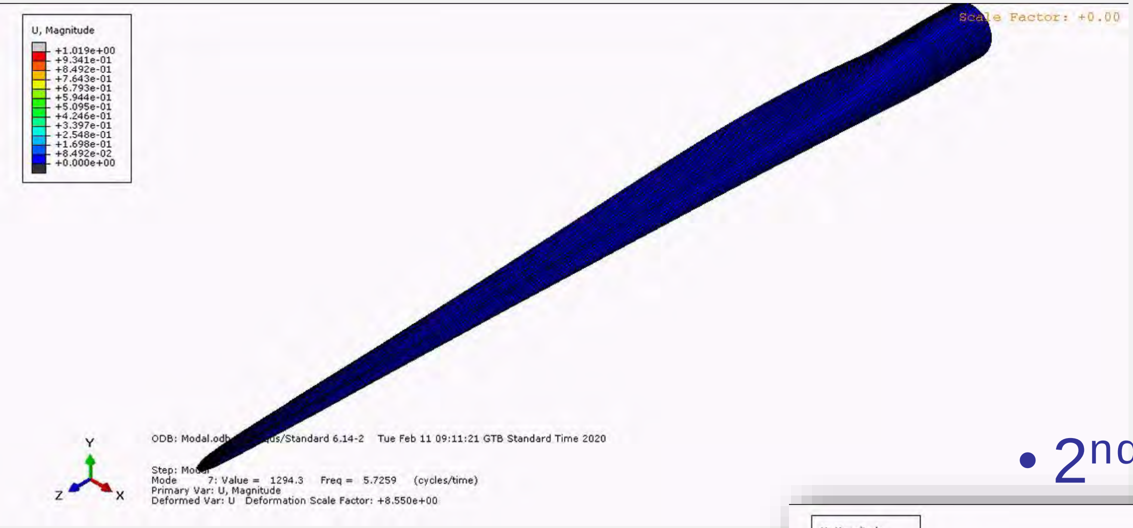


- 2nd Edgewise @2.431 Hz



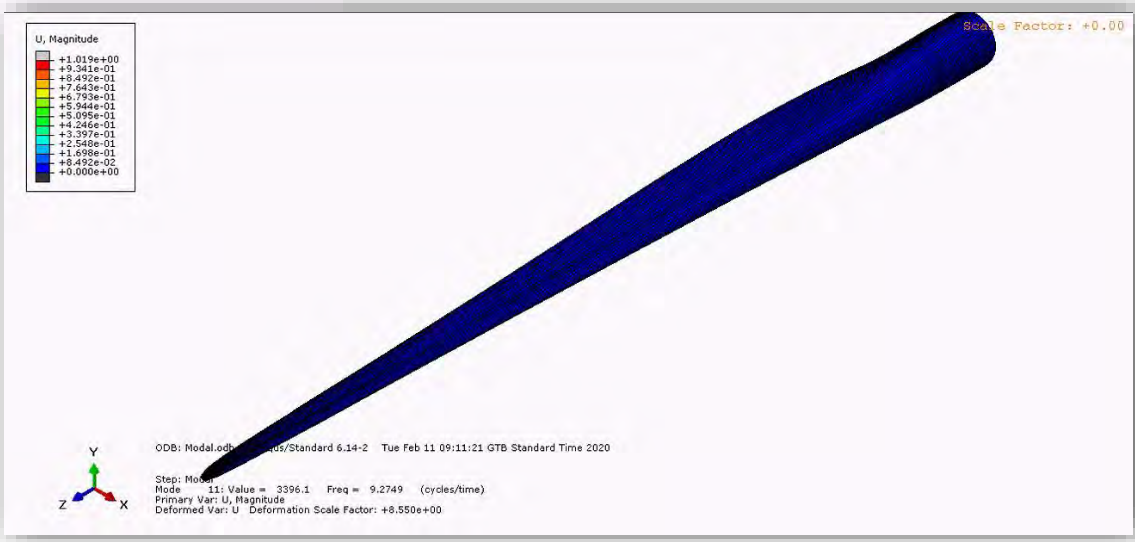
Torsional Eigenmodes

- 1st Torsional @5.725 Hz



All modes
are excited by
and interact with
unsteady aerodynamic
loads

- 2nd Torsional @9.274 Hz



Modelling Challenges in Computational Blade Structural Dynamics

Blade Structural Dynamics

$$[M]\ddot{u}(t)+[C]\dot{u}(t)+[K]u(t)=F(t)$$

Blade Unsteady Aerodynamics

Blade Aeroelastic Model

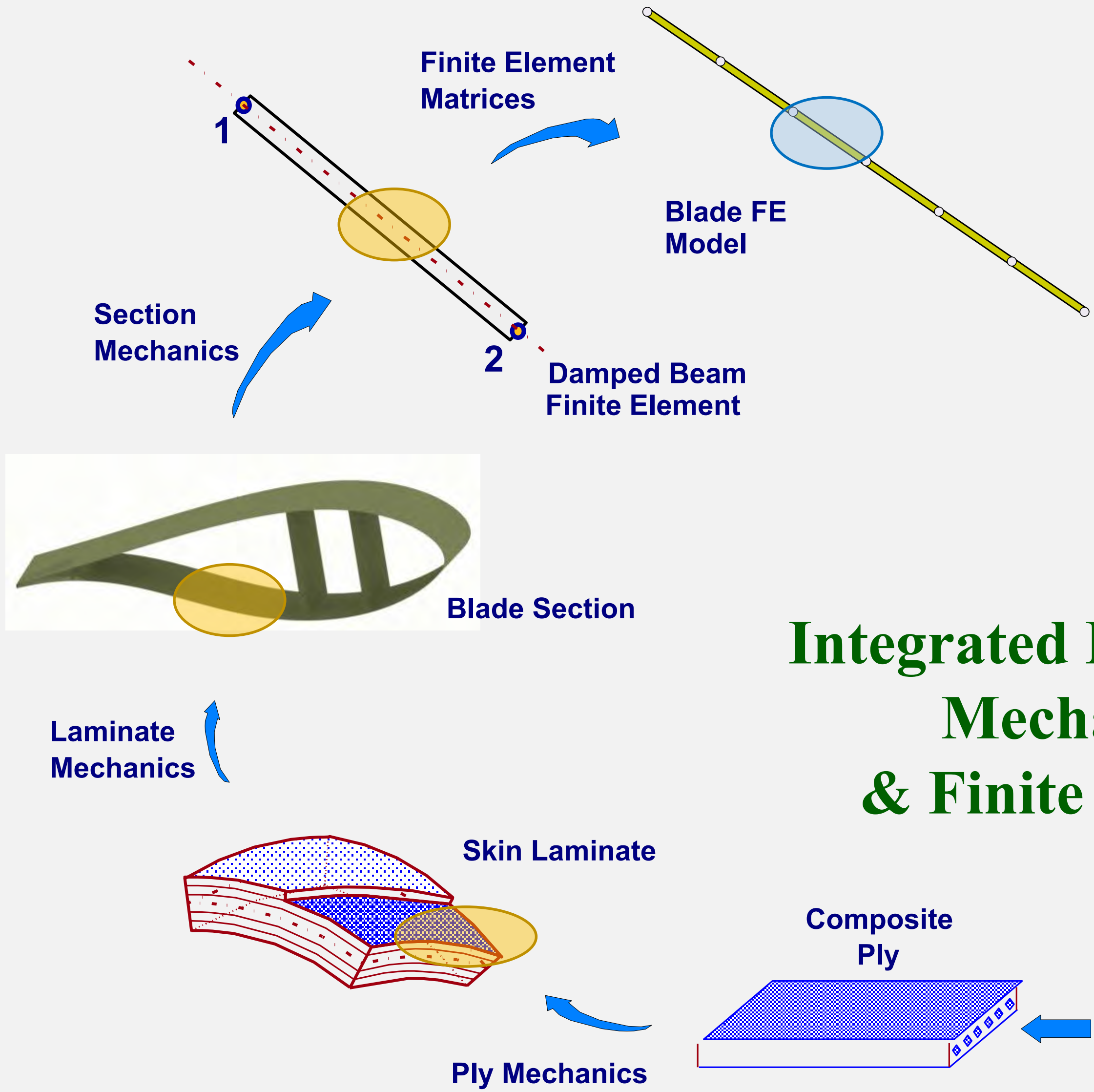
Large Size Coupled Interdisciplinary problem in time domain

- Needs:
  - Reduce the size of the Structural Dynamics Model
  - Predict Structural Damping
  - Non-linear effects due to large blade deformantions
  - Robust & Efficient Computational FEA Framework for Blade Structural Dynamics

Presentation Topics

- Part 1: Reduced-order, Multi-Scale Beam FE model for WTB
- Part 2: Prediction of structural composite damping
- Part 3: Geometric Nonlinear Effects

Composite Blade Analysis Cycle



Small size

Discretized SD Model

$$[M]\ddot{u}(t)+[C]\dot{u}(t)+[K]u(t)=F(t)$$

Integrated Multi-Scale Mechanics & Finite Element

Composite Ply Scale

Minimal Material Input:

- 5 Elastic coefficients
- 4 Damping coefficients

Part 1: Reduced-order, Multi-Scale Beam FE model for WTB

Section Kinematics

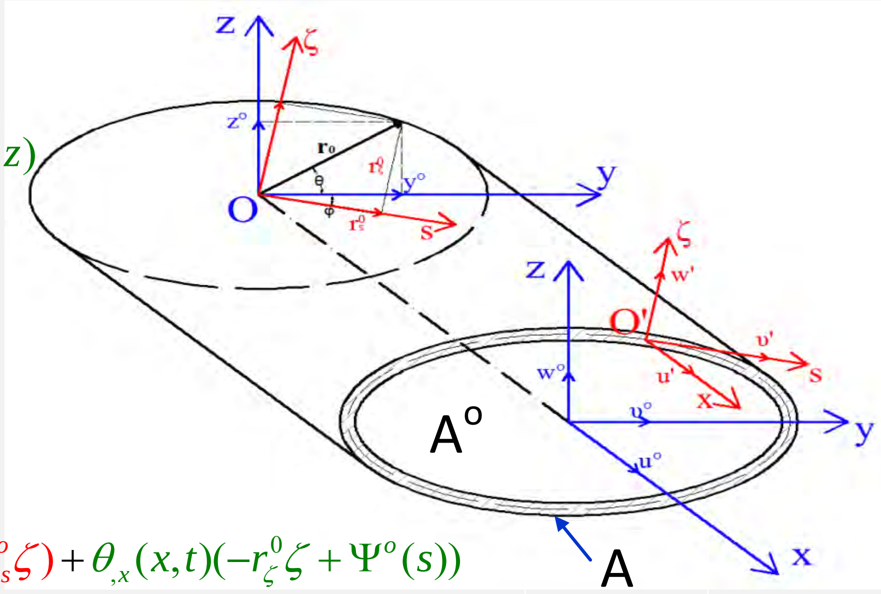
- Cartesian System

$$u(x,y,z,t)=u^o(x,t)+z\beta_y(x,t)+y\beta_z(x,t)+\theta_{,x}(x,t)\Psi(y,z)$$

$$\upsilon(x,y,z,t)=\upsilon^o(x,t)-z\theta(x,t).$$

$$w(x,y,z,t)=w^o(x,t)+y\theta(x,t)$$

- Local Curvilinear System



$$u(x,s,\zeta,t)=u^o(x,t)+\beta_y(x,t)(z_o+y^o_{,s}\zeta)+\beta_z(x,t)(y_o-z^o_{,s}\zeta)+\theta_{,x}(x,t)(-r^0_\zeta\zeta+\Psi^o(s))$$

$$\upsilon'(x,s,\zeta,t)=\upsilon^o(x,t)-\theta(x,t)(r^o_\zeta+\zeta)$$

$$w'(x,s,\zeta,t)=w^o(x,t)+\theta(x,t)r^o_s$$

- Assumptions:
  - Admit extension, bending, transverse shear and torsion
  - Plane sections deformed in extension and bending remain plane
  - Secondary warping due to torsion
  - 6 generalized displacements: $u^o, v^o, w^o, \beta_x, \beta_y, \theta$

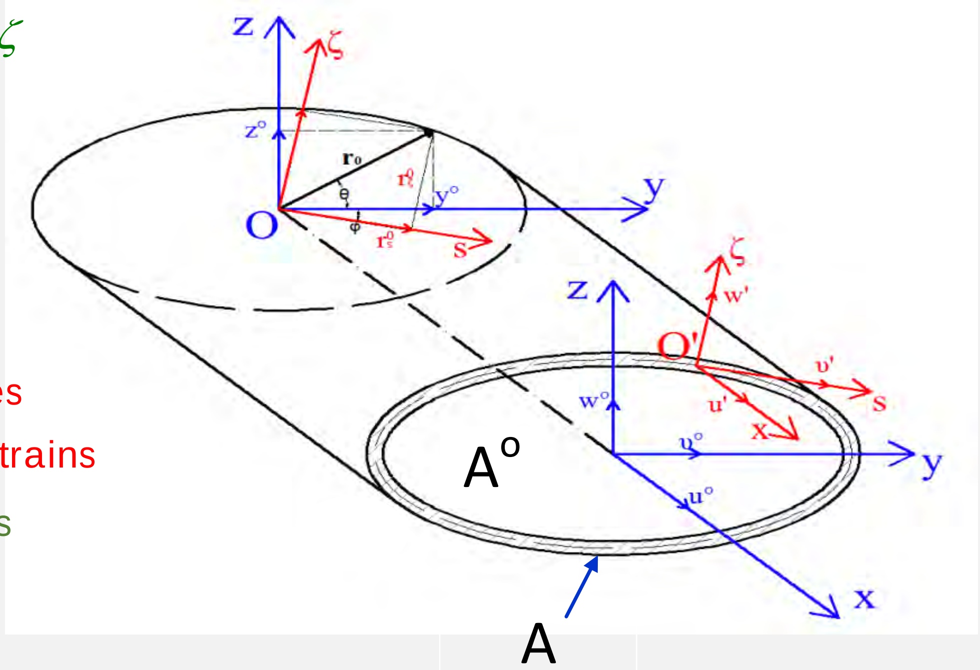
Section Strains

- Local Curvilinear System

$$\varepsilon_x(x,s,\zeta) = \varepsilon_x^0(x) + k_{xy}(x)(z_o + y_{,s}^o\zeta) + k_{xz}(x)(y_o - z_{,s}^o\zeta) + \theta_{,xx}(x)(-r_\zeta^0\zeta + \Psi^o(s))$$

$$\varepsilon_{xs}(x,s,\zeta) = \varepsilon_{xz}^0(x)z_{,s}^0 + \varepsilon_{xy}^0(x)y_{,s}^0 + \varepsilon_{xs}^{0t}(x,s) - 2\theta_{,x}\zeta$$

$$\varepsilon_{x\zeta}(x,s,\zeta) = \varepsilon_{xz}^0(x)y_{,s}^0 - \varepsilon_{xy}^0(x)z_{,s}^0$$



- 6 Generalized Strains

$\varepsilon_x^o = u_{,x}^o$ Axial strain

$k_{xy} = \beta_{y,x}$, $k_{xz} = \beta_{z,x}$ Bending curvatures

$\varepsilon_{xy}^0 = \upsilon_{,x}^0 + \beta_z$, $\varepsilon_{xz}^o = w_{,x}^o + \beta_y$ Transverse shear strains

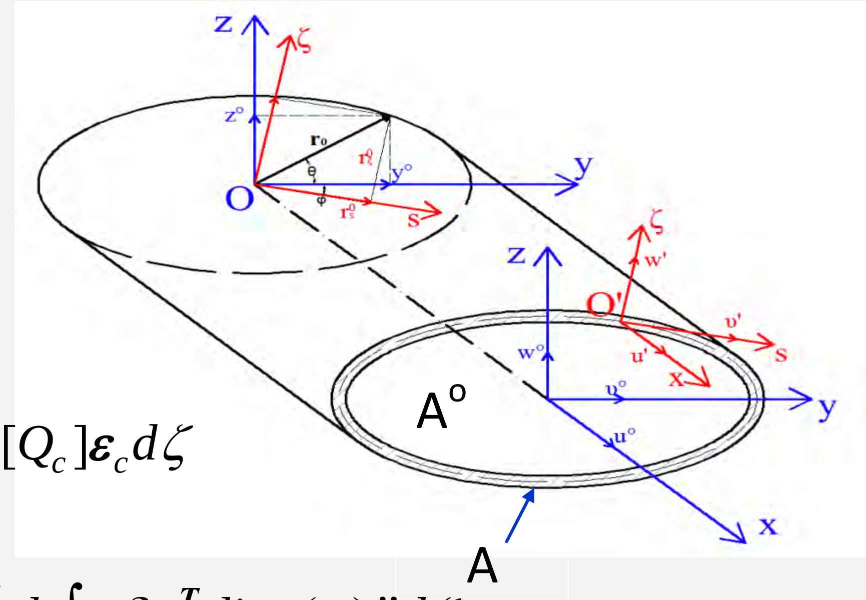
$k_\theta = \theta_{,x}$, $k_{\theta\theta} = \theta_{,xx}$ Twisting curvatures

- Assumptions:
  - $K_{\theta\theta}$ is assumed to be negligible
  - The torsional strain $\varepsilon_{xs}{}^{ot}$ and warping function Ψ^o are evaluated analytically by solving the torsion stress equilibrium equation on the $s\zeta$ plane using a torsional strain function Φ.

Equations of Motion

- Principle of Virtual Works

$$\int_0^L dx \int_A (-\delta H + \delta T - \delta W_d)\, ds\, d\zeta + \oint_\Gamma \delta \bar{\boldsymbol{u}}^T \bar{\boldsymbol{\tau}}\, d\Gamma = 0$$



- Section Energies & Works

Strain Energy

$$\delta H_s = \int_A \delta \boldsymbol{\varepsilon}_c^T \boldsymbol{\sigma}_c\, ds\, d\zeta = \oint ds \int_h \delta \boldsymbol{\varepsilon}_c^T [Q_c] \boldsymbol{\varepsilon}_c\, d\zeta$$

Inertial forces

$$\delta T_s = \int_A -\delta \boldsymbol{u}^T diag(\rho) \ddot{\boldsymbol{u}}\, d\xi\, d\eta = \oint ds \int_h -\delta \boldsymbol{u}^T diag(\rho) \ddot{\boldsymbol{u}}\, d\zeta$$

Surface Tractions

$$\delta W_\tau = \oint_\Gamma \delta \bar{u}^T \bar{\tau}\, d\Gamma = \int_0^L dx \oint \delta \bar{u}^T \bar{\tau}\, ds$$

Blade Section Stiffness

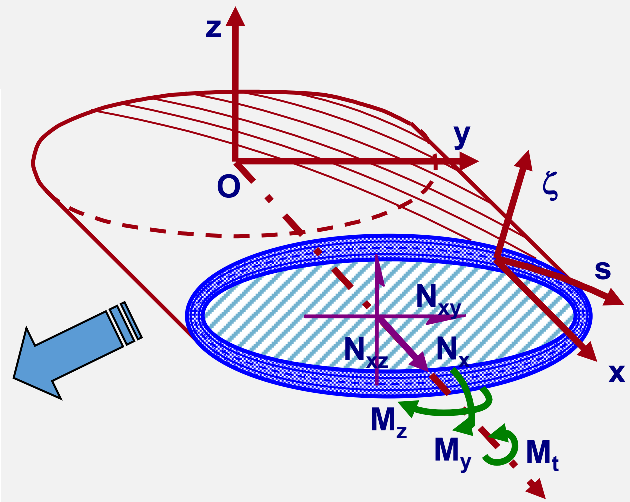
Extension – Shear Coupling

Coupling Stiffness

$$
\begin{Bmatrix} N_x^o \\ N_{xy}^o \\ N_{xz}^o \\ M_{xy} \\ M_{xz} \\ M_\theta \end{Bmatrix}
=
\begin{bmatrix}
A_{11}^0 & \bar{A}_{15}^0 & \bar{A}_{16}^0 & B_{11}^0 & B_{12}^0 & \bar{B}_{16}^0 \\
\bar{A}_{51}^0 & A_{55}^0 & A_{56}^0 & \bar{B}_{51}^0 & \bar{B}_{52}^0 & B_{56}^0 \\
\bar{A}_{61}^0 & A_{65}^0 & A_{66}^0 & \bar{B}_{61}^0 & \bar{B}_{62}^0 & B_{66}^0 \\
B_{11}^0 & B_{51}^0 & \bar{B}_{61}^0 & D_{11}^0 & D_{12}^0 & \bar{D}_{16}^0 \\
\bar{B}_{12}^0 & \bar{B}_{52}^0 & B_{62}^0 & D_{21}^0 & D_{22}^0 & \bar{D}_{26}^0 \\
\bar{B}_{16}^0 & \bar{B}_{56}^0 & B_{66}^0 & \bar{D}_{61}^0 & \bar{D}_{62}^0 & D_{66}^0
\end{bmatrix}
\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_{xy}^o \\ \varepsilon_{xz}^o \\ k_{xy} \\ k_{xz} \\ k_\theta \end{Bmatrix}
$$

Flexural & Torsional Stiffness

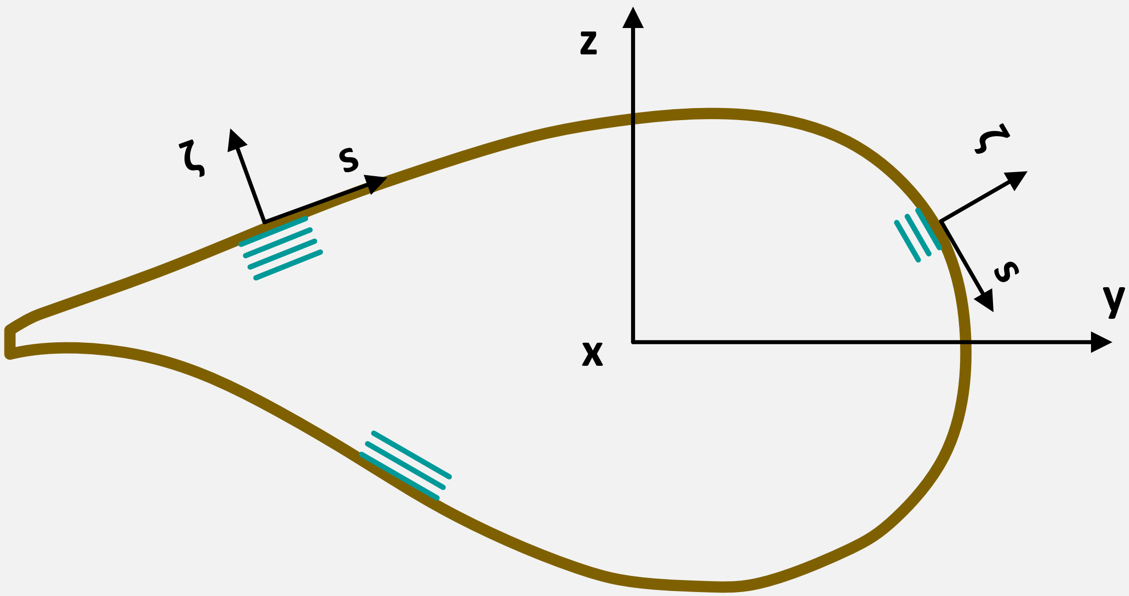
Bending – Torsion Coupling



Overbar indicates terms which include also material coupling effect (or skin laminate) at skin level due to rotated plies →$Q_{16} \neq 0$ or A_{16}, B_{16}, $D_{16} \neq 0$

Calculation of Select Stiffness terms

Section Example



Example of section:

- Laminate/element coordinate system indicated

&

- Skin laminates with local laminate stiffness matrices [A], [B], [D]

Flapping Stiffness	$D_{11}^0 = \oint (A_{11}(z^0)^2 + 2z^0 y_{,s} B_{11} + (y_{,s}^0)^2 D_{11}) ds$
Flapping-Torsion Coupling Stiffness	$D_{16}^0 = \oint (-A_h z^0 A_{16} - (A_h y_{,s}^0 + 2z^0) B_{16} - 2y_{,s}^0 D_{16}) ds$
Torsional Stiffness	$D_{66}^0 = \oint (A_h^2 A_{66} + 4 A_h B_{66} + 4 D_{66}) ds$

Blade Section Mass

- Work of Inertial Forces

$$\delta T^{sec} = \oint \left(\delta \mathbf{u}^{\mathbf{0}^{\mathbf{T}}} \mathbf{m}^{\mathbf{A}} \ddot{\mathbf{u}}^{\mathbf{0}} + 2\delta \mathbf{u}^{\mathbf{0}^{\mathbf{T}}} \mathbf{m}^{\mathbf{B}} \ddot{\boldsymbol{\beta}} + \delta \boldsymbol{\beta}^{\mathbf{T}} \mathbf{m}^{\mathbf{D}} \ddot{\boldsymbol{\beta}} \right) \mathrm{d}s$$

Linear Densities	First order coupling inertias (mass imbalance)	2nd order inertias (Rotational mass inertias)

$$\mathbf{m}^{\mathbf{A}} = \begin{bmatrix} m_{11}^{A} & 0 & 0 \\ 0 & m_{11}^{A} & 0 \\ 0 & 0 & m_{11}^{A} \end{bmatrix}, \quad \mathbf{m}^{\mathbf{B}} = \begin{bmatrix} m_{11}^{B} & m_{12}^{B} & m_{13}^{B} \\ 0 & 0 & m_{23}^{B} \\ 0 & 0 & m_{33}^{B} \end{bmatrix}, \quad \mathbf{m}^{\mathbf{D}} = \begin{bmatrix} m_{11}^{D} & m_{12}^{D} & m_{13}^{D} \\ m_{21}^{D} & m_{22}^{D} & m_{23}^{D} \\ m_{31}^{D} & m_{32}^{D} & m_{33}^{D} \end{bmatrix}$$

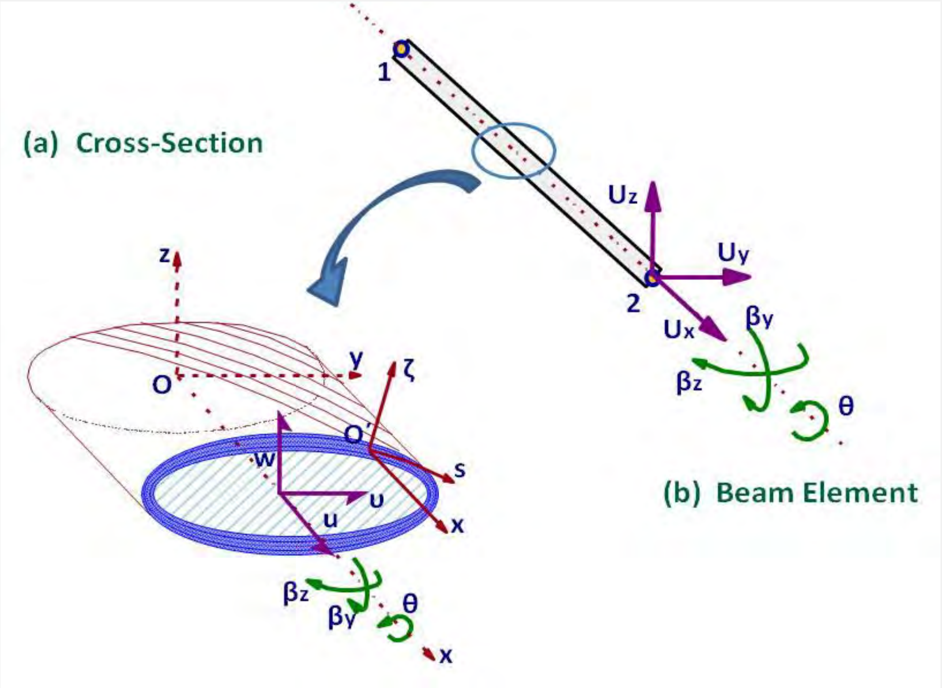
DAMPBEAM 3-D Finite Element

- **6 DOFs per node:** $\mathbf{U}_{\mathbf{e}}^{\mathbf{i}} = \left\{ u^{oi}, \quad \upsilon^{oi}, \quad w^{oi}, \quad \beta_y^i, \quad \beta_z^i, \quad \theta^i \right\}$

- 2-node shear element with linear displacement interpolation functions

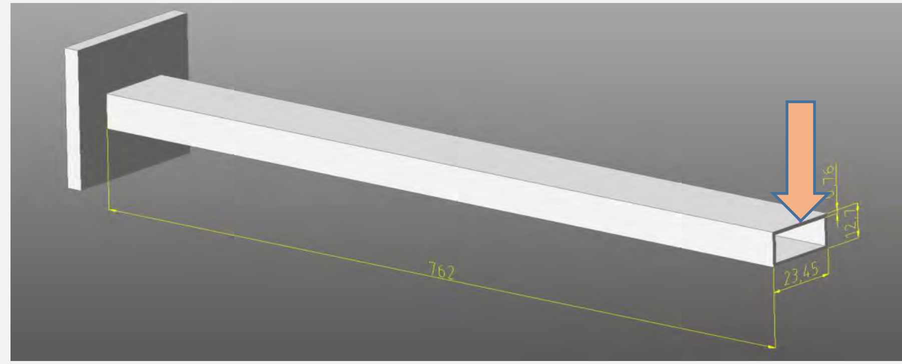
$$\left\langle u^o(x), \upsilon^o(x), w^o(x), \beta_y(x), \beta_z(x), \theta\ (x) \right\rangle \cong \sum_{i=1}^{n} N^i(x) \underset{\sim}{\mathbf{U}}_e^i$$

- Finite Element provides:
    - Stiffness matrix
    - Mass matrix



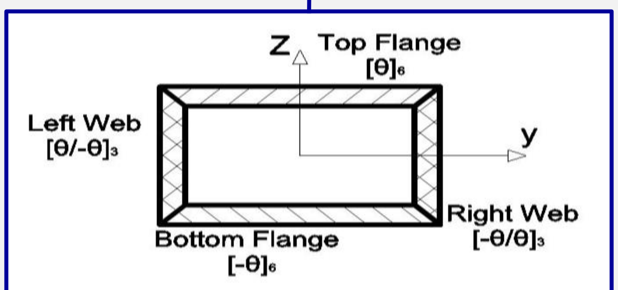
Validation Cases on Box-Section Beam

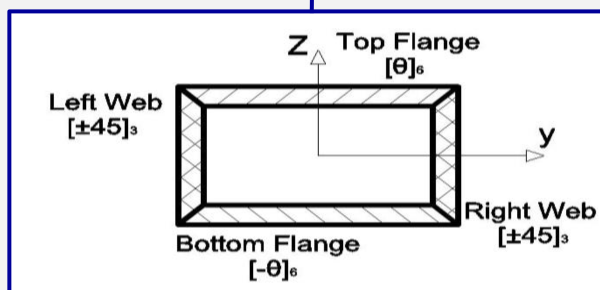
- Material: Carbon / Epoxy
- Vertical Force at tip, F=4.45N
- Dimensions and lay-up proposed by Volovoi & Hodges



Box-Section Beam

CASE I	CASE II

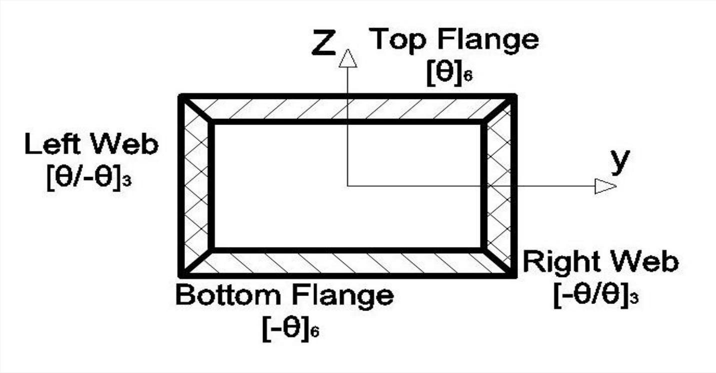


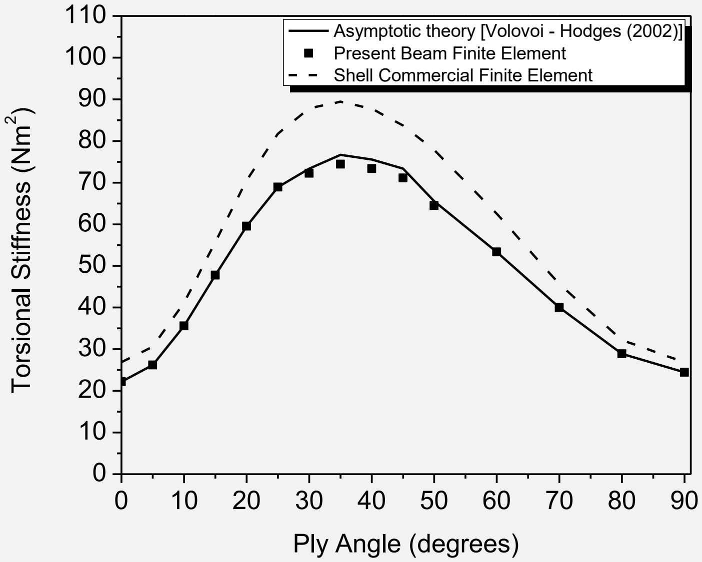


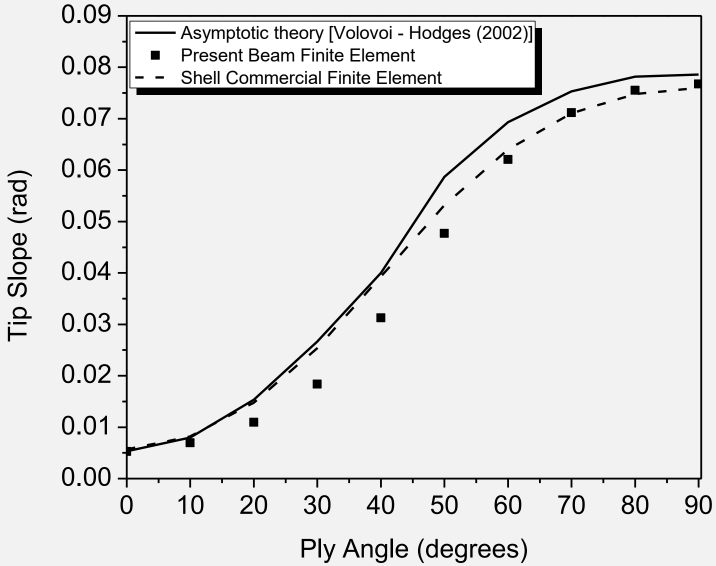
High BTC coupling (Case I) — **Low BTC coupling** (Case II)

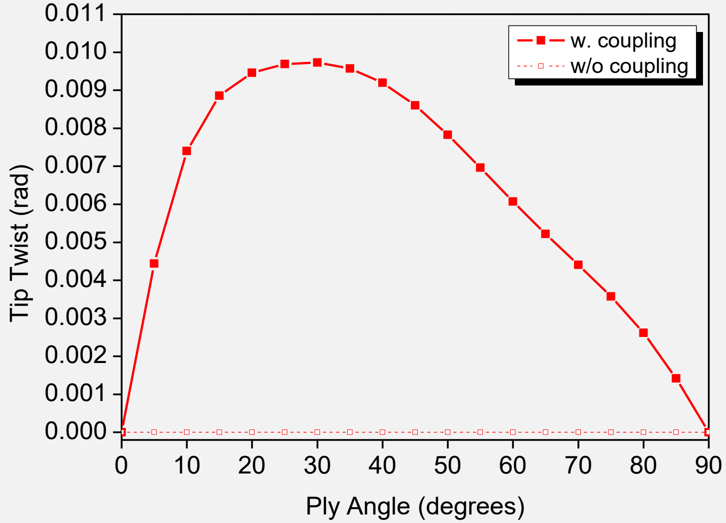
Ply-Angle Coupling Effect on Static Characteristics

CASE I

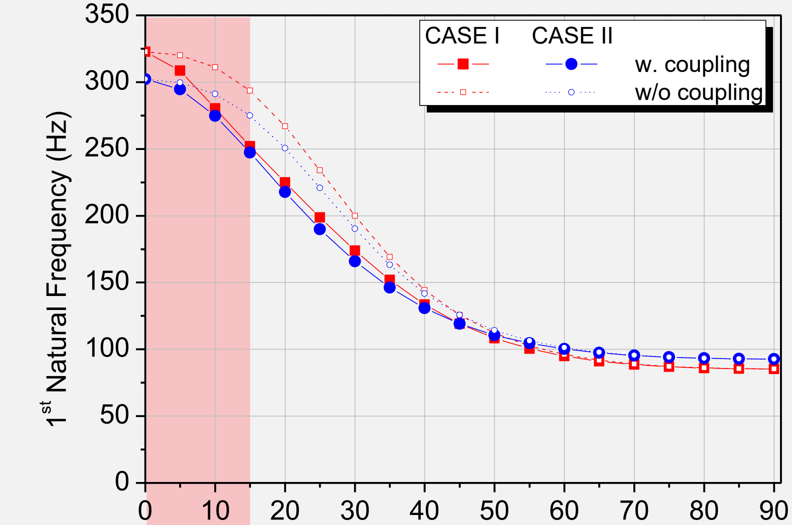


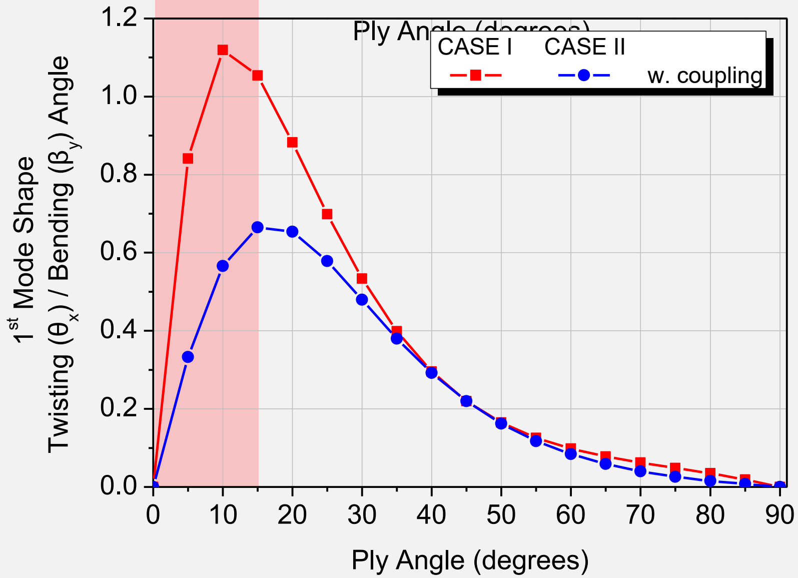




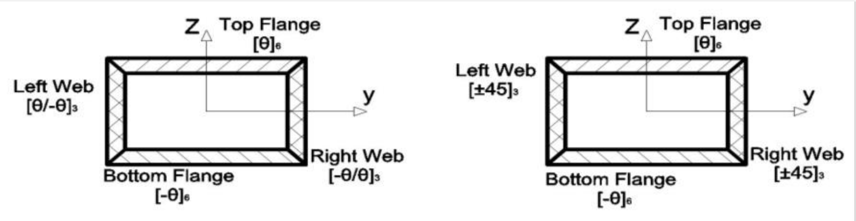


Material Coupling Effect on Dynamic Characteristics



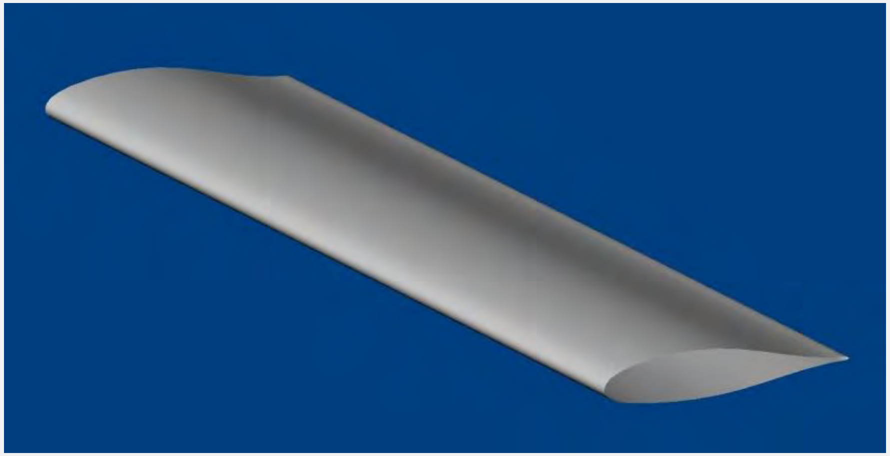


- High material coupling for $\theta = 5^0 - 30^0$ laminate orientations
- CASE I provides higher coupling values



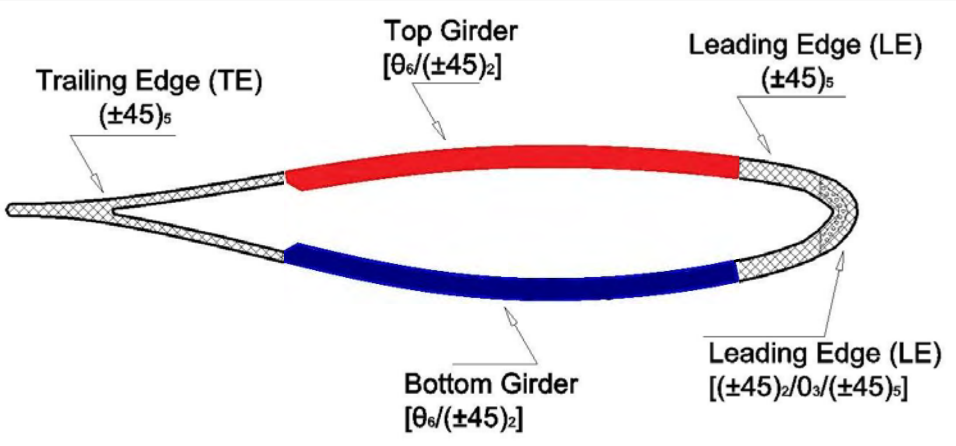
CASE I **CASE II**

Small Model Blade Configuration



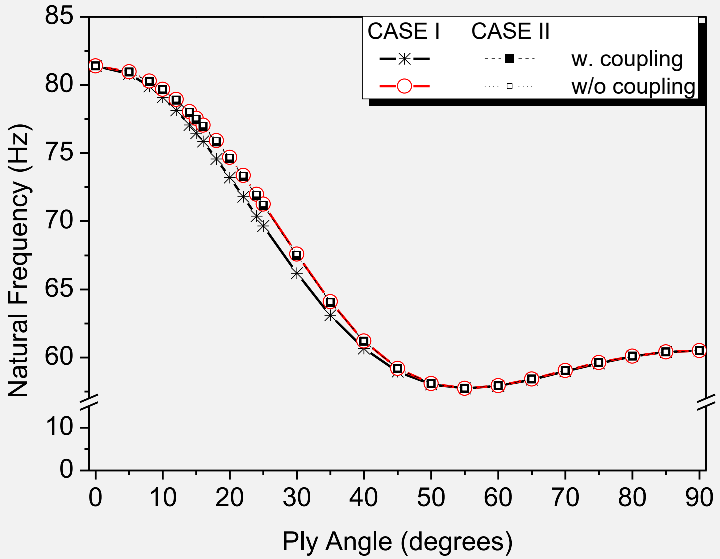
- Glass/Epoxy
- Various Ply Angle Laminations at Girder segments

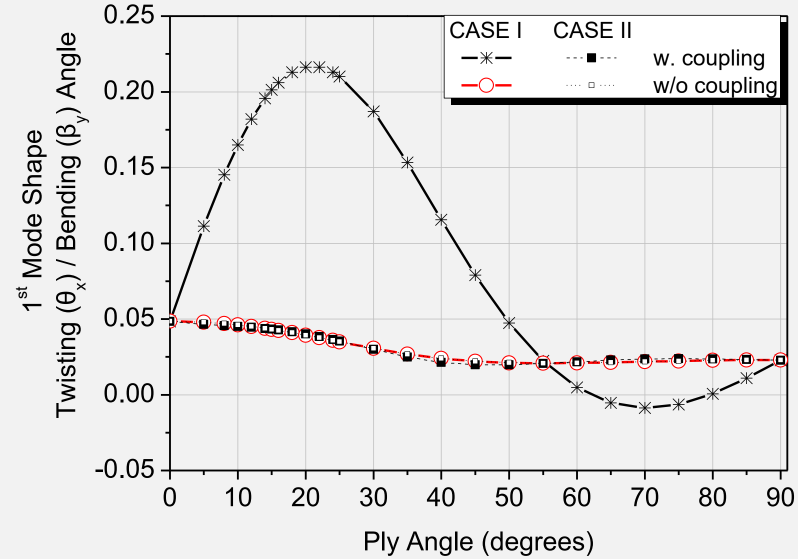
Length (mm)	Chord (mm)	Airfoil Thickness (mm)	Ply Thickness (mm)	Weight (Kg)
1540	283.5	50	0.80	6.0



	LAMINATION		
	Top Girder	Bottom Girder	Lay-up
CASE I	**[θ_6/(±45)$_2$]**	**[-θ_6/(±45)$_2$]**	**Anti-Symmetric**
CASE II	[θ_6/(±45)$_2$]	[θ_6/(±45)$_2$]	Symmetric

Material Coupling Effect on Small Model Blade





- **¨CASE I¨ presents a strong skin coupling terms** $A_{16}, B_{16}, D_{16} \neq 0$
- **CASE II provides almost negligible coupling** $A_{16}, B_{16}, D_{16} \cong 0$
- **The difference is eliminated at θ=0^0 and 90^0 where the material coupling effect is physically vanished.**

Part 2:
Inclusion of Composite Damping

Significance of Composite Damping

- Damping is available
  - Damping mainly provided by polymer matrix
  - Higher damping than most common metals
  - Composite damping is anisotropic
- Composite damping is tailorable
  - fiber orientation
  - stacking sequence
  - structural parameters (thickness, angle of curvature, etc)
- Additional damping capability due to interlaminar shear
  - Adhesive joints
  - Thick Sandwich Laminates
  - Shear damping layer treatment

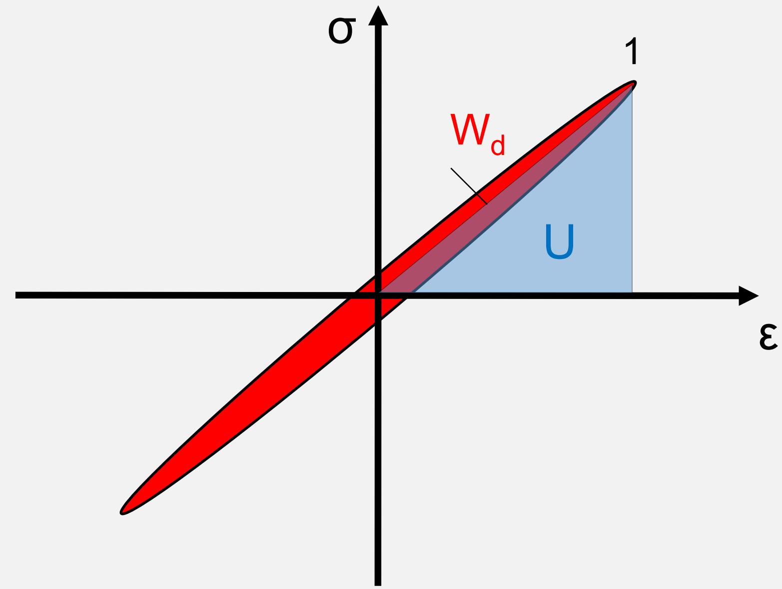
Energy Definition of Damping

Specific Damping Capacity:

$$\psi = \frac{dissipated\ energy\ in\ a\ loading\ cycle}{\max strain\, energy} = \frac{W_d}{U}$$

Loss Coefficient:

$$\eta = \frac{dissipated\ energy\ per\ radian}{\max strain\, energy} = \frac{W_d}{2\pi U}$$



$$\psi = 2\pi\eta$$

Viscoelastic Damping in time domain

- The dynamic stress that resists vibration and finally damps it, depends on the rate of strain.

- Kelvin-Solid Model: Dynamic stress depends linearly on the rate of strain

$$\sigma = E\varepsilon + E_d\dot{\varepsilon}$$

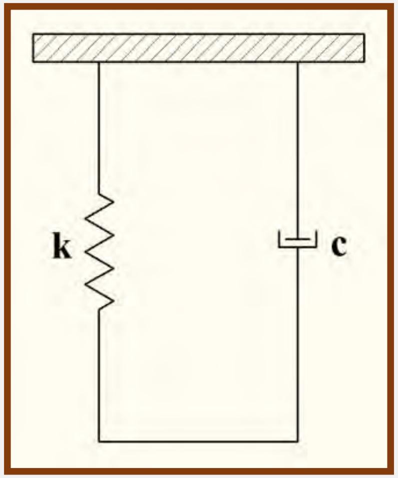
- System equation of motion (free vibration)

$$\boxed{m\ddot{x} + c\dot{x} + k = 0}$$

Damping ratio

$$\boxed{\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n}}$$

Kelvin-solid Model



Viscous damping increases linearly with frequency

Viscoelastic Damping in frequency domain

The complex modulus of elasticity gives the elastic behavior of a linear viscoelastic material in the frequency domain:

$$E(\omega) = E^{'}(\omega) + jE^{''}(\omega)$$

$$\tan\delta(\omega) = \eta(\omega) = \frac{E^{''}(\omega)}{E^{'}(\omega)}$$

A means for calculating damping

The dissipated energy in a viscoelastic material in a vibrating cycle is:

$$W_d = \int_0^{2\pi} E^{''}\varepsilon d\varepsilon$$

The maximum strain energy stored in the material in a cycle is:

$$U = \frac{1}{2}E^{'}\varepsilon_{\max}^2$$

$$\psi = 2\pi\tan\delta \Rightarrow \tan\delta = \eta$$

Relations Among Damping Values

Assuming harmonic vibration, the dissipated energy in a vibrating cycle is:

$$W_d = 4\int_0^{\pi/2} c\dot{x}dx = \pi c \omega x^2$$

$$\psi = \frac{\pi c \omega x^2}{\frac{1}{2}kx^2} = 2\pi \frac{2\zeta\omega}{\omega_n} = 4\pi\zeta \frac{\omega}{\omega_n}$$

Not constant relationship, depends linearly on frequency.

In a system that vibrates exclusively at the eigenfrequency:

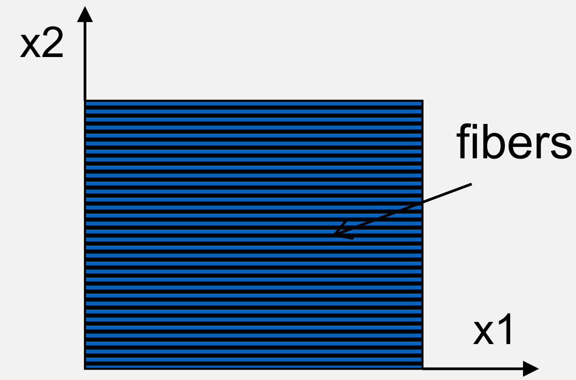
$$\psi_n = 2\pi\eta_n = 4\pi\zeta_n$$

Damping in the Material System

Assumptions:

i. A typical continuous fiber ply configuration

ii.Each ply has viscoelastic behavior



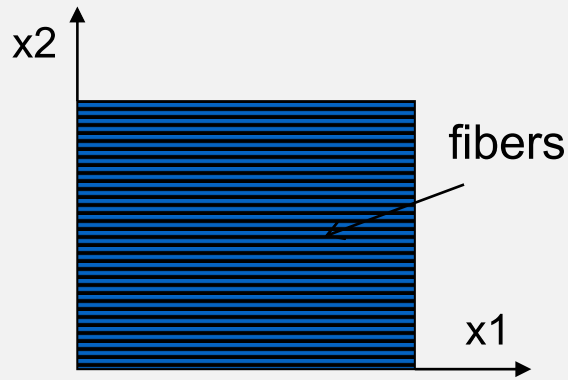
Composite Damping is described by 4 independent damping coefficients:

- $n_{l1}=n_{l11}$: Longitudinal Damping Coefficient $\longrightarrow$ $\psi_{l1} = \dfrac{W_{d_{l11}}}{1/2\sigma_{l11}\varepsilon_{l11}}$
- $n_{l2}=n_{l22}$: Transverse in-plane Damping Coefficient
- $n_{l6}=n_{l12}$: Shear in-plane Damping Coefficient
- $n_{l4}=n_{l23}$: Shear out of-plane Damping Coefficient

Composite Ply Damping Described by Damping Matrix $[n_l]$

- On-axis ply damping matrix

$$[n_l] = \begin{bmatrix} n_{l1} & 0 & 0 & 0 & 0 & 0 \\ 0 & n_{l2} & 0 & 0 & 0 & 0 \\ 0 & 0 & n_{l3} & 0 & 0 & 0 \\ 0 & 0 & 0 & n_{l4} & 0 & 0 \\ 0 & 0 & 0 & 0 & n_{l5} & 0 \\ 0 & 0 & 0 & 0 & 0 & n_{l6} \end{bmatrix}$$



Elastic Properties	Damping Coefficients (%)
E_{11} 23.27 GPa	ζ_{l1} 0.321
E_{12} 7.899 GPa	ζ_{l2} 1.17
G_{12} 3.185 GPa	ζ_{l6} 1.44
ν_{12} 0.344	

- Governing Material Equations

$$\boldsymbol{\sigma}_{i} = ([\mathbf{Q}_{ij}] + j[\mathbf{Q}_{im}][\boldsymbol{\eta}_{mj}])\boldsymbol{\varepsilon}_{j}$$

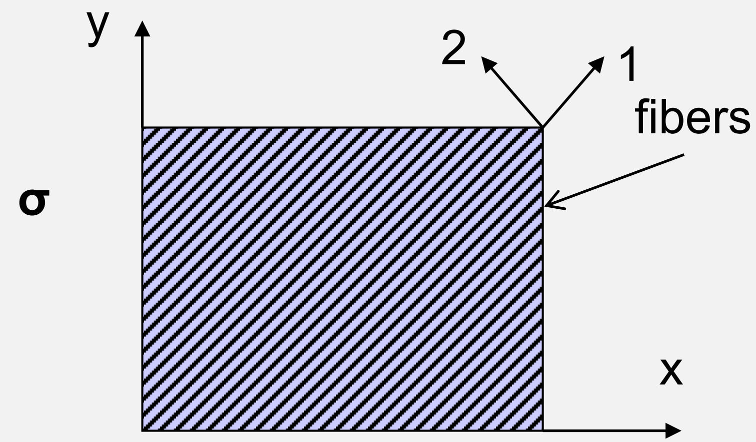
$$W_d = \frac{1}{2} 2\pi \boldsymbol{\varepsilon}^{T}[\boldsymbol{\eta}]\boldsymbol{\sigma} = \frac{1}{2} 2\pi \boldsymbol{\varepsilon}^{T}[\boldsymbol{\eta}][\mathbf{Q}]\boldsymbol{\varepsilon}$$

Off-Axis Damping

Material coordinate system O_{123} is rotated by angle θ regarding the system $O_{\xi\eta\zeta}$.

Damping in the system $O_{\xi\eta\zeta}$ is described by the matrix $[\eta_c]$, derived from the following transformation:

$$[\eta_C] = [R]^T [\eta_l][R]^{-T}$$



2nd order tensor rotation matrix:

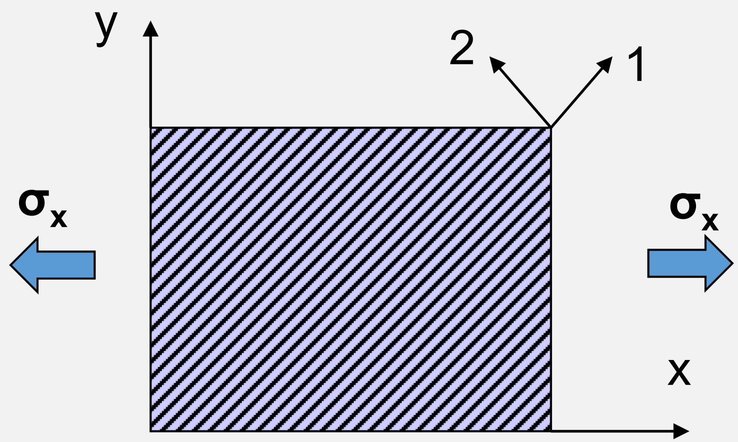
$$[R] = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & n & -n & 0 \\ 0 & 0 & 0 & -m & m & 0 \\ -mn & mn & 0 & 0 & 0 & m^2 - n^2 \end{bmatrix}, m = \cos\theta, n = \sin\theta$$

Off-axis ply damping matrix

$$[n_C] = \begin{bmatrix} n_{C11} & n_{C12} & 0 & 0 & 0 & n_{C16} \\ n_{C21} & n_{C22} & 0 & 0 & 0 & n_{C26} \\ 0 & 0 & n_{C33} & 0 & 0 & 0 \\ 0 & 0 & 0 & n_{C44} & n_{C45} & 0 \\ 0 & 0 & 0 & n_{C54} & n_{C55} & 0 \\ n_{C61} & n_{C62} & 0 & 0 & 0 & n_{C66} \end{bmatrix}$$

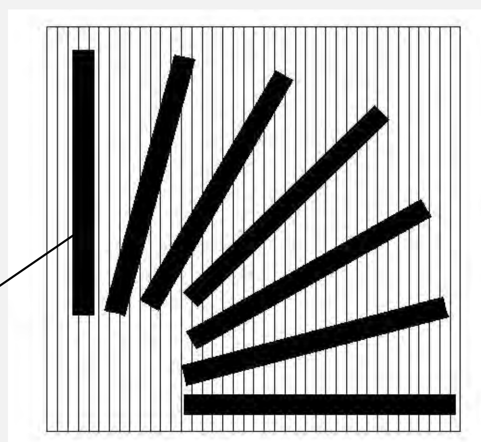
Terms representing the coupling of damping along the axes of the natural system $O_{\xi\eta\zeta}$

Rotated Composite Ply Damping

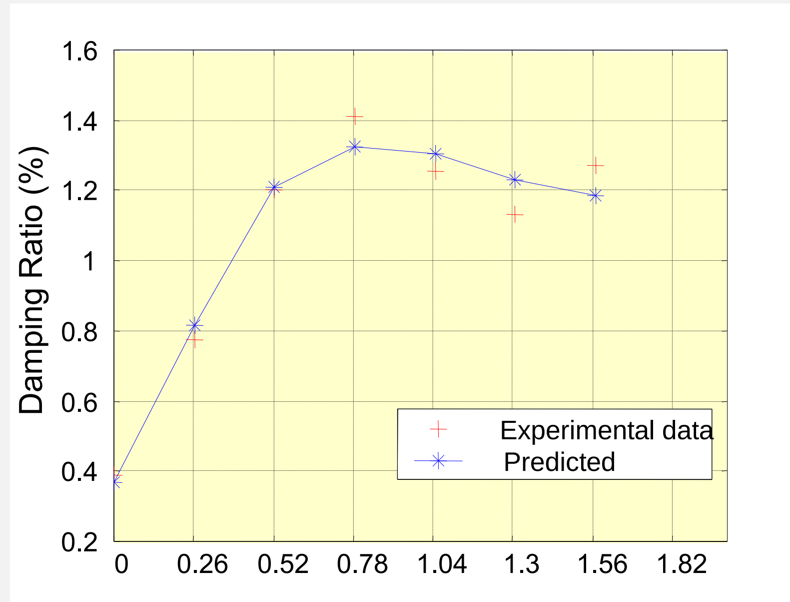


UD composite Plate

Single Ply Beams

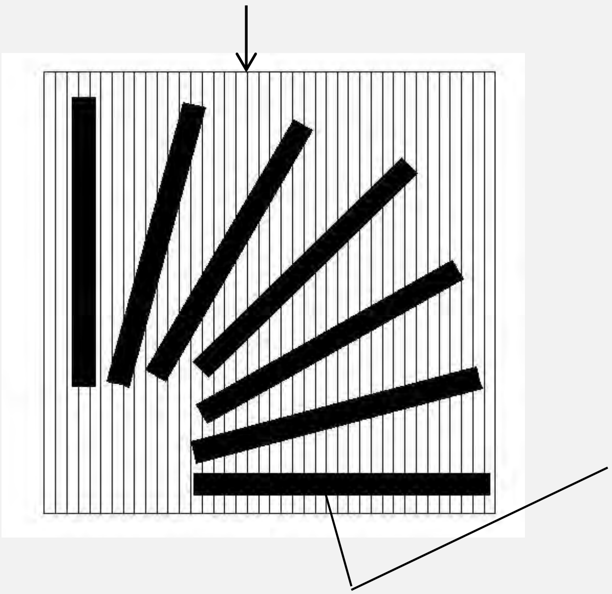


Axial Damping of an Angle Ply [θ] Composite

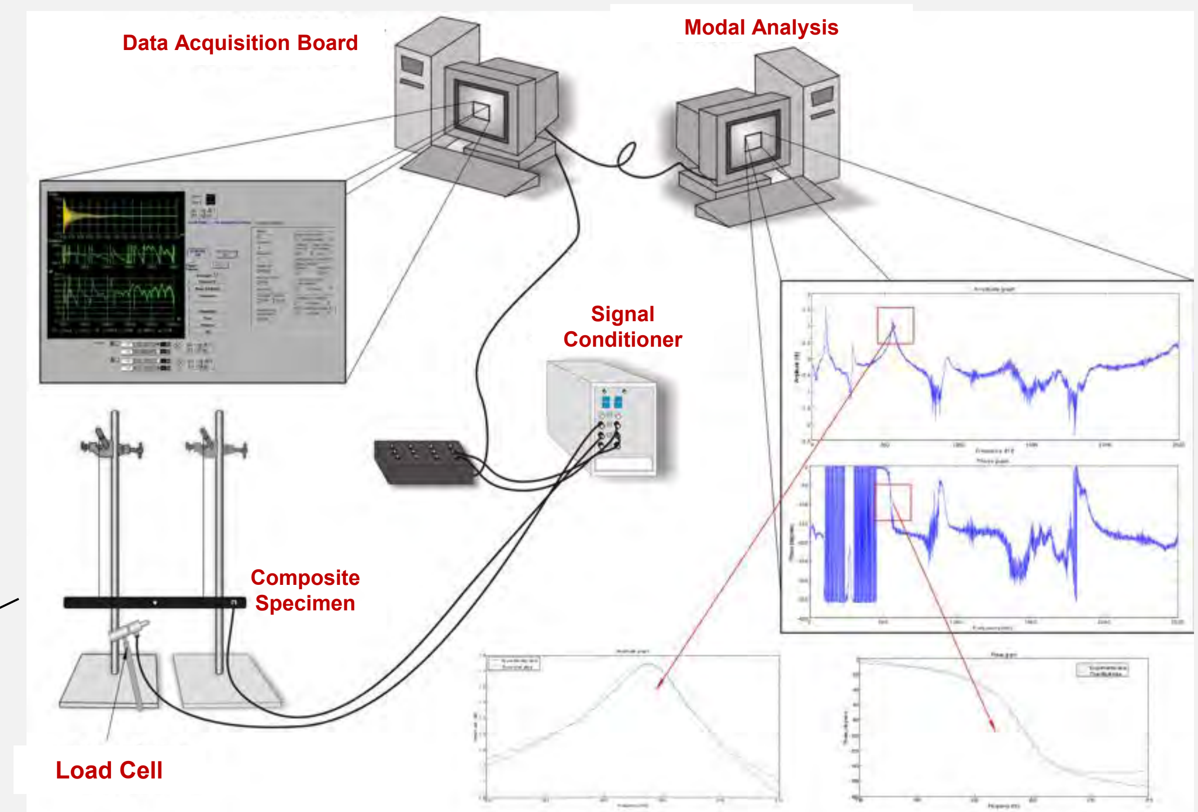


Fiber Orientation Angle θ (rads)

Measurement of Damping and Elastic Coefficients



Single Ply Beams tested in free bending



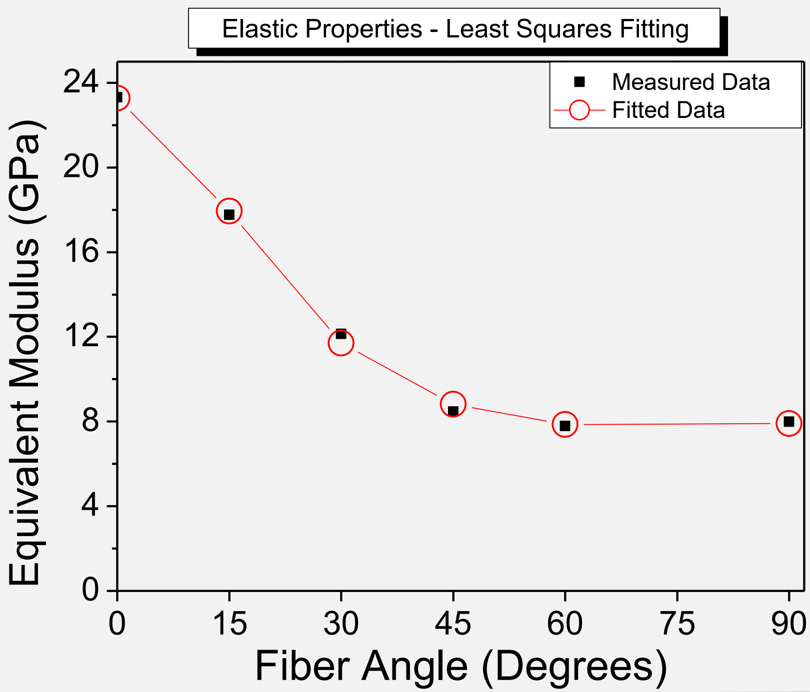
Extraction of modal damping & frequency values

Extraction of Elastic & Damping Coefficients

- **From Experiment:**
  - **Modal frequencies**
- **Calculation of Equivalent Modulus**
- **From ply mechanics**

$$E_C = 1/S_{c11} = f\left(E_{l11}, E_{l22}, G_{l12}, v_{l12}, \theta\right)$$

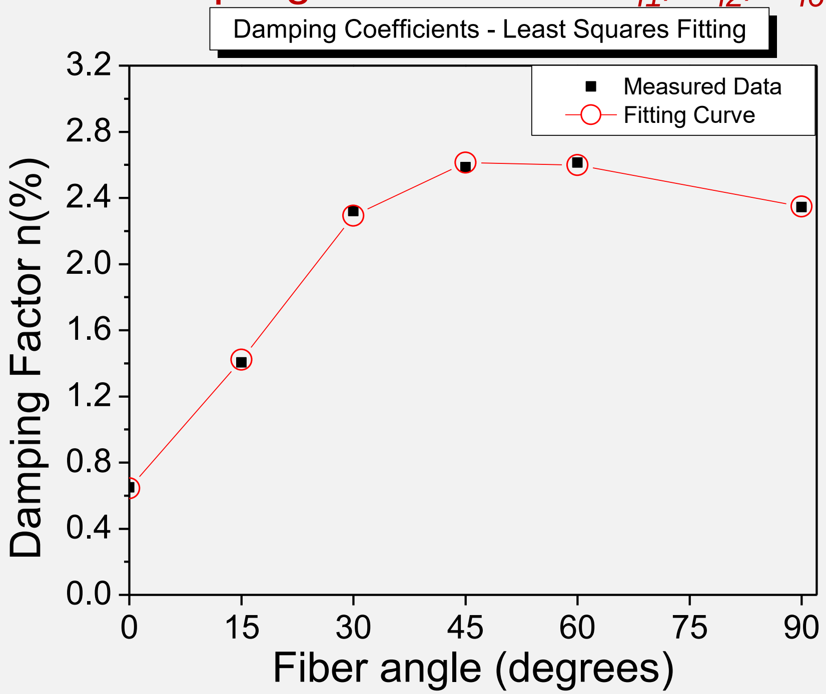
- **Least Squares Fit: *E_{11}, E_{12}, G_{12}, v_{12}***



- **From Experiment: Off-axis flexural damping ratio n_{cx}**
- **From Composite Mechanics:**

$$n_C(\theta) = f\left(\left[n_l\right], \left[Q_l\right], \theta\right)$$

- Least squares fit provides in-plane damping coefficients n_{l1}, n_{l2}, n_{l6}



Extraction of Elastic Properties

- Input from Experiment:
  - Modal frequencies

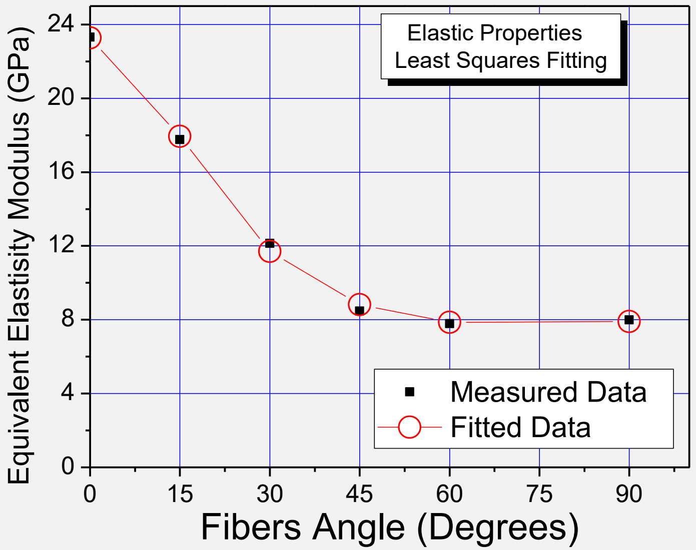
$$E_C = \frac{\bar{m} \cdot L^4}{I \cdot C_n^2} \cdot \left(2\pi\omega_n\right)$$

- Equivalent modulus E_C is calculated from each modal frequency (beam solution, Euler)

$$[S_c] = [R]^T [S_L][R]$$

$$E_C = 1/S_{c11} = f\left(E_{l11}, E_{l22}, G_{l12}, v_{l12}, \theta\right)$$

- From Composite Mechanics & Least Squares Fit
  - The material elastic properties E_{11}, E_{22}, G_{12} and $\mathbf{v}_{12}$ are extracted



Extraction of Damping Coefficients

Input:

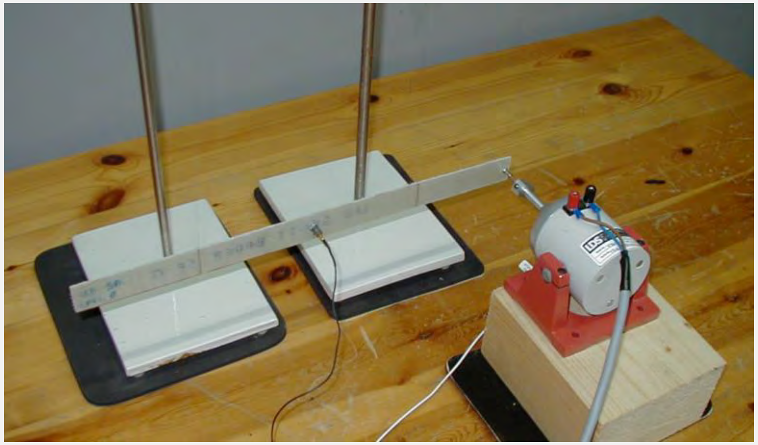
- Composite elastic properties E_{11}, E_{22}, G_{12} and $\mathbf{v}_{12}$
- Measured off-axis flexural damping ratio values n_c

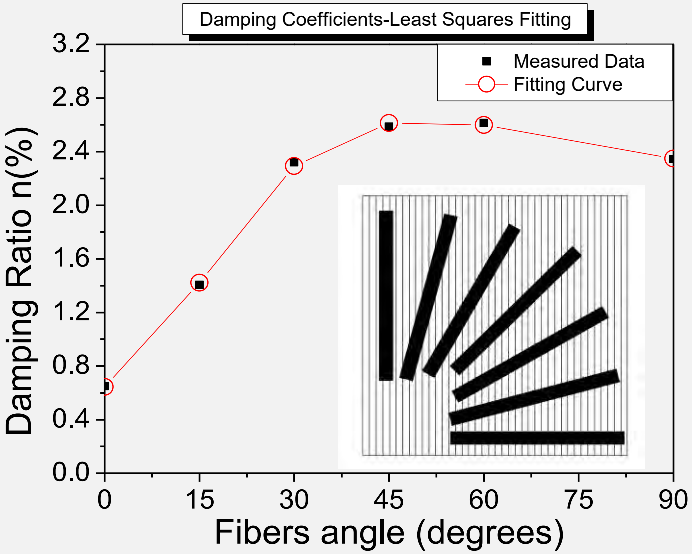
From Composite Mechanics:

- Analytical relationship of off-axis flexural damping to damping coefficients

$$n_C(\theta) = f\left(\left[n_l\right],\left[Q_l\right],\theta\right)$$

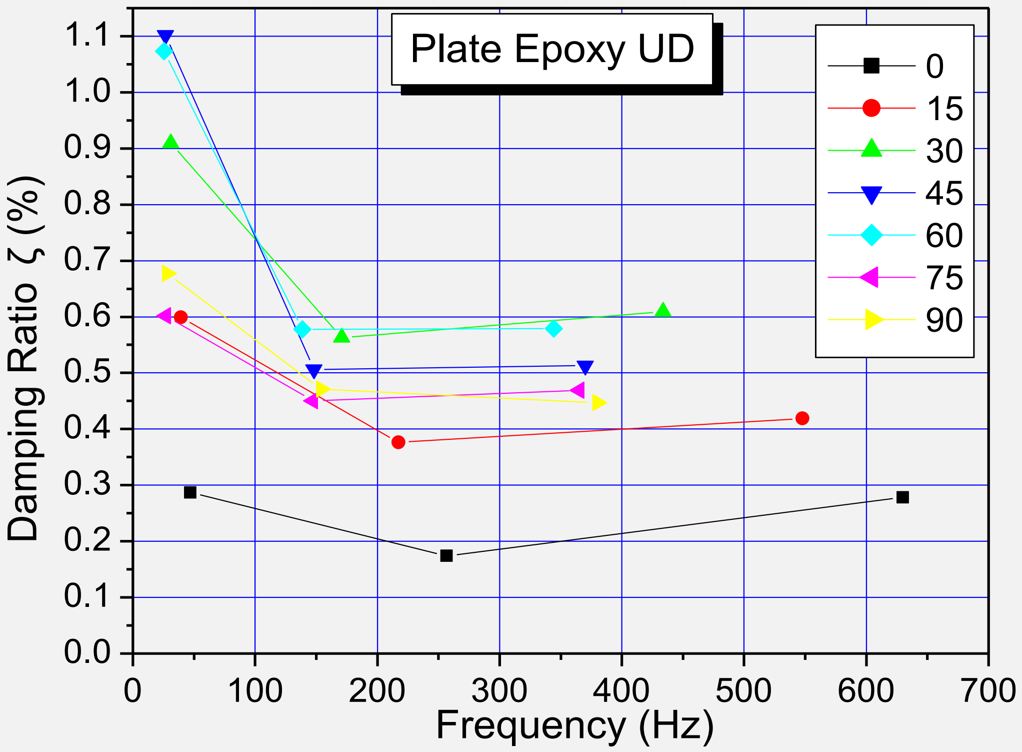
- Least squares fit provides in-plane damping coefficients n_{l1}, n_{l2}, n_{l6} of the composite

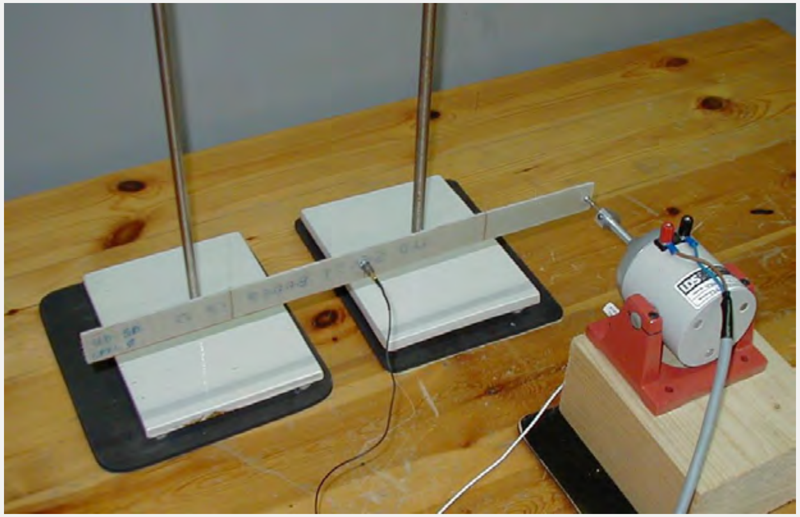




Experimental Results

Glass/Epoxy(Ampreg20) Composite





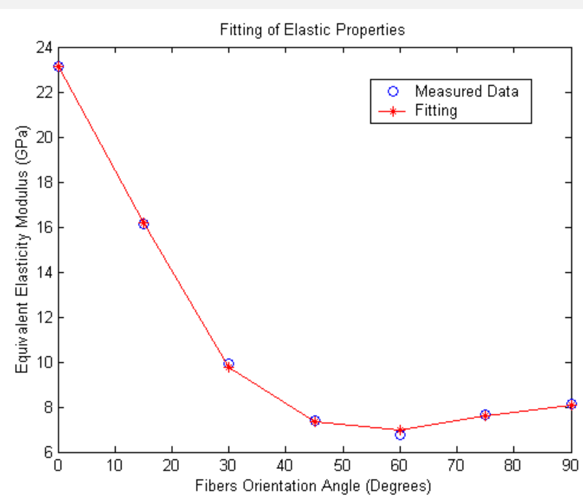
- Damping was found to be sensitive to the modal frequency
- Relationship of damping to frequency is also characterized

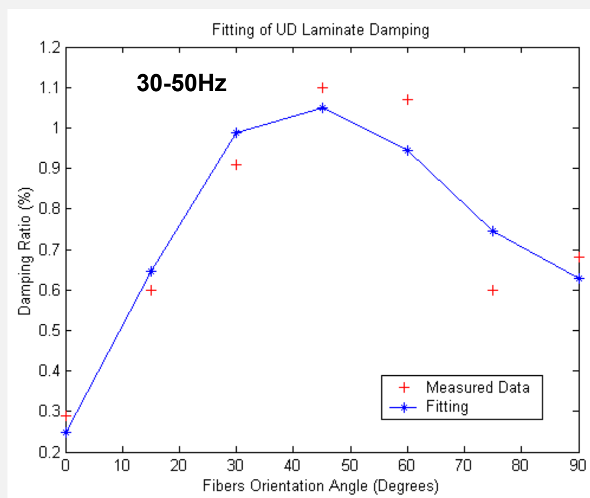
Measured Elastic and Damping Coefficients

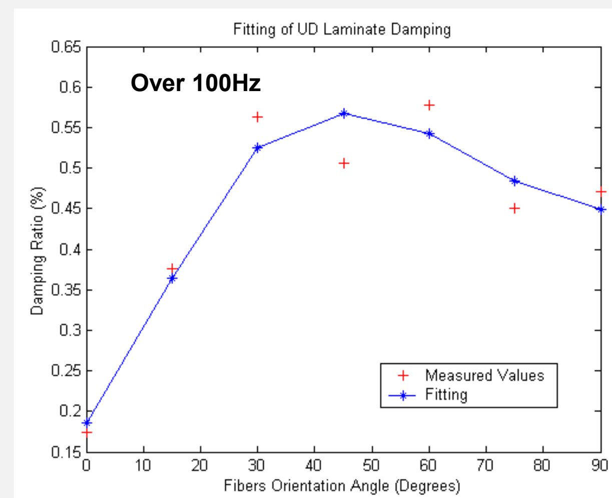
- Elastic & Damping coefficients extracted vs. frequency

	Elastic Properties
E_{11}	23.1 GPa
E_{12}	8.1 GPa
G_{12}	2.5 GPa
ν_{12}	0.333

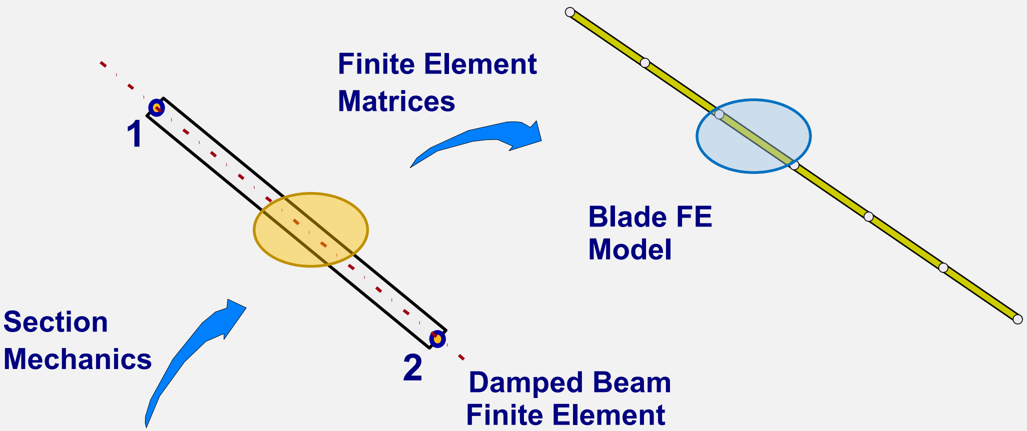
	Damping Coefficients	
	Modal Frequencies<100Hz	*Modal Frequencies>100Hz*
Ψ_{11}	0.031	0.023
Ψ_{12}	0.079	0.056
Ψ_{16}	0.153	0.079







Composite Blade Analysis Cycle

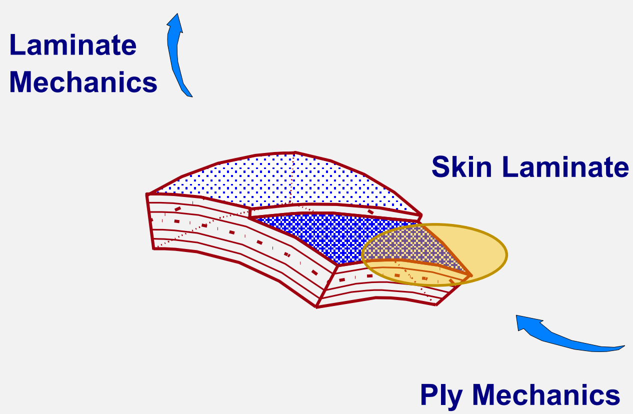


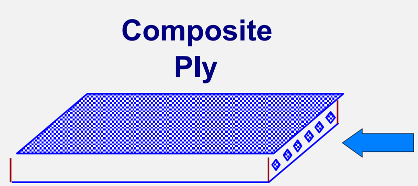
Small size

Discretized SD Model

$$[M]\ddot{u}(t)+[C]\dot{u}(t)+[K]u(t)=F(t)$$

Integrated Multi-Scale Mechanics & Finite Element





Composite Ply Scale

Minimal Material Input:

- 5 Elastic coefficients
- **4 Damping coefficients**

Dissipated Skin Laminate Strain Energy

Dissipated strain energy per unit area during a vibration cycle:

$$W_{d_L} = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}} \mathbf{S}_i^{\,T}[\mathbf{Q}_{d_{c_{ij}}}]\mathbf{S}_j dz \qquad \text{where: } [\mathbf{Q}_{d_c}] = [\mathbf{Q}_c][\boldsymbol{\eta}_c]$$

kinematic assumption + integrating through the thickness:

$$\delta W_{d_L} = \delta S_i^0 A_{d_{ij}} S_j^0 + \delta S_i^0 B_{d_{ij}} k_j + \delta k_j B_{d_{ji}} S_i^0 + \delta k_i D_{d_{ij}} k_j$$

$$\left\langle A_{d_{ij}}, B_{d_{ij}}, D_{d_{ij}} \right\rangle = \sum_{m=1}^{n} \int_{\zeta_m}^{\zeta_{m+1}} Q_{ij}^m [\eta_{j\lambda}] \left\langle 1, \zeta, \zeta^2 \right\rangle d\zeta \quad i, j, \lambda = 1, 2, 6$$

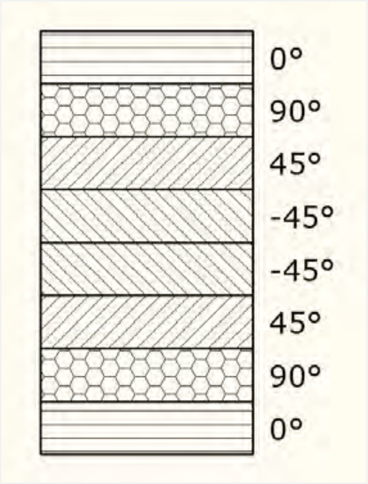
$$A_{d_{ij}} = \sum_{m=1}^{n} \int_{\zeta_m}^{\zeta_{m+1}} Q_{ij}^m [\eta_{j\lambda}] d\zeta \quad i, j, \lambda = 4, 5$$

- A_d extension damping matrix,
- B_d extension-bending (coupling) matrix
- D_d bending damping matrix

Laminate Level

Modeling Modal Damping of Laminated Gr/Ep Beam

✓Lamination: [θ/90+θ/45+θ/-45+θ]$_s$

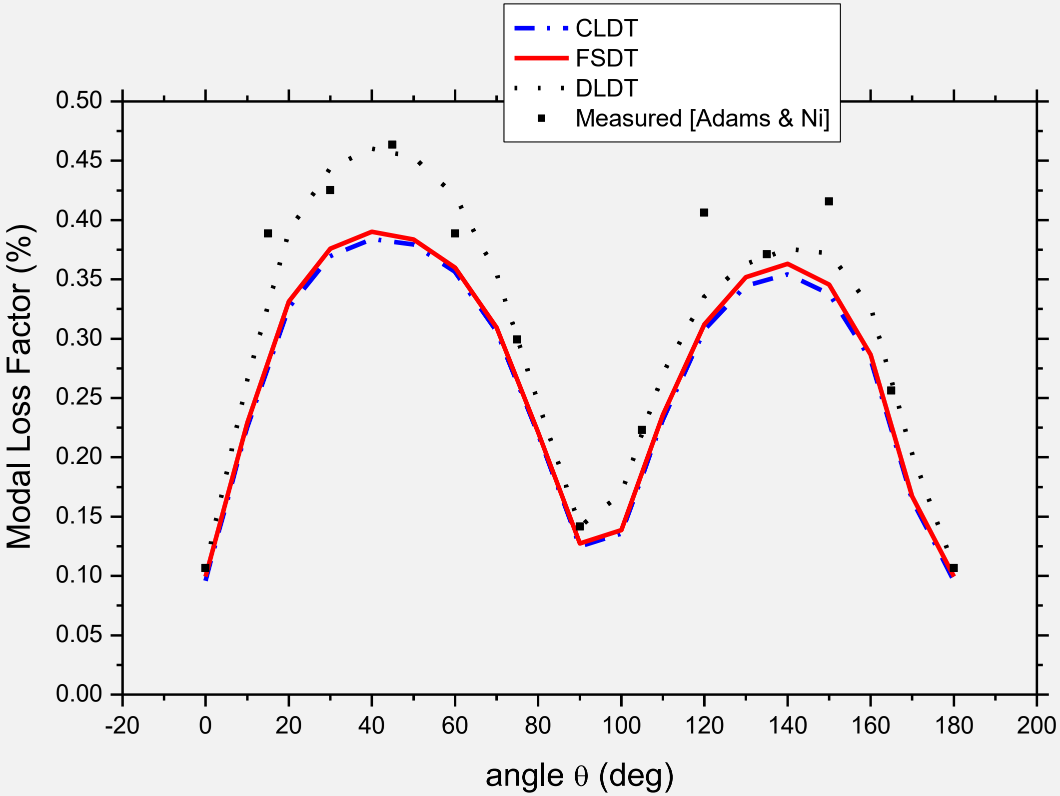


✓Support: Free-Free

✓Length: 200mm

✓Width: 12mm

✓Thickness: 1.6mm



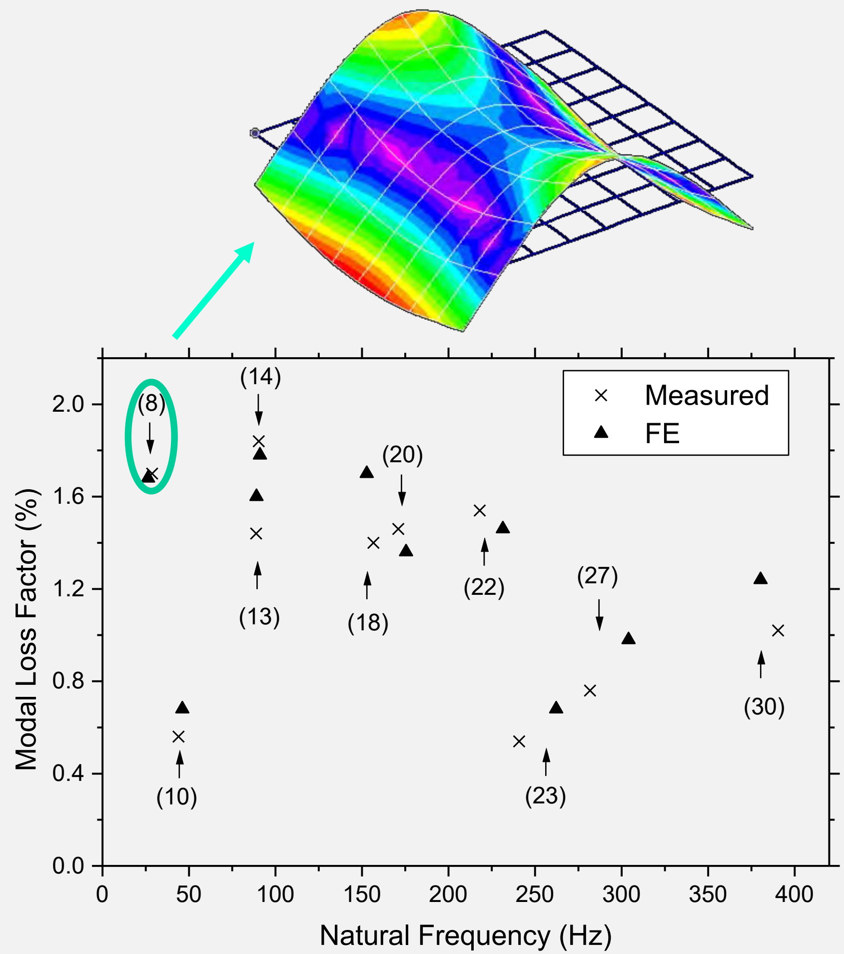
- CLDT: **C**lassical **L**aminate **D**amping **T**heory
- FSDT: **F**irst order **S**hear **D**eformation **T**heory
- DLDT: **D**iscrete **L**ayer **D**amping **T**heory

Modeling Measured Frequencies & Modal Loss Factors of GI/PI Plate

- ✓Support: Free-Free
- ✓Lamination: $[0]_3$
- ✓Length: 455mm
- ✓Width: 455mm
- ✓Thickness: 2.3mm

Damping was measured with modal analysis tests

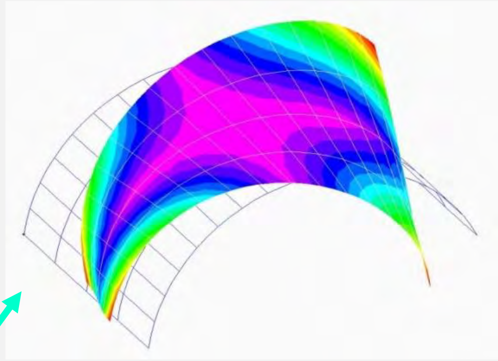
mode shape of plate

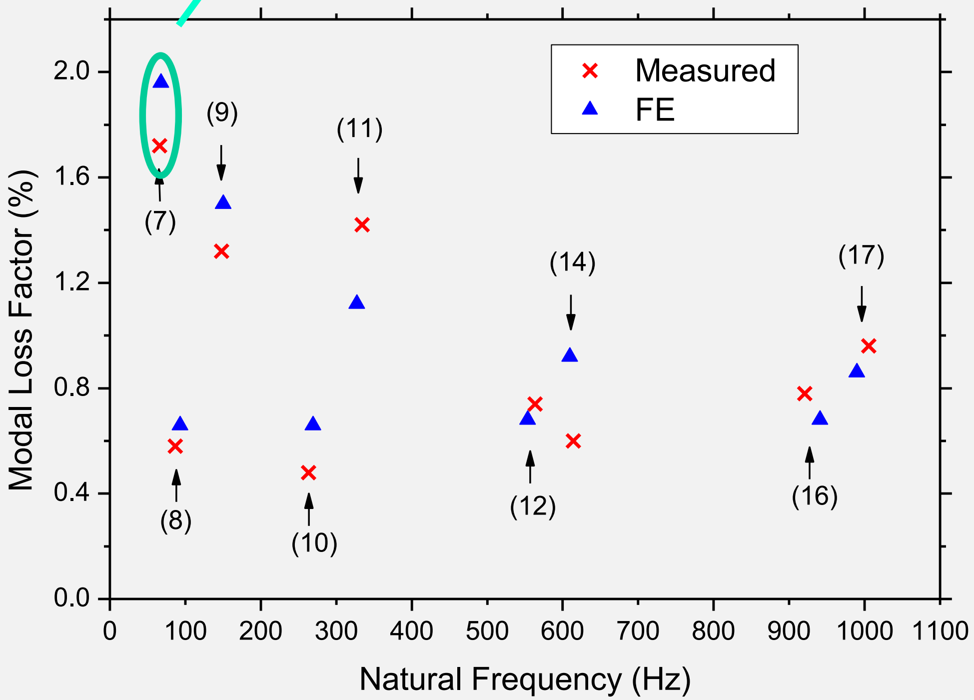


Modeling Frequencies & Modal Loss Factors of Gl/Pl Cylindrical Panel

- ✓Support: Free-Free
- ✓Lamination: $[0]_3$
- ✓Length: 355mm (circumferential direction)
- ✓Width: 204mm
- ✓Thickness: 3.3mm

mode shape of panel





Equations of Motion

$$\int_0^L \mathrm{d}x \int_A (-\delta H_s + \delta T_s - \delta W_d)\mathrm{d}s\mathrm{d}\zeta + \oint_\Gamma \delta\bar{\mathbf{u}}^{\mathbf{T}}\bar{\boldsymbol{\tau}}\mathrm{d}\Gamma = 0$$

- ***H***, is the Strain Energy → $\delta H_s = \int_A \delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}\boldsymbol{\sigma}_\mathbf{c}\mathrm{d}s\mathrm{d}\zeta = \oint \mathrm{d}s \int_h \delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}[\mathbf{Q}_\mathbf{c}]\boldsymbol{\varepsilon}_\mathbf{c}\mathrm{d}\zeta$
- ***T***, is the Kinetic Energy → $\delta T_s = \int_A -\delta\mathbf{u}^\mathbf{T}\mathrm{diag}(\boldsymbol{\rho})\ddot{\mathbf{u}}\mathrm{d}\xi\mathrm{d}\eta = \oint \mathrm{d}s \int_h -\delta\mathbf{u}^\mathbf{T}\mathrm{diag}(\boldsymbol{\rho})\ddot{\mathbf{u}}d\zeta$
- ***W***
- $\boldsymbol{W_d}$, is the Dissipated Energy → $\delta W_{ds} = \int_A \delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}\mathbf{Q}_\mathbf{c}\boldsymbol{\eta}_\mathbf{c}\boldsymbol{\varepsilon}_\mathbf{c}\mathrm{d}s\mathrm{d}\zeta = \oint \mathrm{d}s \int_h \delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}\mathbf{Q}_\mathbf{c}\boldsymbol{\eta}_\mathbf{c}\boldsymbol{\varepsilon}_\mathbf{c}\mathrm{d}\zeta$
- $\bar{\boldsymbol{\tau}}$ the surface tractions on the free surface Γ
- ***L***, is the length of the beam

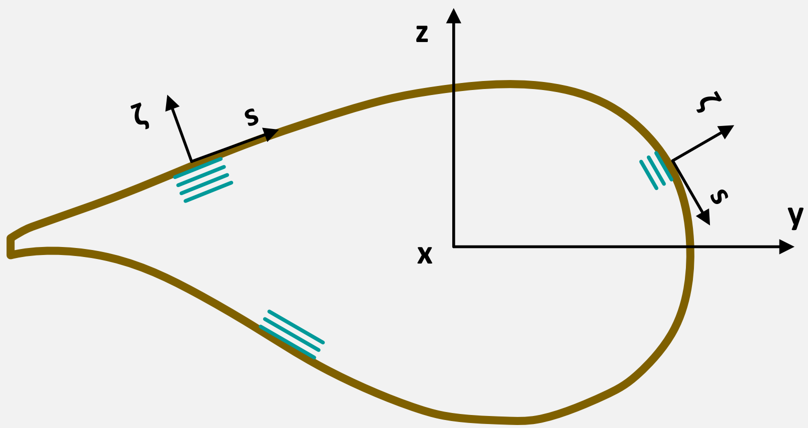
$\boldsymbol{\varepsilon_c}$, $\boldsymbol{\sigma_c}$: Off-axis strains and stresses of a composite ply

$\boldsymbol{Q_c}$, $\boldsymbol{\eta_c}$: Equivalent off-axis stiffness and damping (loss-factor) matrices of the composite ply

$\boldsymbol{\rho}$: Density of each ply

$\boldsymbol{h}$: Thickness of the skin laminate

Section Example



Equivalent Blade Section Stiffness

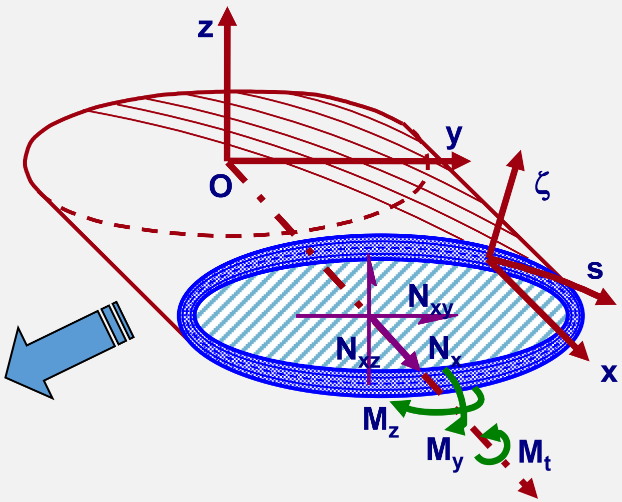
Extension – Shear Coupling

Coupling Stiffness

$$\begin{Bmatrix} N_x^{o} \\ N_{xy}^{o} \\ N_{xz}^{o} \\ M_{xy} \\ M_{xz} \\ M_{\theta} \end{Bmatrix} = \begin{bmatrix} A_{11}^{0} & \bar{A}_{15}^{0} & \bar{A}_{16}^{0} & B_{11}^{0} & B_{12}^{0} & \bar{B}_{16}^{0} \\ \bar{A}_{51}^{0} & A_{55}^{0} & A_{56}^{0} & \bar{B}_{51}^{0} & \bar{B}_{52}^{0} & B_{56}^{0} \\ \bar{A}_{61}^{0} & A_{65}^{0} & A_{66}^{0} & \bar{B}_{61}^{0} & \bar{B}_{62}^{0} & B_{66}^{0} \\ B_{11}^{0} & B_{51}^{0} & \bar{B}_{61}^{0} & D_{11}^{0} & D_{12}^{0} & \bar{D}_{16}^{0} \\ \bar{B}_{12}^{0} & \bar{B}_{52}^{0} & B_{62}^{0} & D_{21}^{0} & D_{22}^{0} & \bar{D}_{26}^{0} \\ \bar{B}_{16}^{0} & \bar{B}_{56}^{0} & B_{66}^{0} & \bar{D}_{61}^{0} & \bar{D}_{62}^{0} & D_{66}^{0} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{o} \\ \varepsilon_{xy}^{o} \\ \varepsilon_{xz}^{o} \\ k_{xy} \\ k_{xz} \\ k_{\theta} \end{Bmatrix}$$

Bending – Torsion Coupling

Flexural & Torsional Stiffness

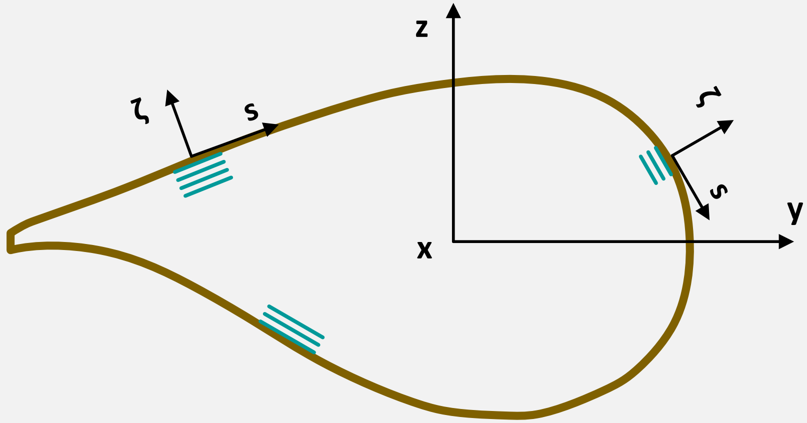


Overbar indicates terms which include material coupling effect (or skin laminate) at skin level due to rotated plies → $Q_{16} \neq 0$ or A_{16}, B_{16}, $D_{16} \neq 0$

Equivalent Blade Section Damping Terms

The section damping matrices, including coupling terms derived from the dissipated energy equation are:

Section Example



$$\delta W_{ds} = \int_A \delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}\mathbf{Q}_\mathbf{c}\boldsymbol{\eta}_\mathbf{c}\boldsymbol{\varepsilon}_\mathbf{c}\mathrm{d}s\mathrm{d}\zeta = \oint \mathrm{d}s \int_h \delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}\mathbf{Q}_\mathbf{c}\boldsymbol{\eta}_\mathbf{c}\boldsymbol{\varepsilon}_\mathbf{c}\mathrm{d}\zeta$$

$$\delta W_{ds} = \left\{\delta\varepsilon^{0^T} \quad \delta \mathrm{k}^T\right\} \begin{bmatrix} \left[\mathrm{A}_\mathrm{d}^0\right] & \left[\mathrm{B}_\mathrm{d}^0\right] \\ \left[\mathrm{B}_\mathrm{d}^0\right]^\mathrm{T} & \left[\mathrm{D}_\mathrm{d}^0\right] \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix}$$

$$\left[\mathbf{A}_\mathbf{d}^\mathbf{0}\right] = \begin{bmatrix} A_{d11}^0 & \bar{A}_{d15}^0 & \bar{A}_{d16}^0 \\ \bar{A}_{d51}^0 & A_{d55}^0 & A_{d56}^0 \\ \bar{A}_{d61}^0 & A_{d65}^0 & A_{d66}^0 \end{bmatrix}, \quad \left[\mathbf{B}_\mathbf{d}^\mathbf{0}\right] = \begin{bmatrix} B_{d11}^0 & B_{d12}^0 & \bar{B}_{d16}^0 \\ \bar{B}_{d51}^0 & \bar{B}_{d52}^0 & B_{d56}^0 \\ \bar{B}_{d61}^0 & \bar{B}_{d62}^0 & B_{d66}^0 \end{bmatrix}, \quad \left[\mathbf{D}_\mathbf{d}^\mathbf{0}\right] = \begin{bmatrix} D_{d11}^0 & D_{d12}^0 & \bar{D}_{d16}^0 \\ D_{d21}^0 & D_{d22}^0 & \bar{D}_{d26}^0 \\ \bar{D}_{d61}^0 & \bar{D}_{d62}^0 & D_{d66}^0 \end{bmatrix}$$

Overbar red terms depend directly on the extension-shear coupling damping terms of the skin laminate A_{d16}, B_{d16}, D_{d16}.

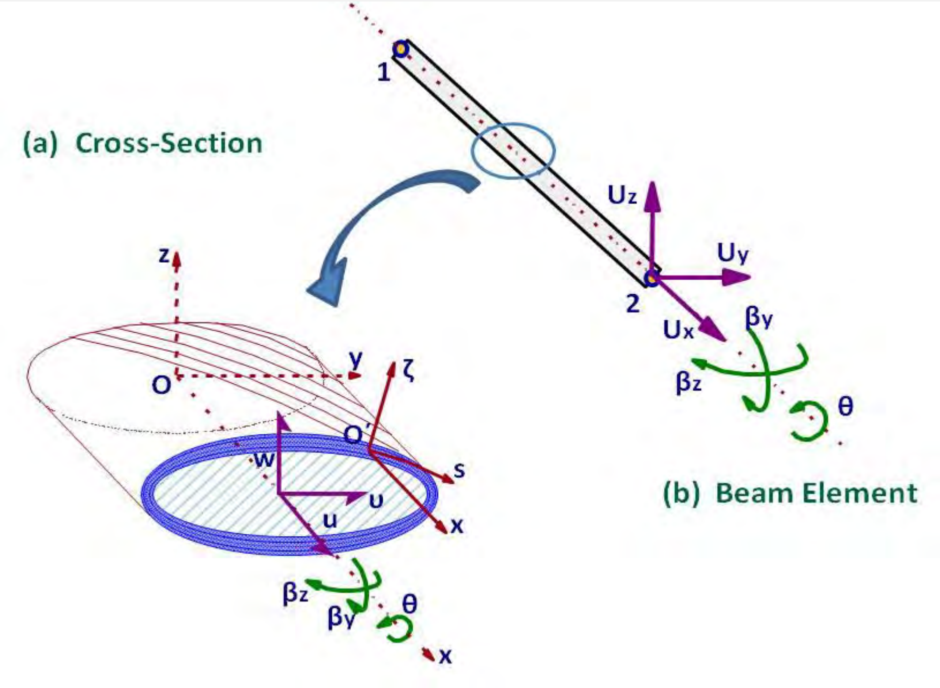
DAMPBEAM 3-D <u>Linear</u> Finite Element

- **6 DOFs per node:** $\mathbf{U}_{\mathbf{e}}^{\mathbf{i}} = \left\{ u^{oi}, \quad \upsilon^{oi}, \quad w^{oi}, \quad \beta_y^i, \quad \beta_z^i, \quad \theta^i \right\}$

- 2-node shear element with linear displacement interpolation functions

$$\left\langle u^o(x), \upsilon^o(x), w^o(x), \beta_y(x), \beta_z(x), \theta\ (x) \right\rangle \cong \sum_{i=1}^{n} N^i(x) \underset{\sim}{\mathbf{U}}_e^i$$

- Finite Element provides:
  - Stiffness matrix
  - Damping matrix
  - Mass matrix



Structural Equations of Motion

• Assuming harmonic vibration

• Combining equations of motion with equations of energy and relations between strains and displacements

• Gathering together the terms which correspond to common nodes between the elements

$$\boxed{-\omega^2[M]\mathbf{U} + j[C]\mathbf{U} + [K]\mathbf{U} = F(\omega)}$$

Equations of Motion

where:

[M] → mass matrix

[K] → stiffness matrix

[C] → damping matrix

[U] → displacement and rotation vector of the structure

[F] → force vector

Synthesized by the corresponding matrices of the elements

Structural Damping

1. By solving the equations of motion through the complex eigenvalues p_m of the system

$$\zeta_m = -\mathrm{Re}(p_m)/\| p_m \|$$ **Damping ratio of the m-th mode**

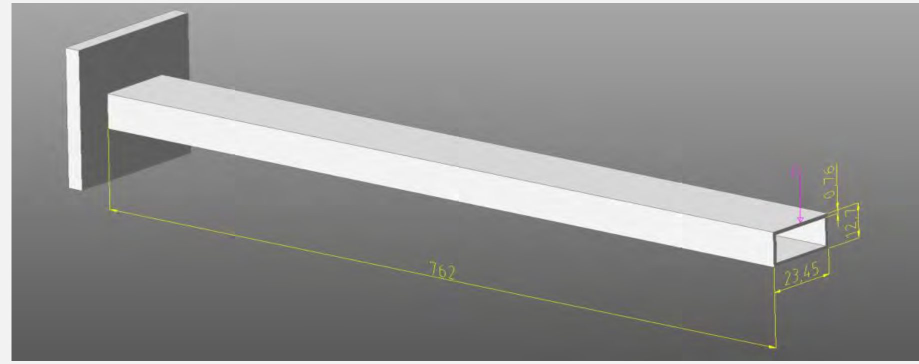
2. By using the dissipated modal strain energy method

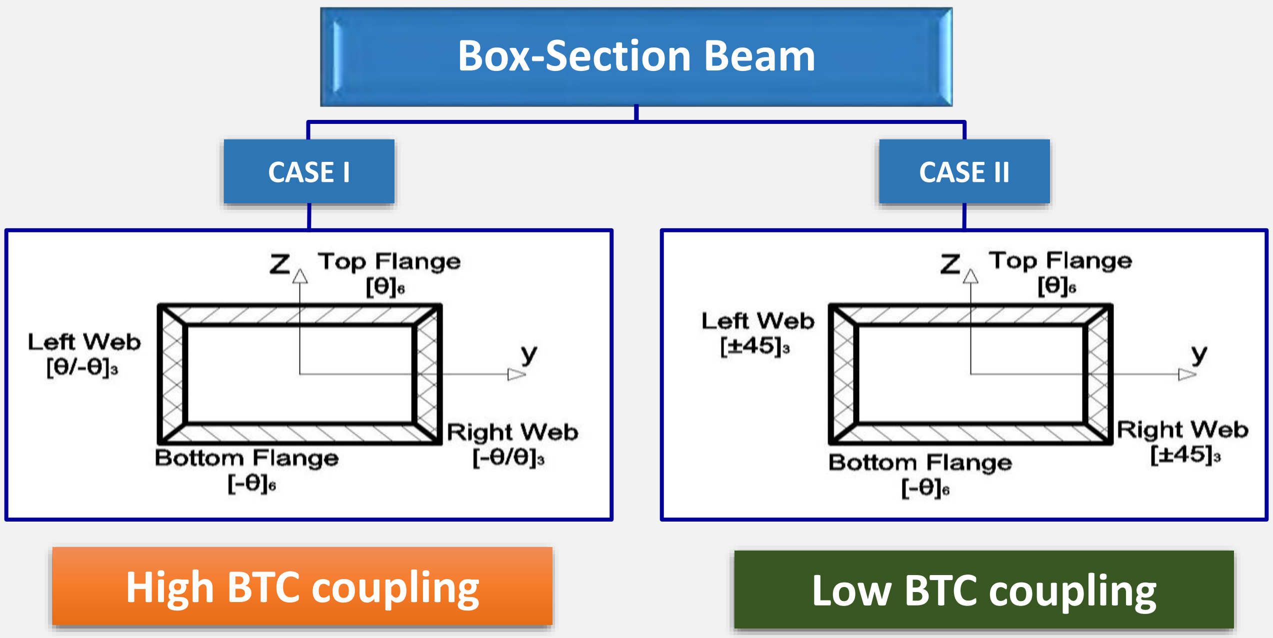
- Accurate in cases with small damping
- Assumes that the natural frequencies and eigenmodes of the system with damping do not differ from those of the undamped system

$$\eta_m = \frac{1}{2\pi} \frac{\int_A W_{d_{Lm}} dA}{\int_A W_{Lm} dA} = \frac{1}{2\pi} \frac{\mathbf{U}_m^T [C] \mathbf{U}_m}{\mathbf{U}_m^T [\mathrm{K}] \mathbf{U}_m}$$ **Loss coefficient of the m-th eigenmode**

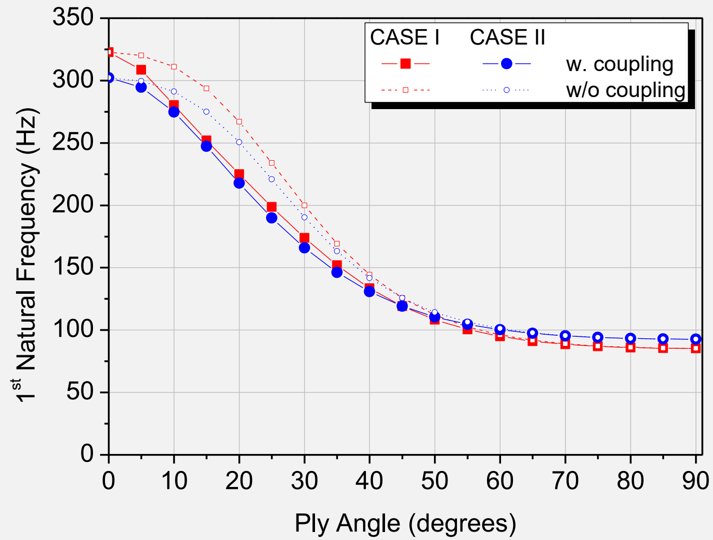
Validation Cases on Box-Section Beam

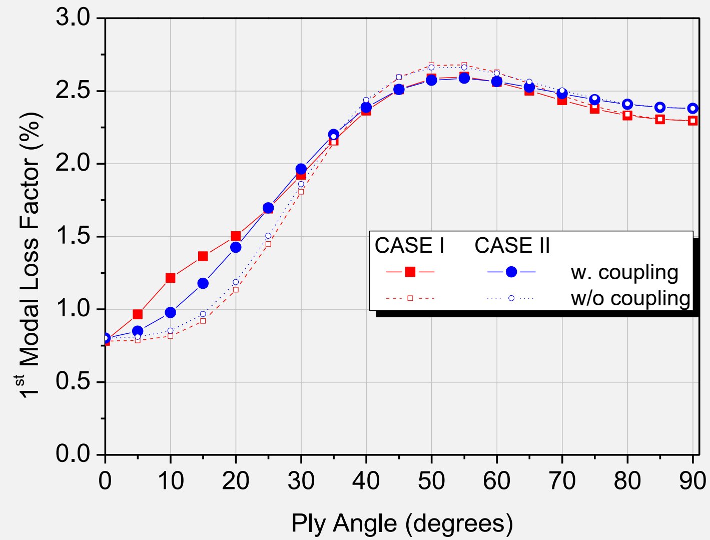
- Material: Carbon / Epoxy
- Vertical Force at tip, F=4.45N
- Dimensions and lay-up proposed by Volovoi & Hodges

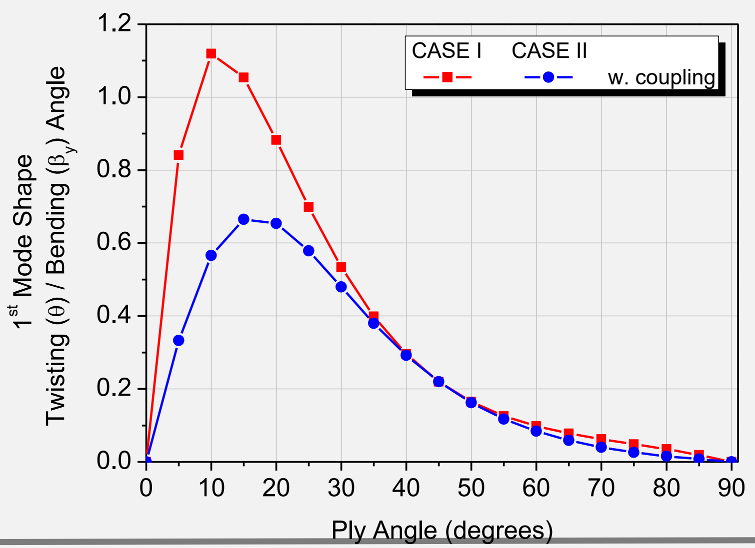




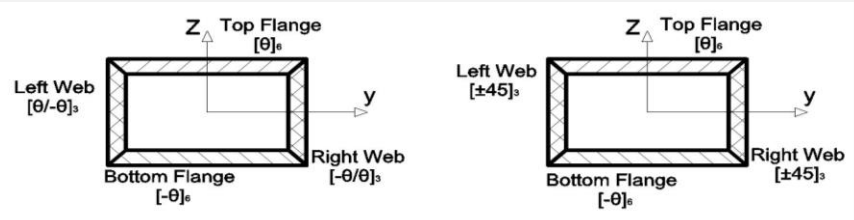
Material Coupling Effect on Dynamic Characteristics of Carbon/Epoxy Beams





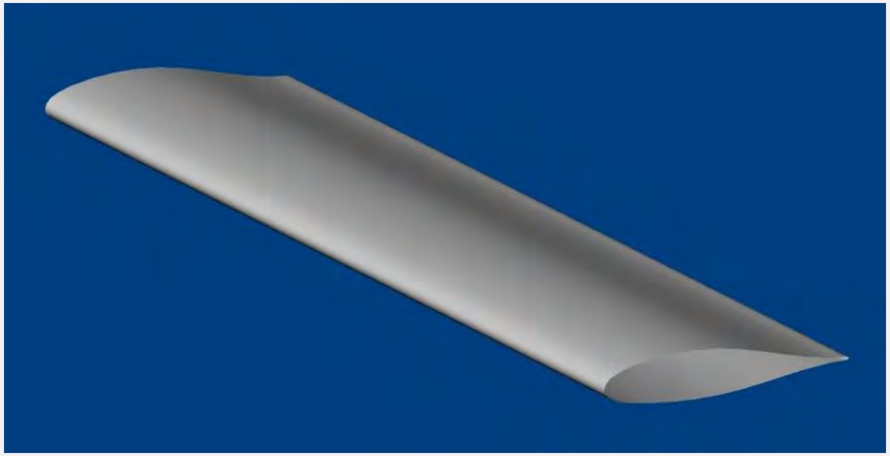


- High material coupling for $\theta = 5^0 - 30^0$ laminate orientations
- CASE I provides higher coupling values



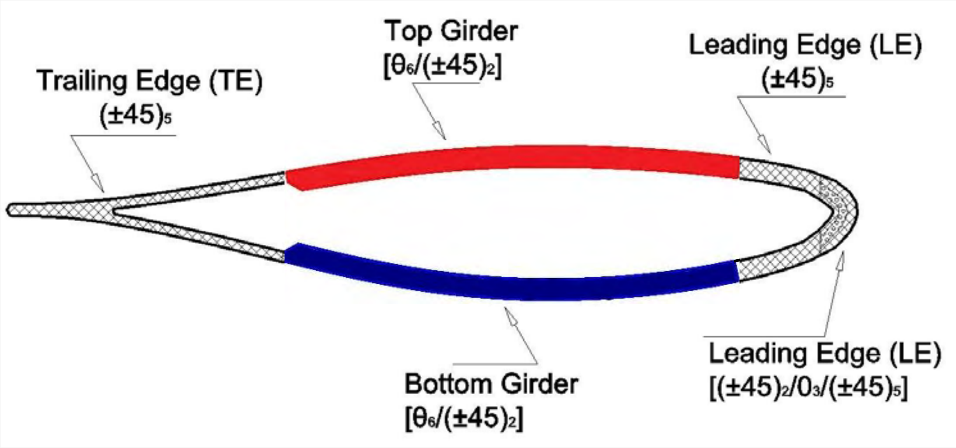
CASE I **CASE II**

Small Model Blade Configuration



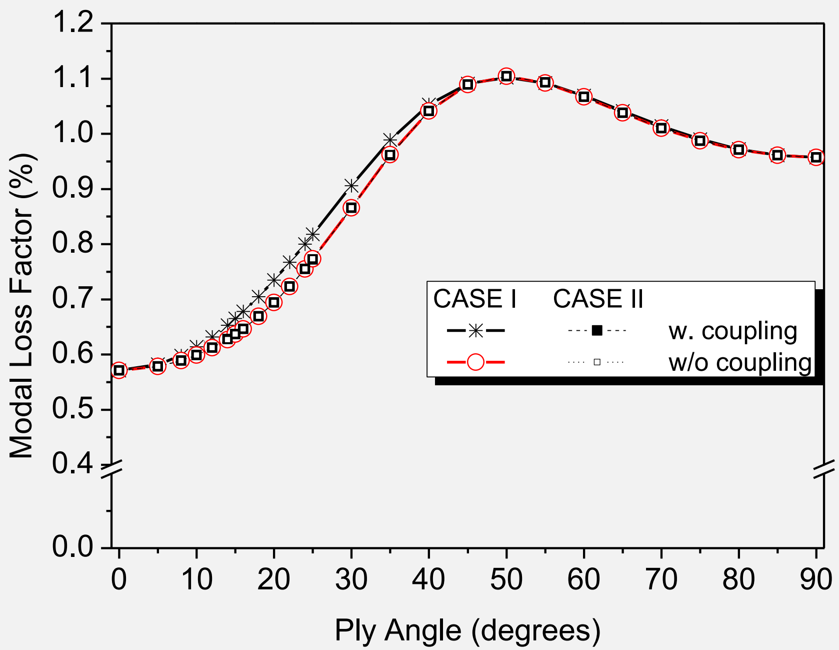
- Glass/Epoxy
- Various Ply Angle Laminations at Girder segments

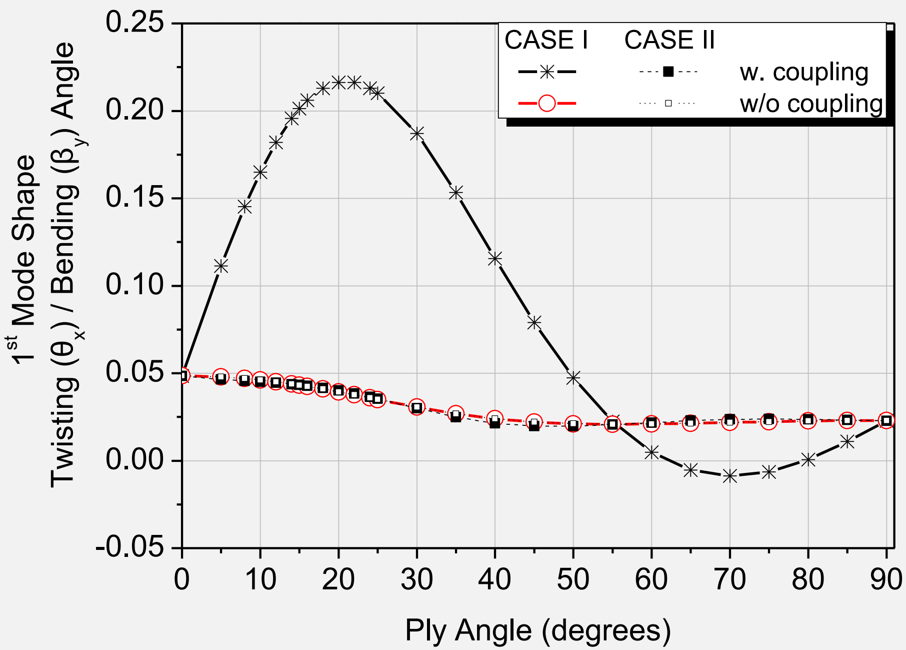
Length (mm)	Chord (mm)	Airfoil Thickness (mm)	Ply Thickness (mm)	Weight (Kg)
1540	283.5	50	0.80	6.0



	LAMINATION		
	Top Girder	**Bottom Girder**	**Lay-up**
CASE I	**$[\theta_6/(\pm45)_2]$**	**$[-\theta_6/(\pm45)_2]$**	**Anti-Symmetric**
CASE II	$[\theta_6/(\pm45)_2]$	$[\theta_6/(\pm45)_2]$	Symmetric

Material Coupling Effect on Small Model Blade

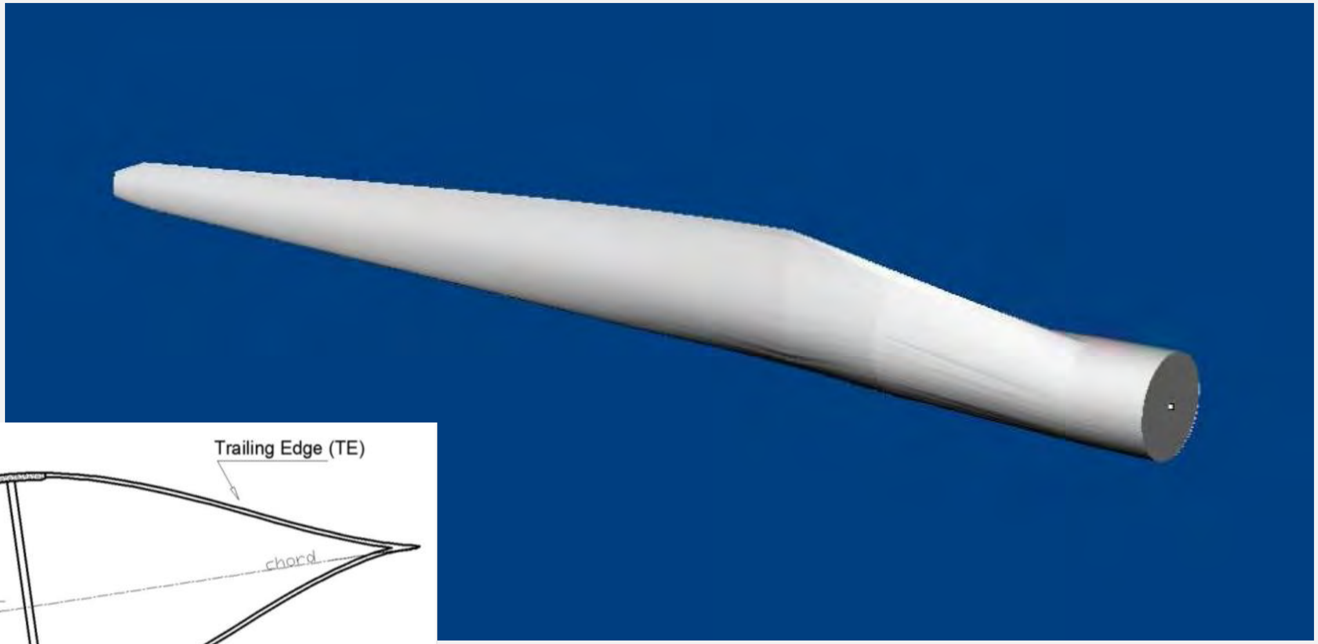


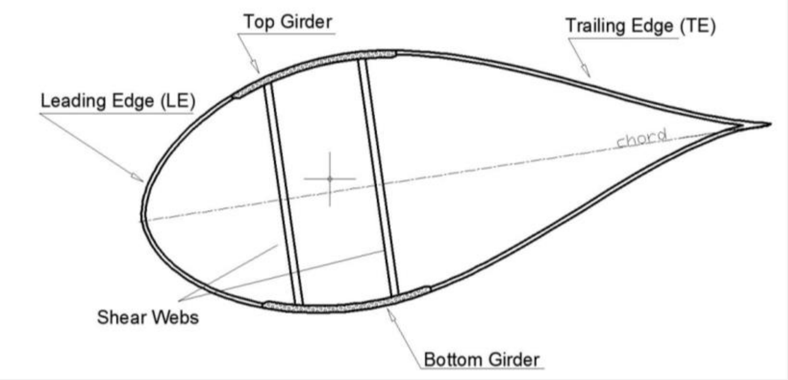


- For the 1st mode shape, ¨CASE I¨ presents a strong coupling terms, whereas CASE II provides almost negligible coupling.
- The difference is eliminated at $\theta=0^0$ and 90^0 where the material coupling effect is physically vanished.

19m Realistic Wind-Turbine Blade Design

- Glass/Polyester composite sandwich laminations at cross-section Leading (LE) and Trailing Edge (TE)
- Two Shear Webs along its length
- Two Girder segments reinforced with multiple UD layers





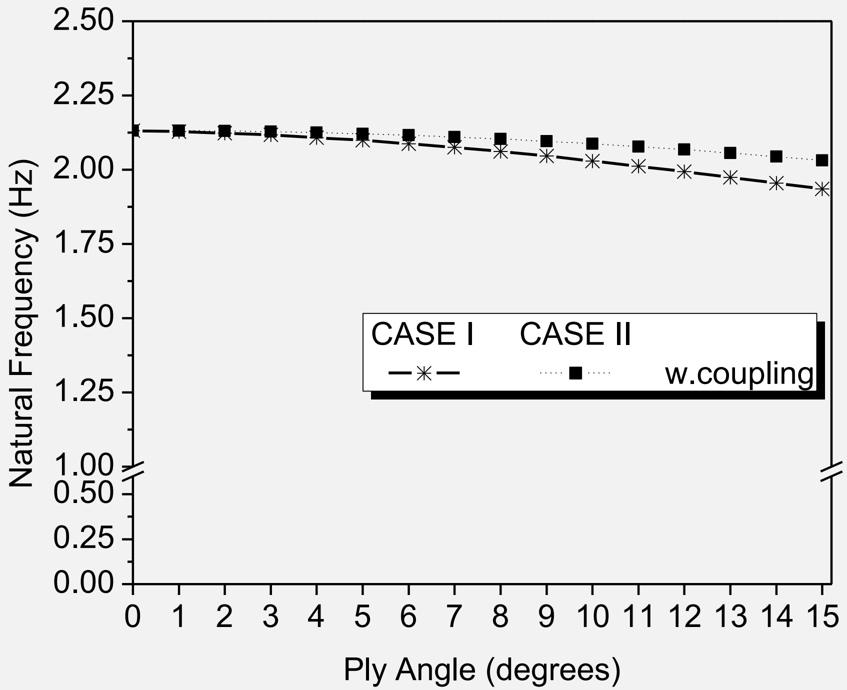
19m Glass/Polyester Blade

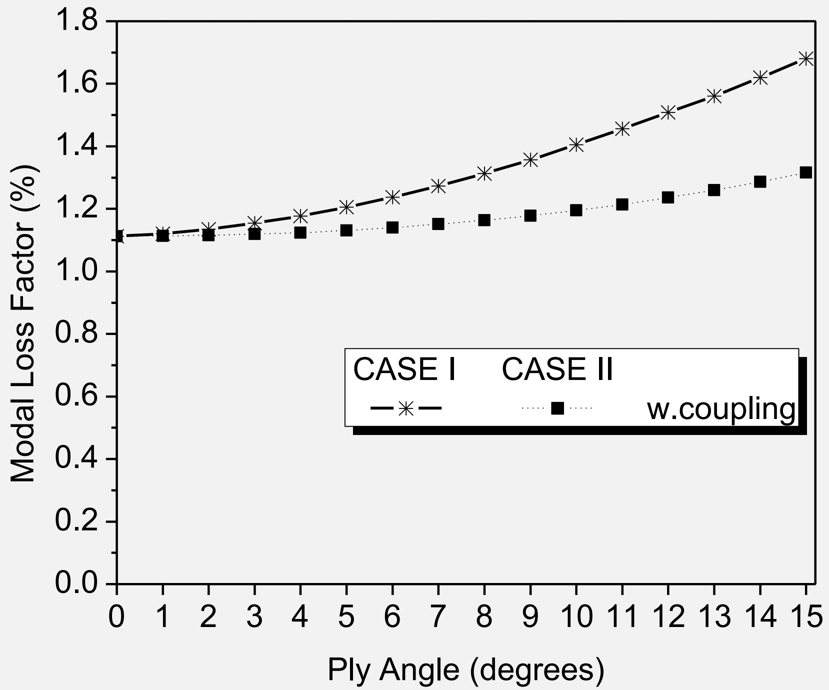
Predicted and Measured Blade Modal Damping & Frequencies

Mode	Beam Element Prediction	Measured (CRES)
Modal Frequencies (Hz):		
1st Flapwise	2.15	1.98
1st Edgewise	3.59	3.35
Modal Loss Factor η (%):		
1st Flapwise	0.56	0.78
1st Edgewise	0.60	0.87

Experimental Modal Tests conducted at Facilities of CRES, Athens, Greece (DAMPBLADE)

Prediction of Material Coupling Effect on 19m Wind-Turbine Blade





	UD Composite Plies		
	Top Girder	**Bottom Girder**	**Lay-up**
CASE I	**[θ_n]**	**[-θ_n]**	Anti-Symmetric
CASE II	[θ_n]	[θ_n]	Symmetric

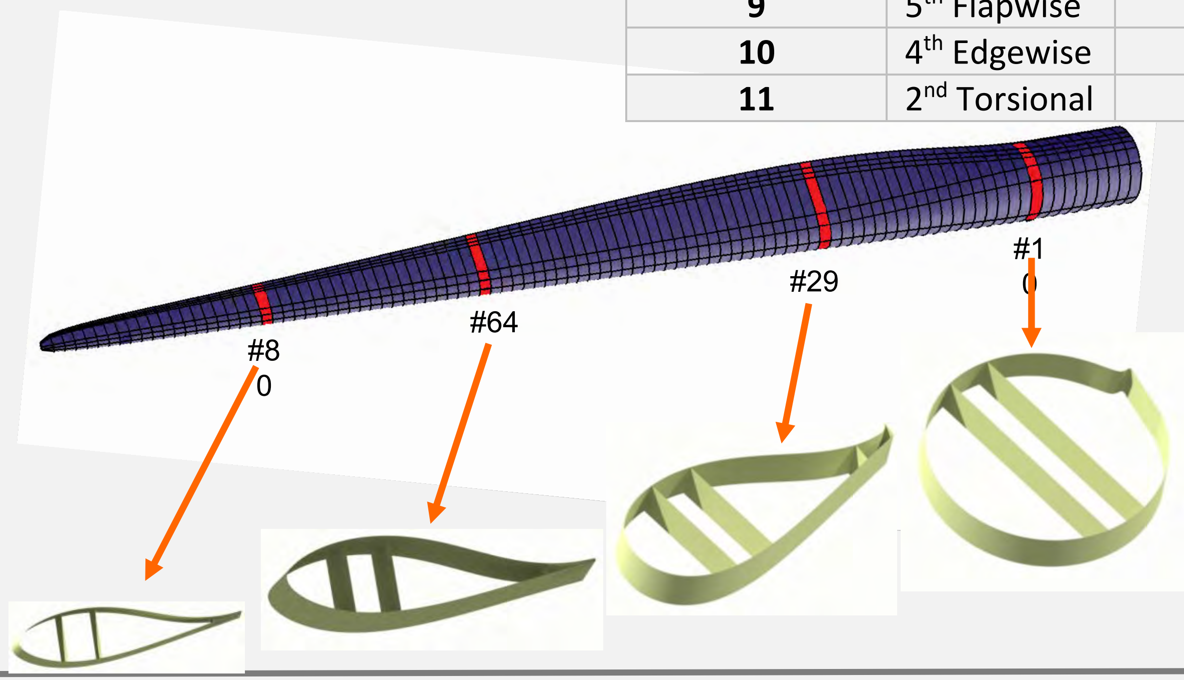
Inclusion of new material coupling terms contributes to:

- Lower Frequency Values
- Higher Damping Values

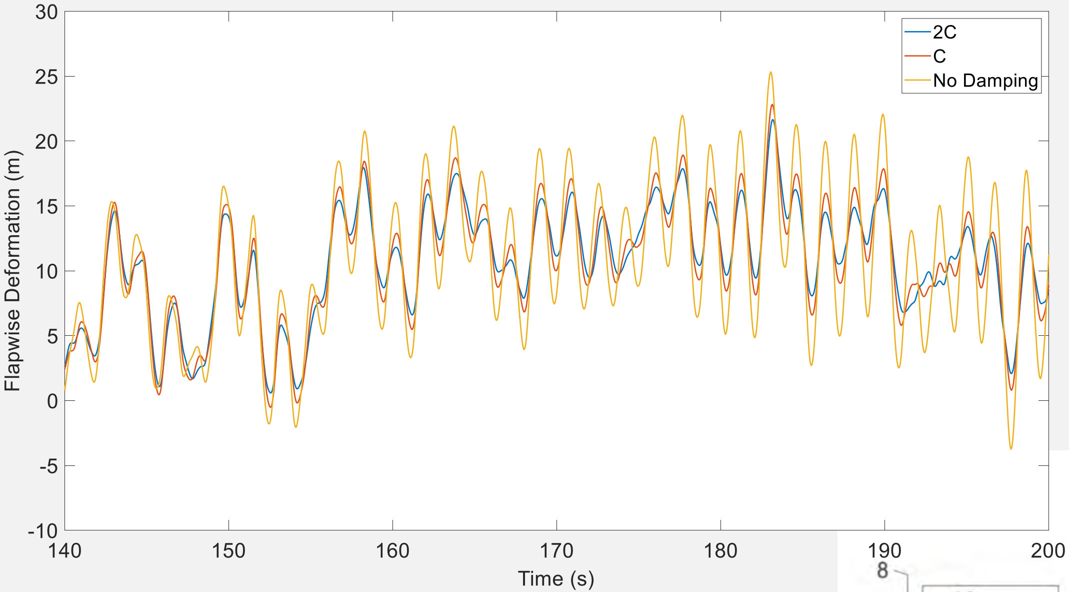
Effect of Damping on Dynamic Response of Modified 89m WT Blade

- 9.6 rpm (0.16Hz) rated speed
- 10 MW rated rotor power
- Uniform inflow with wind speed of 11 m/s

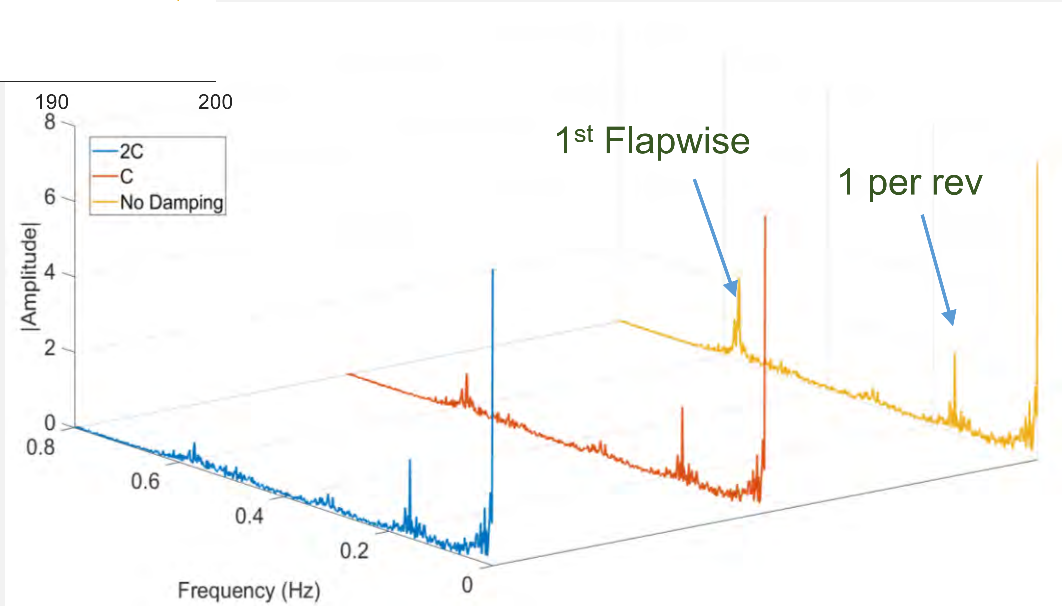
Mode Number	Mode Type	Frequency (Hz)	ζ (%)
1	1st Flapwise	0.583	0.084
2	1st Edgewise	0.938	0.095
3	2nd Flapwise	1.614	0.088
4	2nd Edgewise	2.787	0.102
5	3rd Flapwise	3.352	0.094
6	4th Flapwise	5.621	0.101
7	3rd Edgewise	5.899	0.110
8	1st Torsional	6.494	0.328
9	5th Flapwise	8.546	0.107
10	4th Edgewise	10.011	0.123
11	2nd Torsional	10.817	0.300



Damping Effect on Flapwise deformation @ tip of the wing (x=89m)

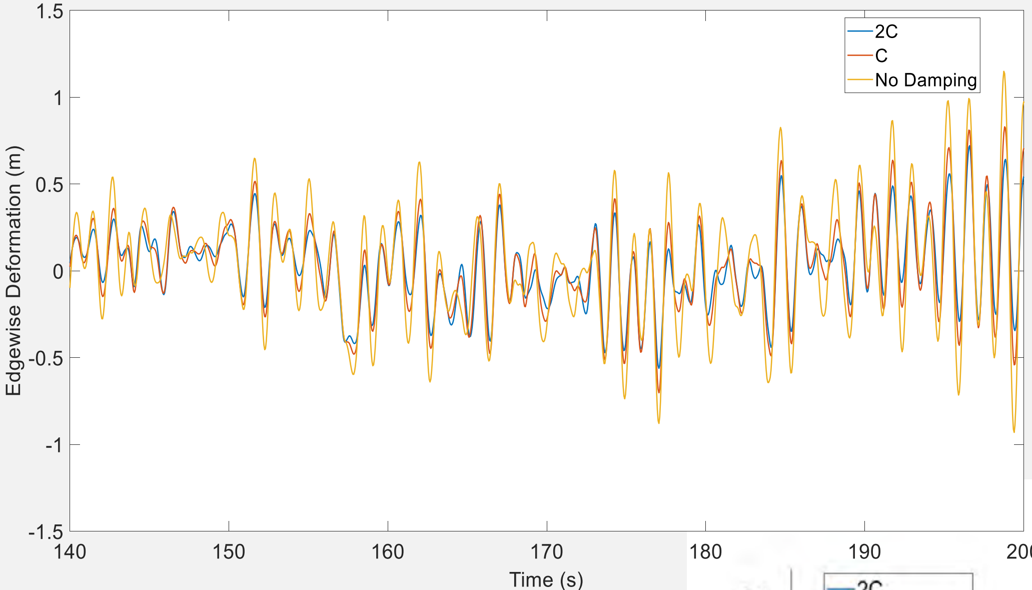


- Unsteady aerodynamics time-series loads were calculated with aeroelasticity model (NTUA)
- These are the loads used for normal operation fatigue damage

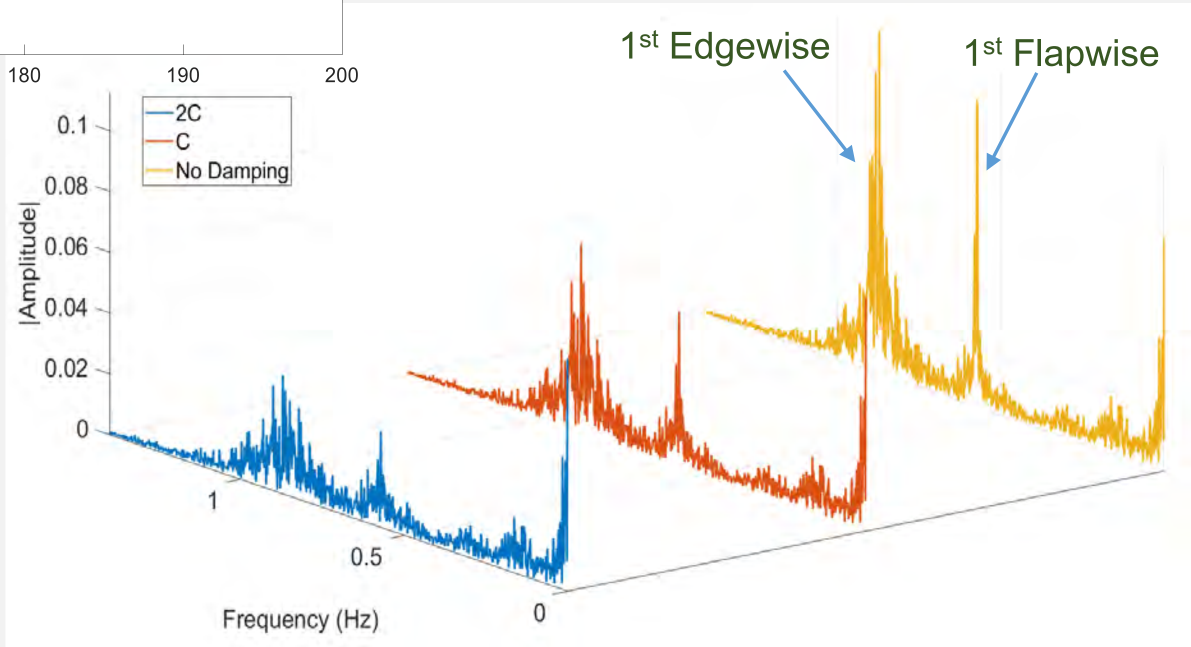


- Mean wind speed of 11 m/s
- Normal turbulence model case corresponding to DLC 1.2 of IEC

Damping Effect on Edgewise deformation @ tip of the wing (x=89m)



- Unsteady aerodynamics time-series loads were calculated with aeroelasticity model (NTUA)
- Loads used for normal operation fatigue damage



- Mean wind speed of 11 m/s
- Normal turbulence model case corresponding to DLC 1.2 of IEC

Part 3: Geometric Nonlinear Effects

MHI Vestas V164-9.5 MW
http://www.mhivestasoffshore.com/worlds-most-powerful-available-wind-turbine-gets-major-power-boost/
Accessed: 01/03/2019

Nonlinear Beam Section Mechanics

➢ Kinematic assumptions:

$$u(x,s,\zeta)=u^o(x)+\beta_y(x)(z_o+y^o_{,s}\zeta)+\beta_z(x)(y_o-z^o_{,s}\zeta)+\theta_{,x}(x)(-r^0_\zeta\zeta+\Psi^o(s))$$

$$\upsilon(x,s,\zeta)=\upsilon^o(x)-\theta(x)(r^o_\zeta+\zeta)$$

$$w(x,s,\zeta)=w^o(x)+\theta(x)r^o_s$$

➢ Green – Lagrange Section Strains:

$$\varepsilon_x=u_{,x}+\frac{1}{2}\upsilon^2_{,x}+\frac{1}{2}w^2_{,x}$$

$$\varepsilon_{xs}=u_{,s}+\upsilon_{,x}$$

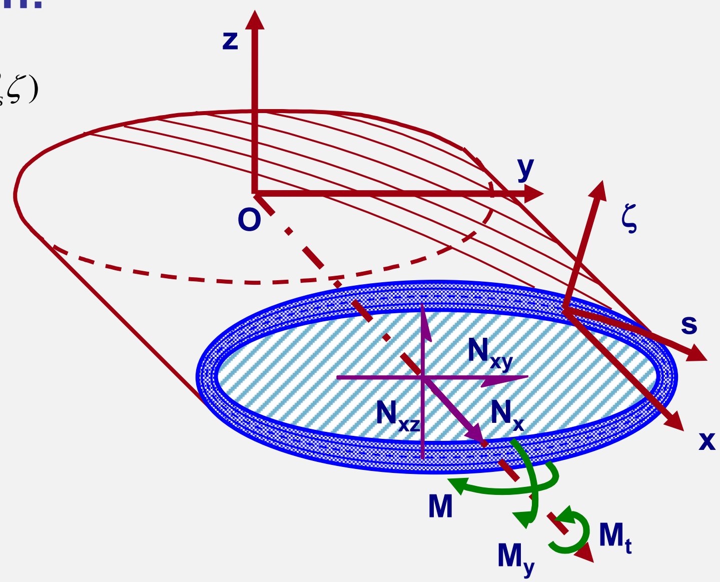
$$\varepsilon_{x\zeta}=u_{,\zeta}+w_{,x}$$

➢ Normal and Shear strains on the cross section:

$$\boldsymbol{\varepsilon}_{\mathbf{x}}(x,s,\zeta,t)=\varepsilon^0_x(x)+\frac{1}{2}\boldsymbol{\upsilon}^{0^2}_{,\mathbf{x}}(\mathbf{x})+\frac{1}{2}\mathbf{w}^{0^2}_{,\mathbf{x}}(\mathbf{x})+k_{xy}(x)(z_o+y^o_{,s}\zeta)+k_{xz}(x)(y_o-z^o_{,s}\zeta)$$
$$+\frac{1}{2}\boldsymbol{\theta}^2_{,\mathbf{x}}(\mathbf{x})\left\{\left(\mathbf{r}^{0^2}_\zeta+\mathbf{r}^{0^2}_s\right)+2\mathbf{r}^0_\zeta\zeta+\zeta^2\right\}-\boldsymbol{\upsilon}^0_{,\mathbf{x}}(\mathbf{x})\boldsymbol{\theta}_{,\mathbf{x}}(\mathbf{x})\left(\mathbf{r}^0_\zeta+\zeta\right)$$
$$+\mathbf{w}^0_{,\mathbf{x}}(\mathbf{x})\boldsymbol{\theta}_{,\mathbf{x}}(\mathbf{x})\left(\mathbf{r}^0_s+\zeta\right)+\boldsymbol{\theta}_{,\mathbf{xx}}(\mathbf{x})\left\{-\mathbf{r}^0_\zeta\zeta+\boldsymbol{\Psi}^o(\mathbf{s})\right\}$$

$$\boldsymbol{\varepsilon}_{\mathbf{xs}}(x,s,\zeta,t)=\varepsilon^0_{xz}(x)z^0_{,s}+\varepsilon^0_{xy}(x)y^0_{,s}+\varepsilon^{0t}_{xs}(x,s)-2\theta_{,x}\zeta$$

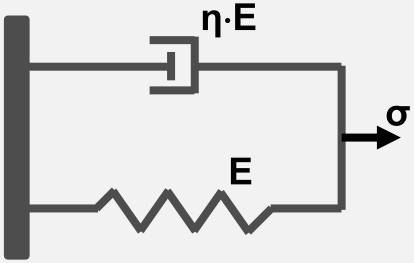
$$\boldsymbol{\varepsilon}_{\mathbf{x}\zeta}(x,s,\zeta,t)=\varepsilon^0_{xz}(x)y^0_{,s}-\varepsilon^0_{xy}(x)z^0_{,s}$$



Equations of Motion

$$-\int_0^L \mathrm{d}x \int_A \delta H \mathrm{d}s\mathrm{d}\zeta + \int_0^L \mathrm{d}x \int_A \delta T \mathrm{d}s\mathrm{d}\zeta + \int_\Gamma \delta \bar{\mathbf{u}}^{\mathbf{T}} \bar{\boldsymbol{\tau}} \mathrm{d}\Gamma = 0$$

Kelvin material model



I. ***Strain Energy Variation***

$$\delta H^{\text{sec}} = \int_A \delta \boldsymbol{\varepsilon}_{\mathbf{c}}^{\mathbf{T}} \boldsymbol{\sigma}_{\mathbf{c}} \mathrm{d}s\mathrm{d}\zeta \quad \text{and} \quad \boldsymbol{\sigma}_{\mathbf{c}} = [\mathbf{Q}_{\mathbf{c}}]\boldsymbol{\varepsilon}_{\mathbf{c}} + [\mathbf{Q}_{\mathbf{d}}]\dot{\boldsymbol{\varepsilon}}_{\mathbf{c}}$$

$$\delta H^{\text{sec}} = \int \mathrm{d}s \int_h \left(\delta\varepsilon_x \left[Q_{11}\varepsilon_x + Q_{16}\varepsilon_{xs} \right] + \delta\varepsilon_{xs} \left[Q_{16}\varepsilon_x + Q_{66}\varepsilon_{xs} \right] + \delta\varepsilon_{x\zeta} \left[Q_{55}\varepsilon_{x\zeta} \right] \right) \mathrm{d}\zeta$$

$$+ \int \mathrm{d}s \int_h \left(\delta\varepsilon_x \left[Q_{11}\dot{\varepsilon}_x + Q_{16}\dot{\varepsilon}_{xs} \right] + \delta\varepsilon_{xs} \left[Q_{16}\dot{\varepsilon}_x + Q_{66}\dot{\varepsilon}_{xs} \right] + \delta\varepsilon_{x\zeta} \left[Q_{55}\dot{\varepsilon}_{x\zeta} \right] \right) \mathrm{d}\zeta$$

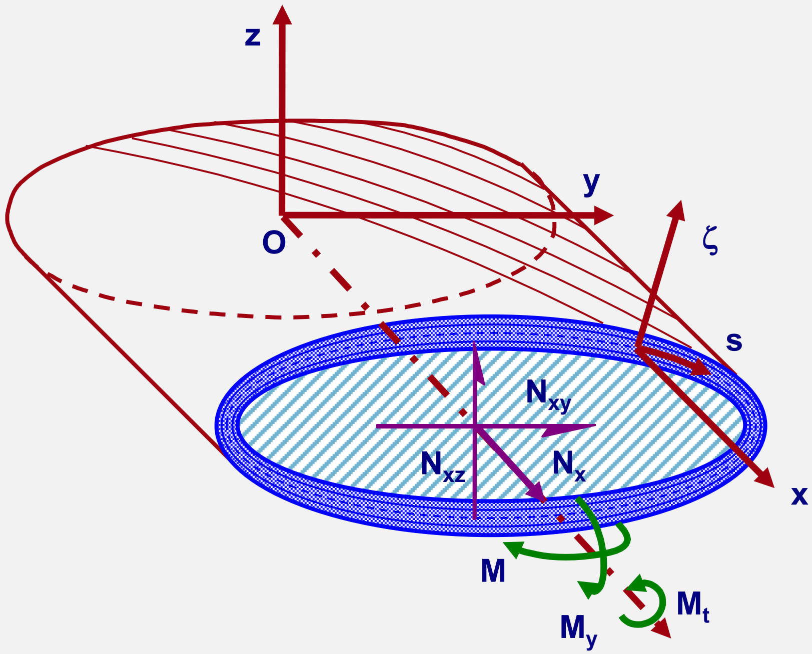
II. *Kinetic Energy Variation* ------→ $\delta T^{\text{sec}} = \int_A -\delta \mathbf{u}^{\mathbf{T}} \mathrm{diag}(\boldsymbol{\rho}) \ddot{\mathbf{u}} \mathrm{d}\xi \mathrm{d}\eta$

- $\boldsymbol{\rho}$ Density of each ply
- $\boldsymbol{h}$ Thickness of the skin laminate
- $\boldsymbol{L}$ Length of the beam
- $\boldsymbol{b}$ Width of the section

Nonlinear Section Stiffness

Section Stiffness Terms:

$$\left[\mathbf{K}_{s_0}\right]+\left[\mathbf{K}_{s_1}(\mathrm{u})\right]+\left[\mathbf{K}_{s_2}\left(\mathrm{u}^2\right)\right]=\left[\mathbf{K}_s\right]$$



The subscripts indicate:

- s: Total section terms
- s_0: Linear terms
- s_1: Nonlinear 1st order terms
- s_2: Nonlinear 2nd order terms

Nonlinear Section Stiffness

1st order nonlinear terms:

$$B_{16}^{nl1} = \theta_{,x}\left\{\oint\left(\frac{1}{2}A_{11}\left(r_\zeta^{0^2}+r_s^{0^2}\right)\right)\mathrm{d}s + \oint\left(B_{11}r_\zeta^{0}\right)\mathrm{d}s + \oint\left(\frac{1}{2}D_{11}\right)\mathrm{d}s\right\}$$

$$\left[\mathbf{K}_{\mathbf{s}_1}\right] = \begin{bmatrix} 0 & A_{15}^{nl1} & A_{16}^{nl1} & 0 & 0 & B_{16}^{nl1} \\ A_{51}^{nl1} & \bar{A}_{55}^{nl1} & \bar{A}_{56}^{nl1} & \bar{B}_{51}^{nl1} & \bar{B}_{52}^{nl1} & \bar{B}_{56}^{nl1} \\ A_{61}^{nl1} & \bar{A}_{65}^{nl1} & \bar{A}_{66}^{nl1} & \bar{B}_{61}^{nl1} & \bar{B}_{62}^{nl1} & \bar{B}_{66}^{nl1} \\ 0 & \bar{B}_{l51}^{nl1} & \bar{B}_{l61}^{nl1} & 0 & 0 & \bar{D}_{16}^{nl1} \\ 0 & \bar{B}_{l52}^{nl1} & \bar{B}_{l62}^{nl1} & 0 & 0 & \bar{D}_{26}^{nl1} \\ B_{l16}^{nl1} & \bar{B}_{l56}^{nl1} & \bar{B}_{l66}^{nl1} & \bar{D}_{61}^{nl1} & \bar{D}_{62}^{nl1} & \bar{D}_{66}^{nl1} \end{bmatrix} \begin{Bmatrix} u_{,x}^0 \\ \upsilon_{,x}^0 \\ w_{,x}^0 \\ k_{xy} \\ k_{xz} \\ \theta_{,x} \end{Bmatrix}$$

$$A_{66}^{nl1} = w_{,x}^0\oint\left(\frac{3}{2}A_{16}z_{,s}^0\right)\mathrm{d}s + \theta_{,x}\oint\left(2A_{16}r_s^0 z_{,s}^0\right)\mathrm{d}s$$

- Bending
- Twisting
- Bending – Torsion
- Extension – Torsion

2nd order nonlinear terms:

$$\left[\mathbf{K}_{\mathbf{s}_2}\right] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{55}^{nl2} & A_{56}^{nl2} & 0 & 0 & B_{56}^{nl2} \\ 0 & A_{65}^{nl2} & A_{66}^{nl2} & 0 & 0 & B_{66}^{nl2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B_{l56}^{nl2} & B_{l66}^{nl2} & 0 & 0 & D_{66}^{nl2} \end{bmatrix} \begin{Bmatrix} u_{,x}^0 \\ \upsilon_{,x}^0 \\ w_{,x}^0 \\ k_{xy} \\ k_{xz} \\ \theta_{,x} \end{Bmatrix}$$

$$A_{66}^{nl2} = w_{,x}^{0^2}\oint\left(\frac{1}{2}A_{11}\right)\mathrm{d}s + w_{,x}^0\theta_{,x}\oint\left(\frac{3}{2}A_{11}r_s^0\right)\mathrm{d}s + \theta_{,x}^2\oint\left(A_{11}r_s^{0^2}\right)\mathrm{d}s$$

Nonlinear Tangential Section Stiffness

1st order nonlinear terms:

$$B^{nl1}_{t16} = w^0_{,x}\oint A_{11} r^0_s \,\mathrm{ds} - \upsilon^0_{,x}\oint\left(A_{11} r^0_\zeta + B_{11}\right)\mathrm{ds}$$
$$+\theta_{,x}\left\{\oint\left(A_{11}\left(r^{0^2}_\zeta + r^{0^2}_s\right) + 2B_{11} r^0_\zeta + D_{11}\right)\mathrm{ds}\right\}$$

$$\left[\mathbf{K}_{\mathbf{Ts}_1}\right] = \begin{bmatrix} 0 & A^{nl1}_{t15} & A^{nl1}_{t16} & 0 & 0 & B^{nl1}_{t16} \\ A^{nl1}_{t51} & \bar{A}^{nl1}_{t55} & \bar{A}^{nl1}_{t56} & \bar{B}^{nl1}_{t51} & \bar{B}^{nl1}_{t52} & \bar{B}^{nl1}_{t56} \\ A^{nl1}_{t61} & \bar{A}^{nl1}_{t65} & \bar{A}^{nl1}_{t66} & \bar{B}^{nl1}_{t61} & \bar{B}^{nl1}_{t62} & \bar{B}^{nl1}_{t66} \\ 0 & \bar{B}^{nl1}_{t51} & \bar{B}^{nl1}_{t61} & 0 & 0 & \bar{D}^{nl1}_{t16} \\ 0 & \bar{B}^{nl1}_{t52} & \bar{B}^{nl1}_{t62} & 0 & 0 & \bar{D}^{nl1}_{t26} \\ B^{nl1}_{t16} & \bar{B}^{nl1}_{t56} & \bar{B}^{nl1}_{t66} & \bar{D}^{nl1}_{t61} & \bar{D}^{nl1}_{t62} & \bar{D}^{nl1}_{t66} \end{bmatrix} \begin{Bmatrix} u^0_{,x} \\ \upsilon^0_{,x} \\ w^0_{,x} \\ k_{xy} \\ k_{xz} \\ \theta_{,x} \end{Bmatrix}$$

$$A^{nl1}_{t56} = \upsilon^0_{,x}\oint\left(A_{16} z^0_{,s}\right)\mathrm{ds} + w^0_{,x}\oint\left(A_{16} y^0_{,s}\right)\mathrm{ds}$$
$$+\theta_{,x}\left\{\oint\left(A_{16} r^0_s y^0_{,s}\right)\mathrm{ds} - \oint\left(A_{16} r^0_\zeta z^0_{,s} + B_{16} z^0_{,s}\right)\mathrm{ds}\right\}$$

- Bending
- Twisting
- Bending –Torsion
- Extension – Torsion

2nd order nonlinear terms:

$$\left[\mathbf{K}_{\mathbf{Ts}_2}\right] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{nl2}_{t55} & A^{nl2}_{t56} & 0 & 0 & B^{nl2}_{t56} \\ 0 & A^{nl2}_{t65} & A^{nl2}_{t66} & 0 & 0 & B^{nl2}_{t66} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B^{nl2}_{t56} & B^{nl2}_{t66} & 0 & 0 & D^{nl2}_{t66} \end{bmatrix} \begin{Bmatrix} u^0_{,x} \\ \upsilon^0_{,x} \\ w^0_{,x} \\ k_{xy} \\ k_{xz} \\ \theta_{,x} \end{Bmatrix}$$

$$A^{nl2}_{t56} = \upsilon^0_{,x} w^0_{,x}\oint\left(A_{11}\right)\mathrm{ds} + \upsilon^0_{,x}\theta_{,x}\oint\left(A_{11} r^0_s\right)\mathrm{ds}$$
$$-w^0_{,x}\theta_{,x}\oint\left(A_{11} r^0_\zeta + B_{11}\right)\mathrm{ds} - \theta^2_{,x}\oint\left(A_{11} r^0_\zeta r^0_s + B_{11} r^0_s\right)\mathrm{ds}$$

Nonlinear Section Damping Terms

$$\delta H_{ds} = \delta H_{ds_0} + \delta H_{ds_1} + \delta H_{ds_2}$$

$$\left[\mathbf{C_{ds}}\right] = \left[\mathbf{C_{ds_0}}\right] + \left[\mathbf{C_{ds_1}}(\mathrm{u})\right] + \left[\mathbf{C_{ds_2}}(\mathrm{u}^2)\right]$$

1st order damping nonlinear terms

$$\left[\mathbf{C_{ds_1}}\right] = \begin{bmatrix} 0 & A^{nl1}_{d15} & A^{nl1}_{d16} & 0 & 0 & B^{nl1}_{d16} \\ A^{nl1}_{d51} & \bar{A}^{nl1}_{d55} & \bar{A}^{nl1}_{d56} & \bar{B}^{nl1}_{d51} & \bar{B}^{nl1}_{d52} & \bar{B}^{nl1}_{d56} \\ A^{nl1}_{d61} & \bar{A}^{nl1}_{d65} & \bar{A}^{nl1}_{d66} & \bar{B}^{nl1}_{d61} & \bar{B}^{nl1}_{d62} & \bar{B}^{nl1}_{d66} \\ 0 & \bar{B}^{nl1}_{d51} & \bar{B}^{nl1}_{d61} & 0 & 0 & \bar{D}^{nl1}_{d16} \\ 0 & \bar{B}^{nl1}_{d52} & \bar{B}^{nl1}_{d62} & 0 & 0 & \bar{D}^{nl1}_{d26} \\ B^{nl1}_{d16} & \bar{B}^{nl1}_{d56} & \bar{B}^{nl1}_{d66} & \bar{D}^{nl1}_{d61} & \bar{D}^{nl1}_{d62} & \bar{D}^{nl1}_{d66} \end{bmatrix} \begin{Bmatrix} \dot{u}^0_{,x} \\ \dot{\upsilon}^0_{,x} \\ \dot{w}^0_{,x} \\ \dot{k}_{xy} \\ \dot{k}_{xz} \\ \dot{\theta}_{,x} \end{Bmatrix}$$

$$A^{nl1}_{d66} = w^0_{,x} \oint \left(2A_{d16} z^0_{,s}\right) \mathrm{ds} + \theta_{,x} \oint \left(2A_{16} r_s^0 z^0_{,s}\right) \mathrm{ds}$$

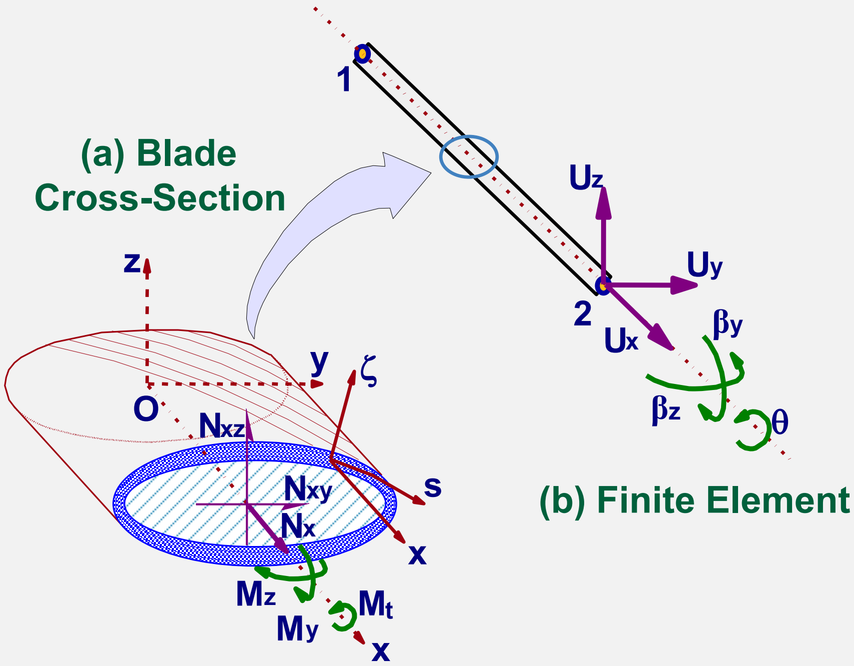
- Bending
- Twisting
- Bending –Torsion
- Extension – Torsion

2nd order damping nonlinear terms

$$\left[\mathbf{C_{ds_2}}\right] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{nl2}_{d55} & A^{nl2}_{d56} & 0 & 0 & B^{nl2}_{d56} \\ 0 & A^{nl2}_{d56} & A^{nl2}_{d66} & 0 & 0 & B^{nl2}_{d66} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B^{nl2}_{d56} & B^{nl2}_{d66} & 0 & 0 & D^{nl2}_{d66} \end{bmatrix} \begin{Bmatrix} \dot{u}^0_{,x} \\ \dot{\upsilon}^0_{,x} \\ \dot{w}^0_{,x} \\ \dot{k}_{xy} \\ \dot{k}_{xz} \\ \dot{\theta}_{,x} \end{Bmatrix}$$

$$A^{nl2}_{d66} = w^{0^2}_{,x} \oint \left(A_{d11}\right) \mathrm{ds} + w^0_{,x}\theta_{,x} \oint \left(2A_{d11} r_s^0\right) \mathrm{ds} + \theta^2_{,x} \oint \left(A_{d11} r_s^{0^2}\right) \mathrm{ds}$$

Updated DAMPBEAM 3-D Finite Element



Total Stiffness matrix

$$\left[K\ (u)\right]=\left[K_0\right]+\left[K_{N_1}(u)\right]+\left[K_{N_2}(u^2)\right]$$

Linearized Stiffness matrix

$$\left[K_T(u)\right]=\left[K_0\right]+\left[K_{T_{N_1}}(u)\right]+\left[K_{T_{N_2}}(u^2)\right]$$

Damping matrix

$$\left[C(u)\right]=\left[C_0\right]+\left[C_{T_{N_1}}(u)\right]+\left[C_{T_{N_2}}(u^2)\right]$$

- 2-node shear element with linear displacement interpolation functions
- 6 DOFs per node: $\mathbf{U}_{\mathbf{e}}^{\mathbf{i}}=\left\{u^{oi},\ \upsilon^{oi},\ w^{oi},\ \beta_y^i,\ \beta_z^i,\ \theta^i\right\}$

Small Vibration about Large Static Deflection

- **Nonlinear Large Static Deflection**:

$$\Psi_s(\mathbf{u}_s) = \left[\mathbf{K}(\mathbf{u}_s)\right]\mathbf{u}_s(\mathbf{t}) - \mathbf{F}_s(\mathbf{t}) = \mathbf{0}$$

Newton – Raphson iterative technique

- **Nonlinear Small Amplitude Free Vibration:** $\mathbf{u}(t) = \mathbf{u}_s + \bar{u}(t)$

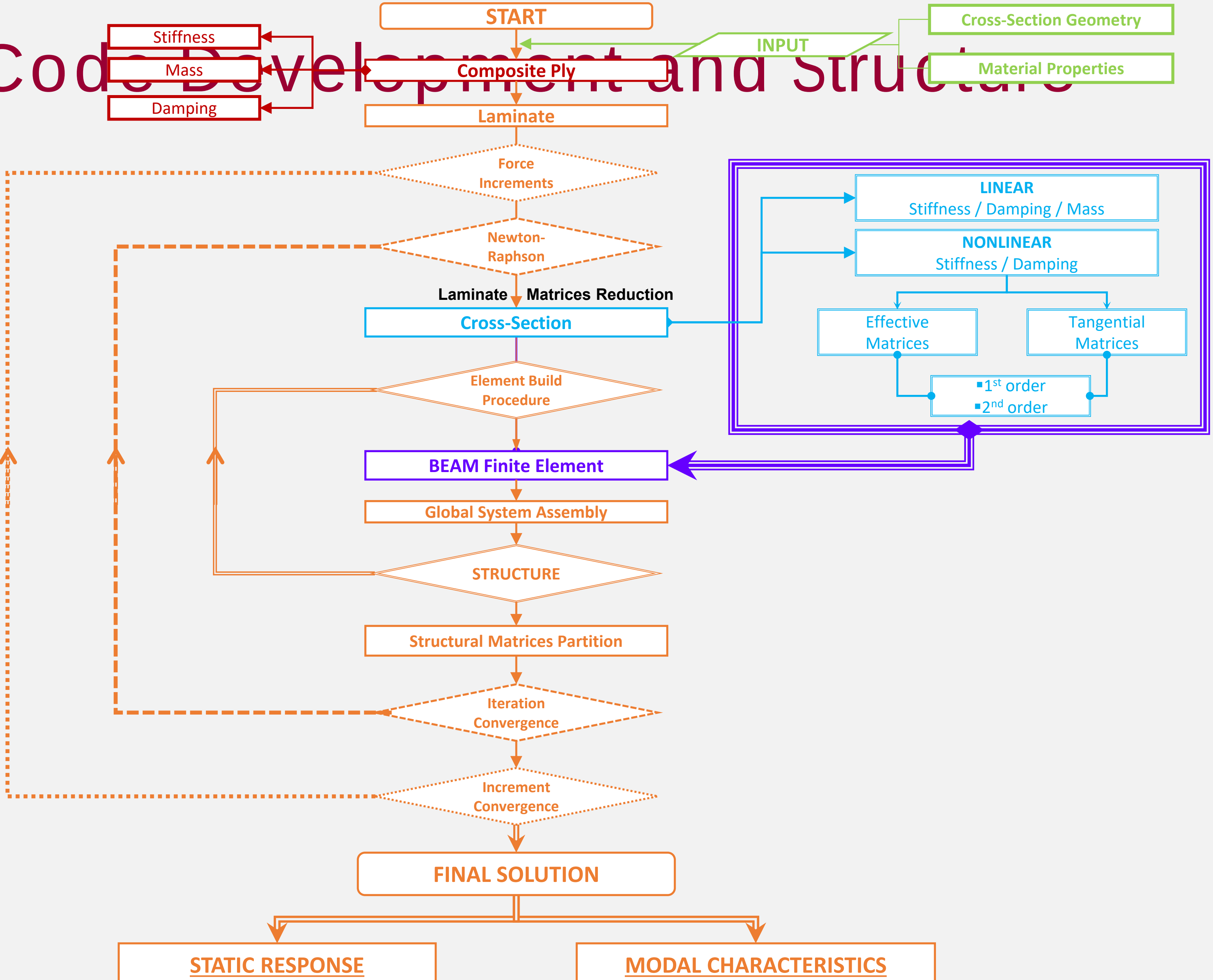
$$\Psi(\bar{\mathbf{u}}, \mathbf{u}_s, \mathbf{t}) = [\mathbf{M}]\ddot{\bar{\mathbf{u}}}(\mathbf{t}) + \left[\mathbf{C}(\mathbf{u}_s)\right]\dot{\bar{\mathbf{u}}}(\mathbf{t}) + \left[\bar{\mathbf{K}}(\mathbf{u}_s)\right]\bar{\mathbf{u}}(\mathbf{t}) - \bar{\mathbf{F}}(\mathbf{t}) = \mathbf{0}$$

❑ The energy approach is used to calculate the structural damping:

- $\boldsymbol{\omega}_\mathbf{m}$ Undamped Frequencies
- $\mathbf{U}_\mathbf{m}$ Mode Shapes
- $\boldsymbol{\eta}_\mathbf{m}$ Modal Loss Factor

$$\boxed{\eta_m = \frac{\omega_m}{2\pi}\frac{\mathbf{U}_\mathbf{m}^\mathbf{T}\left[\mathbf{C}(\mathrm{u}_s)\right]\mathbf{U}_\mathbf{m}}{\mathbf{U}_\mathbf{m}^\mathbf{T}\left[\bar{\mathbf{K}}(\mathrm{u}_s)\right]\mathbf{U}_\mathbf{m}}}$$

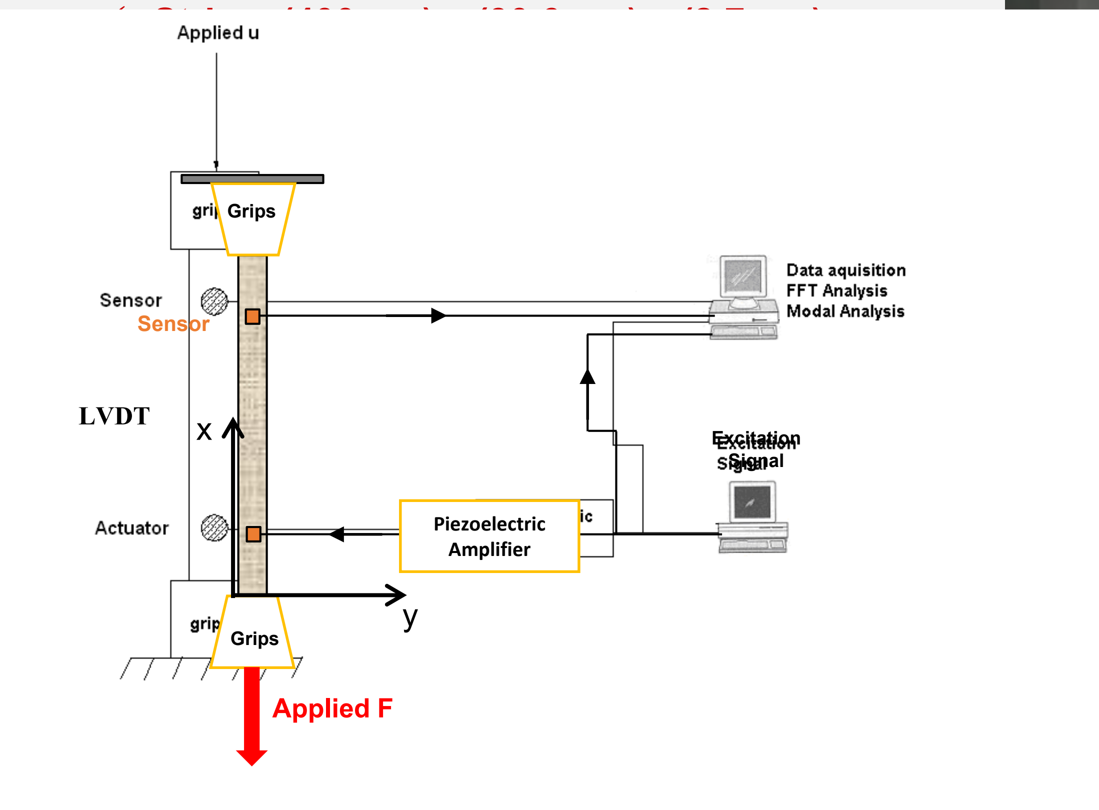
Code Development and Structure



Tension Experiments

Specimens

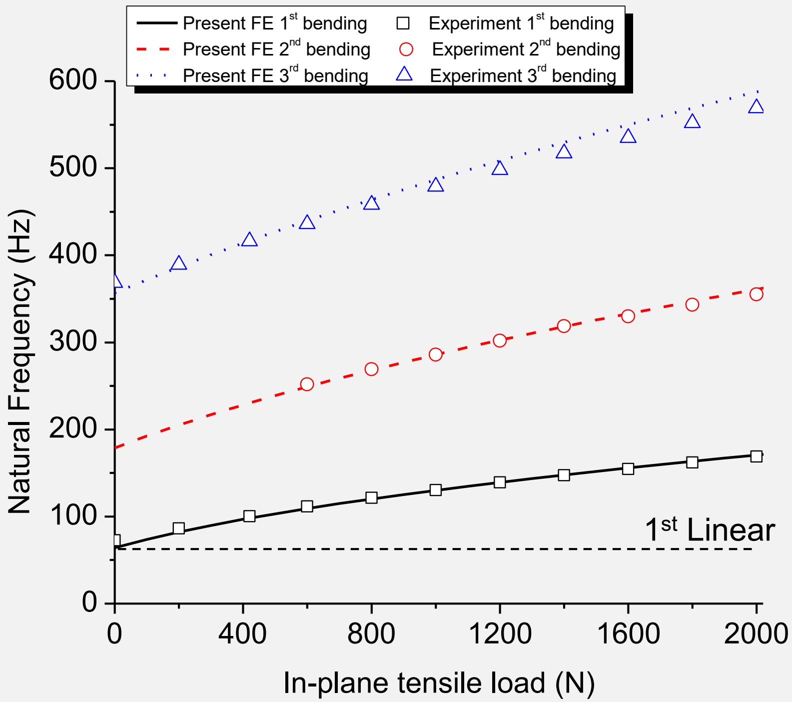
- ✓ UD Glass/Epoxy
- ✓ Manufactured at AML by hand lay-up + vacuum bag method
- ✓ Curing for 12h at 60^0C
- ✓ 2 Laminations: $[0_2/90_2]_s$ and $[90_2/0_2]_s$



Uniaxial Testing Machine

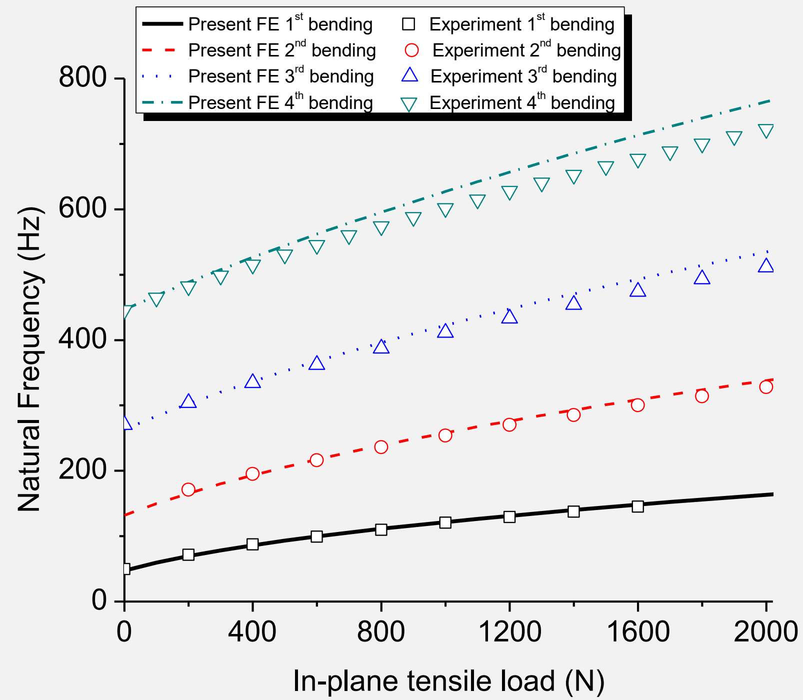
- Constant in-plane displacement *0.02mm/min* rate
- Loading was periodically suspended.
- Strip excited, FRF measured
- Two PZT-5 piezoceramic wafers (sensor & actuator) or
- Impact Hammer and PZT-5 sensor

Numerical Predictions Validation – Modal Frequency

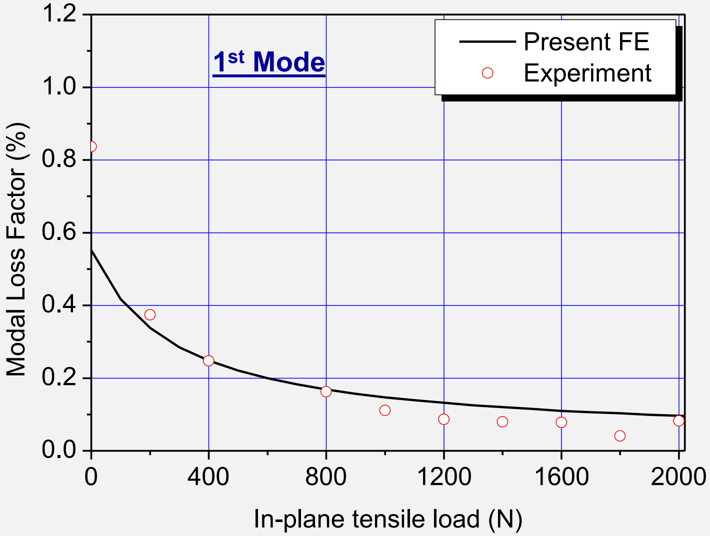


$[0_2/90_2]_s$

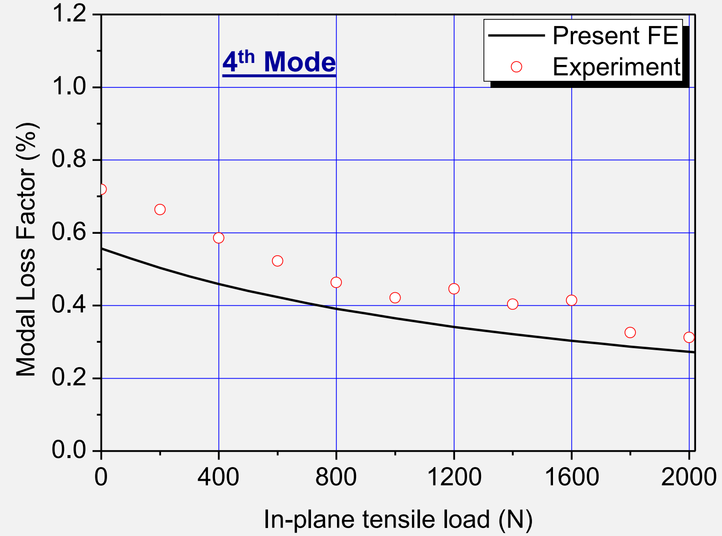
$[90_2/0_2]_s$

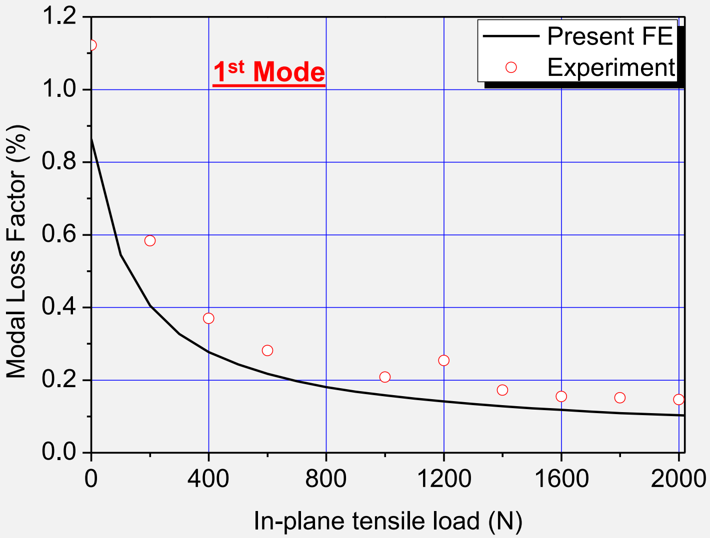


Numerical Predictions Validation – Modal Damping

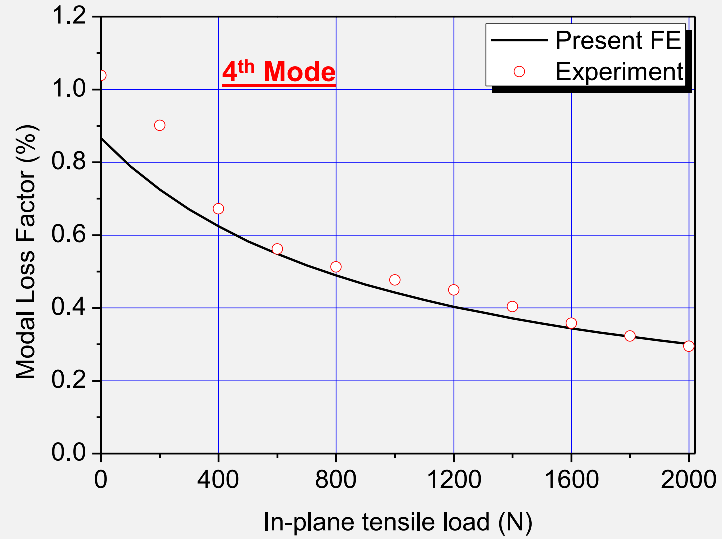


$[0_2/90_2]_s$

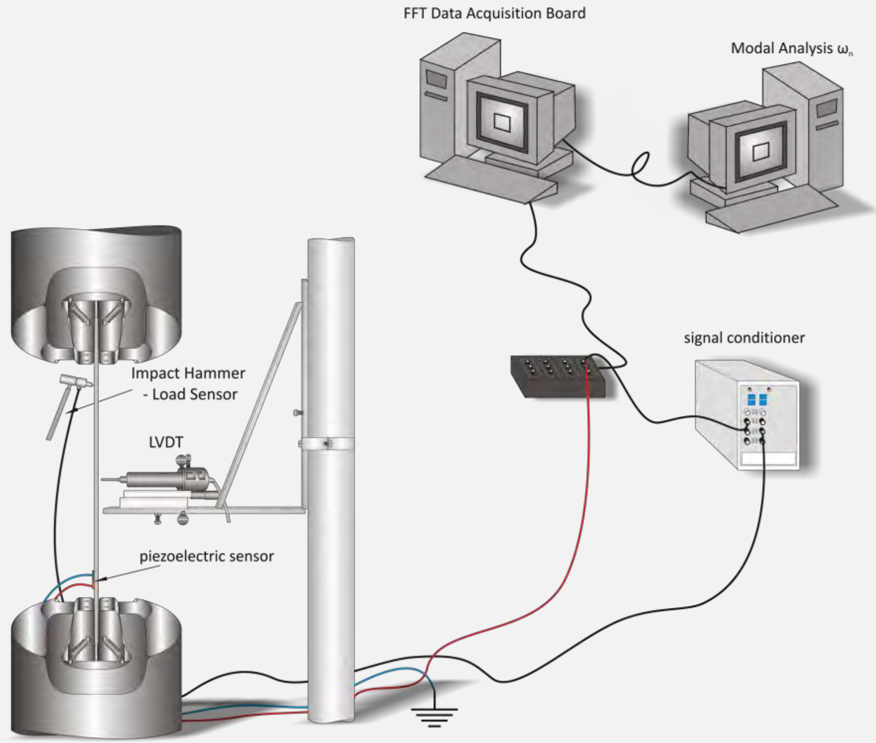




$[90_2/0_2]_s$



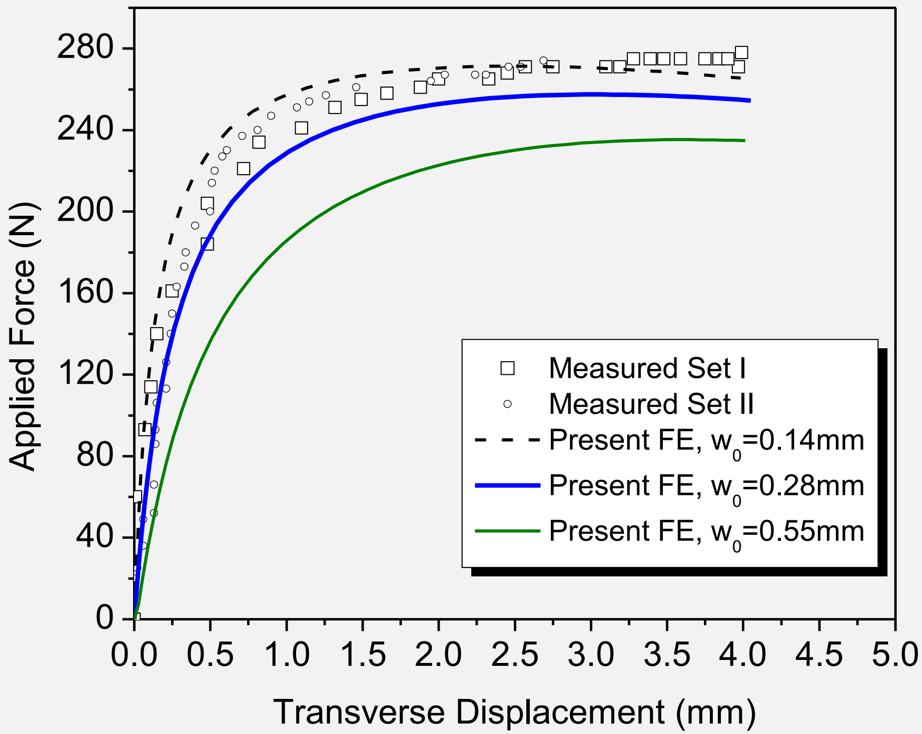
Buckling Experiment



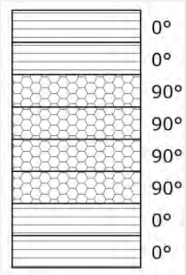
- Applied in-plane compressive displacement at *0.01mm/min* rate
- Actuator: Impact Hammer
- Sensory devices: i) PZT-5 plate (**SET I**) and ii) Accelerometer (**SET II**)
- LVDT at the strip midspan

Predicted Buckling Response

Transverse displacement at stip mid-span



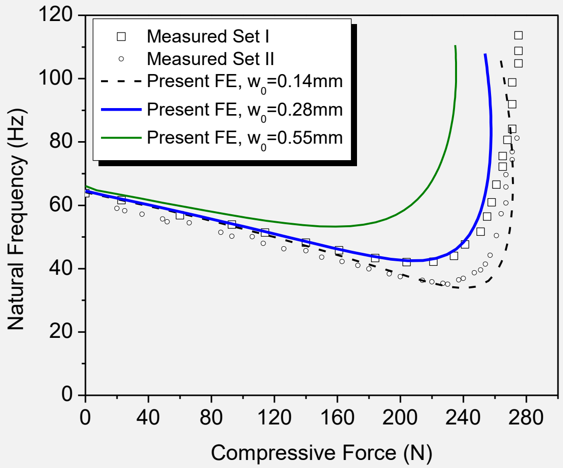
Lamination
$[0_2/90_2]_s$



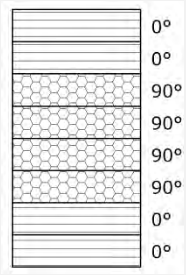
- **Sensitivity of buckling response to the initial imperfection quantified**

Buckling Effect on Modal Frequency and Damping

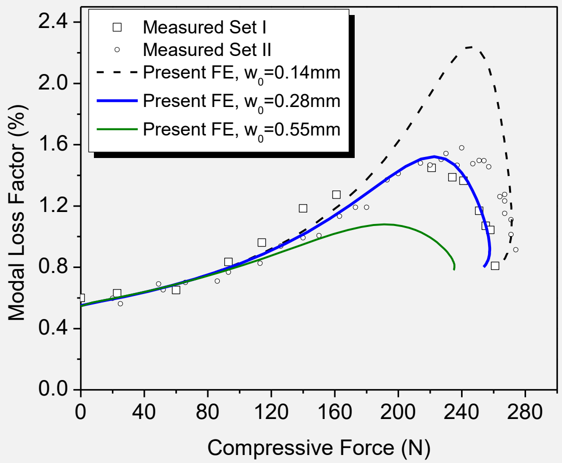
First bending natural frequency



Lamination
$[0_2/90_2]_s$

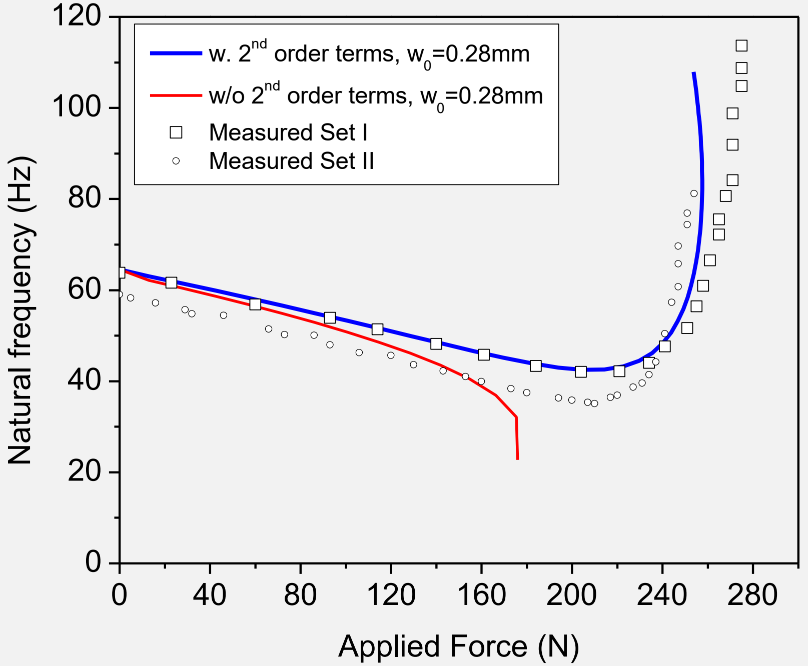


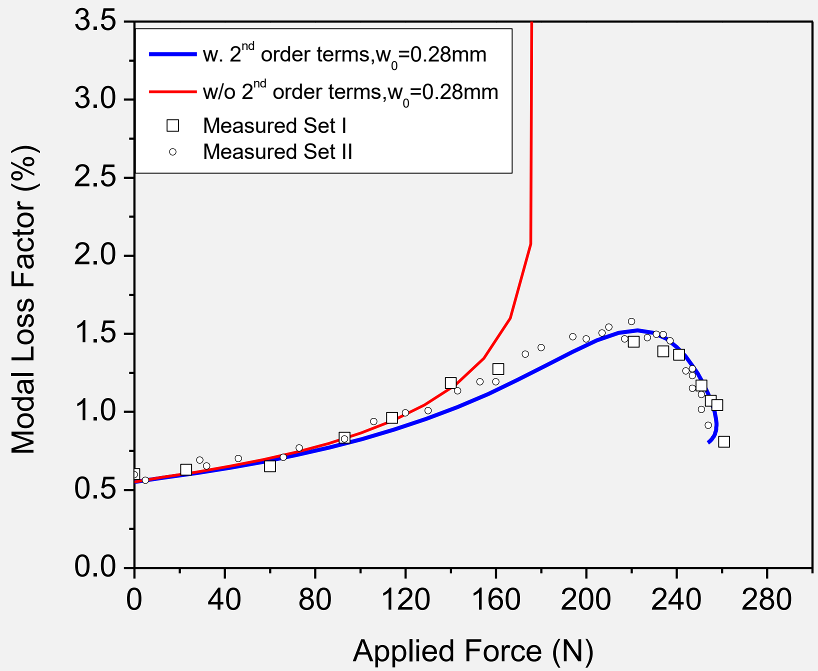
First bending modal loss factor



- **Maximum damping values near the critical buckling point.**
- **Good Correlation between predicted and experimental measurements.**

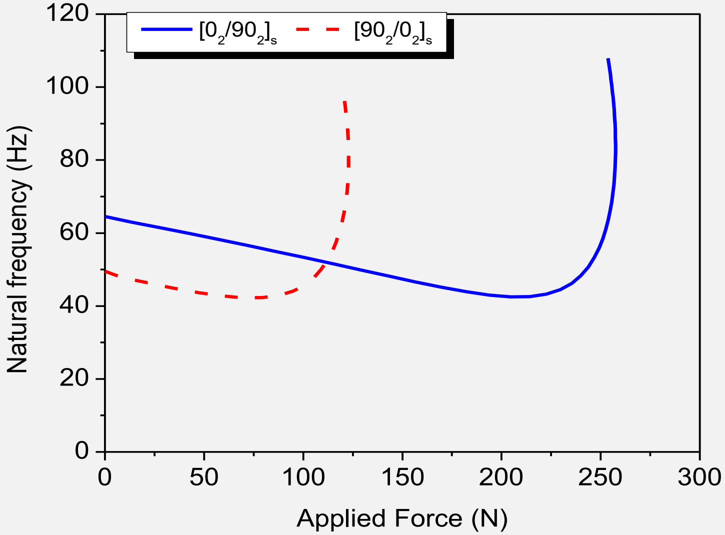
Significance of 2nd order Stiffness and Damping Terms

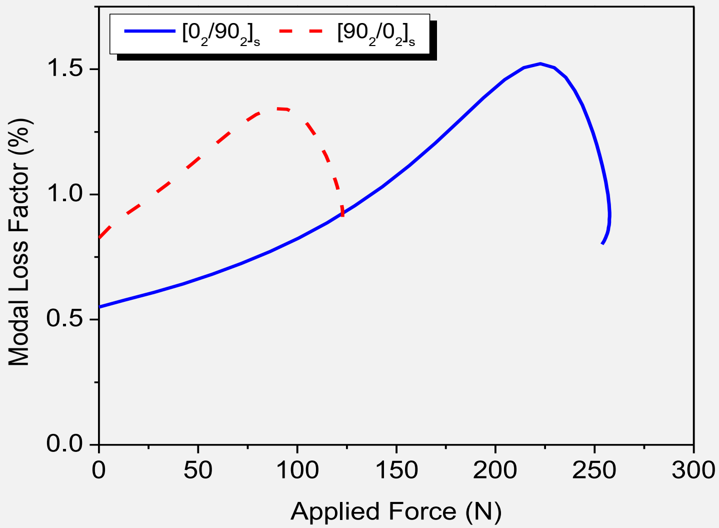


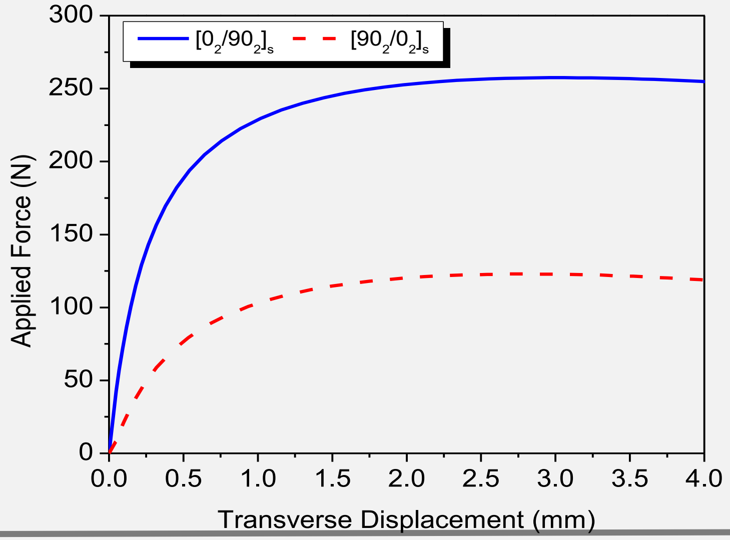


- **Describe the modal characteristic values during the transition from the pre- to the post-buckling region**

$[0_2/90_2]_s$ vs. $[90_2/0_2]_s$ Beam Specimens

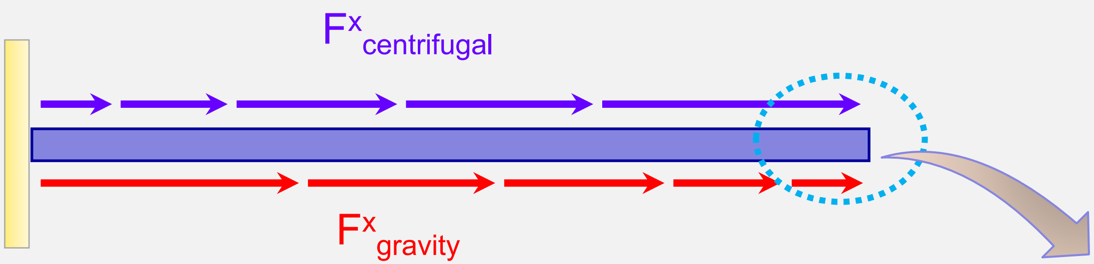




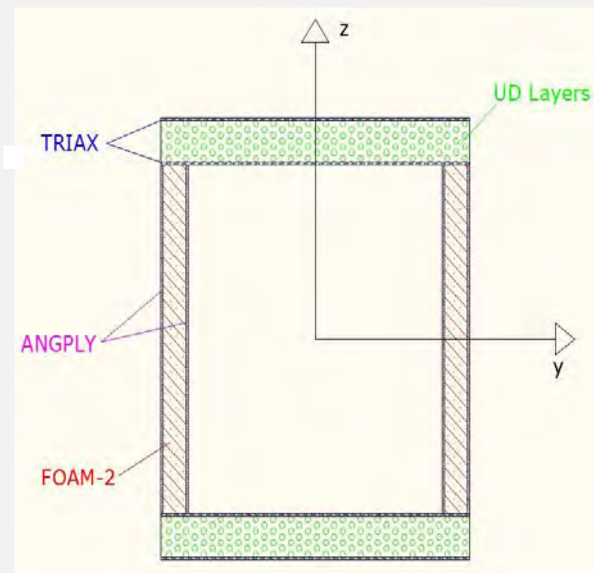


- **The $[90_2/0_2]_s$ beam specimen exhibits:**
  - **more compliant behavior**
  - **lower modal frequency**
  - **lower critical buckling point**

Box-Section Girder Beam of 54m



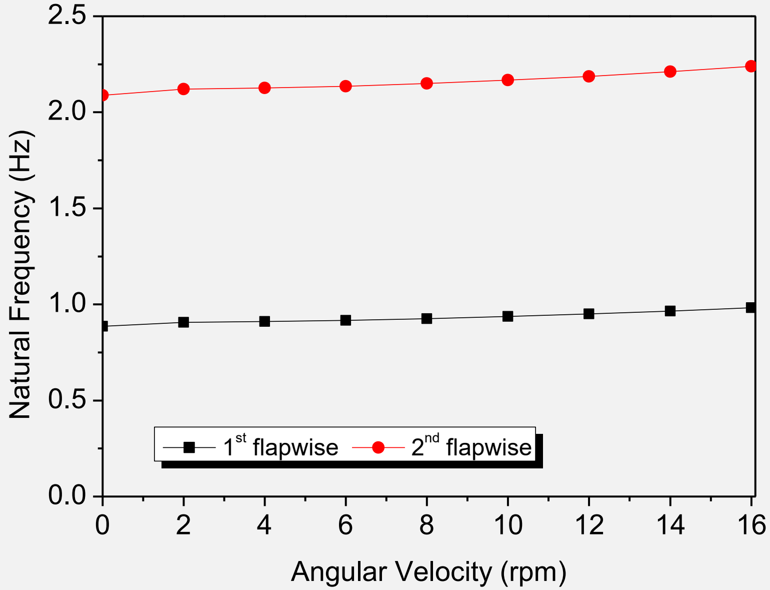
Typical Cross-Section

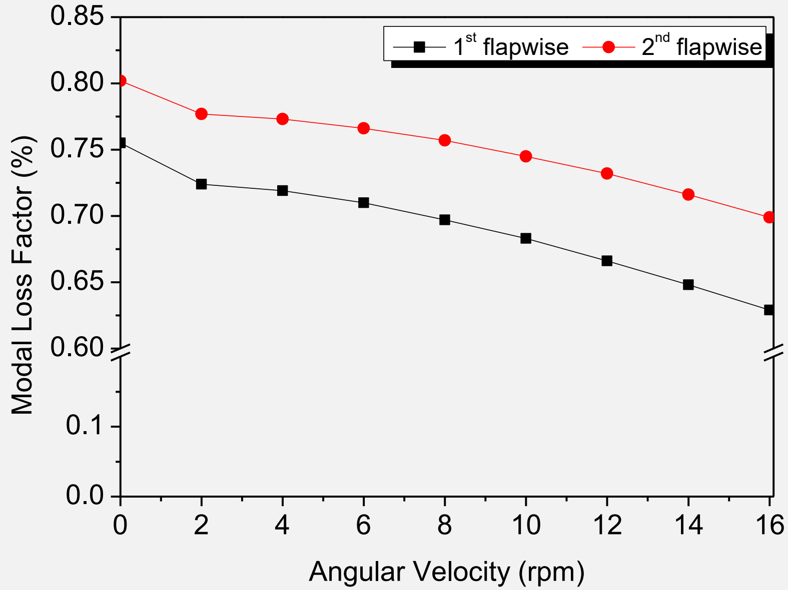


$$\left\{ \mathbf{F}_{b_e}^{centrifugal} \right\} = \frac{\rho A \Omega^2 L_e}{6} \begin{Bmatrix} 3r + L_e \\ 3r + 2L_e \end{Bmatrix}$$

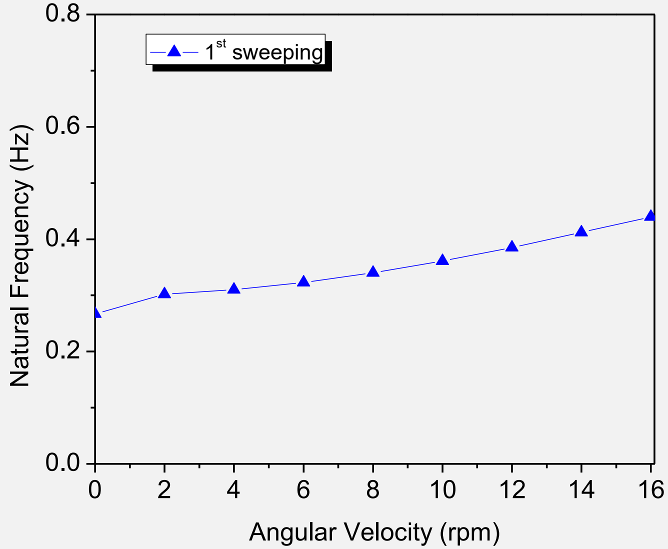
$$\left\{ \mathbf{F}_{b_e}^{granity} \right\} = \frac{\rho g A L_e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

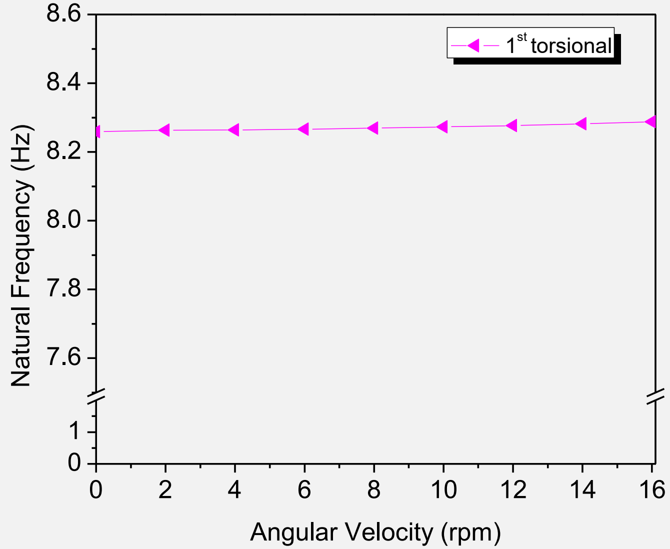
Girder Beam / Modal Characteristics…

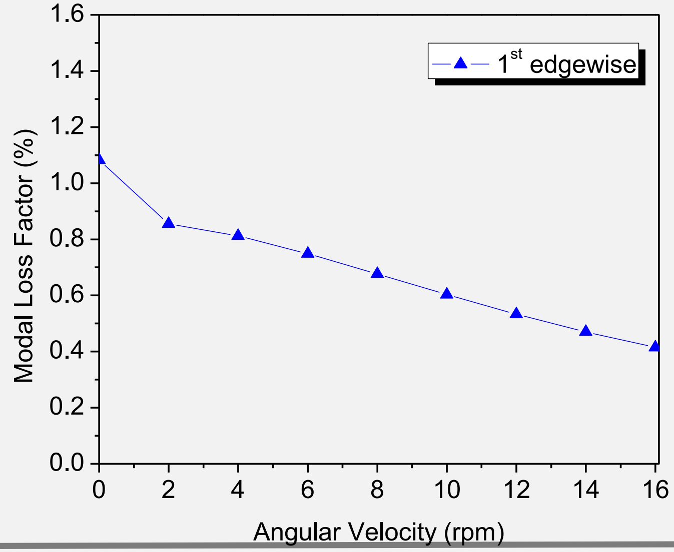


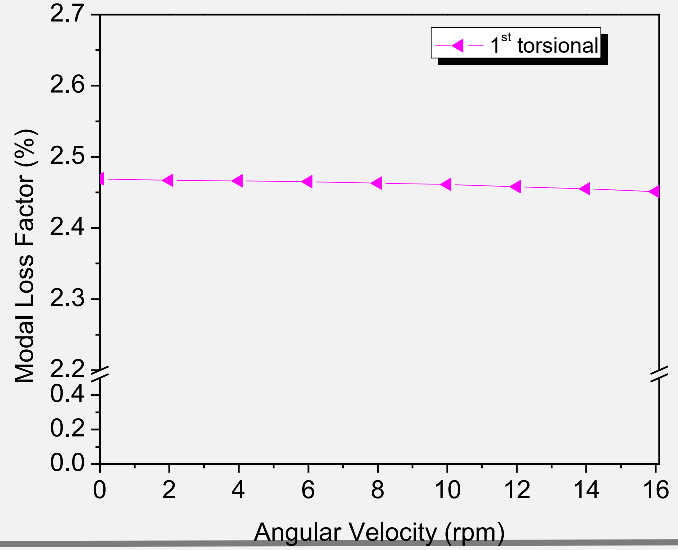


…Girder Beam / Modal Characteristics









Summary

- Overview of WT blade materials and internal geometry
- Composite mechanics and computational structural dynamics models
- Formulation of Reduced Order Beam finite element
- Mechanics and modelling of Bending-Torsion Coupling
- Comprehensive inclusion of Composite Damping into CSD models
- Trade-off between composite damping and stiffness is common feature
- Structural damping also strongly depends on geometry, curvature and mode shape
- Effect of composite damping on time- and frequency- of 89m dynamic response composite blade excited by unsteady aerodynamic loads.
- Nonlinear mechanics and beam finite-element framework
- Initial stress and geometric nonlinearity appear to significantly modify modal frequencies and damping.

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Thank you!
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Tak!
Ευχαριστώ!