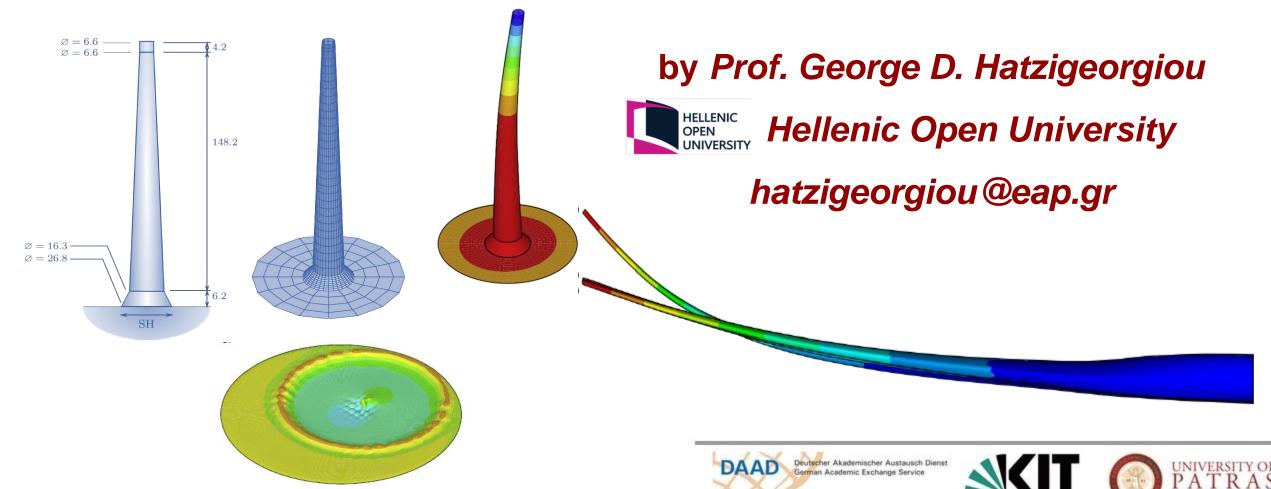
# **How Wind Turbine Engineers Confront**

**Structural Nonlinearities?** 



## <u>Outline</u>

Background

□ Nonlinear behavior of tower

□ Nonlinear behavior of foundation

□ Nonlinear behavior of soil

- □ Nonlinear behavior of flanges
- □ Nonlinear behavior of blades







#### **Background**



Wind turbines consist of a plethora of subcomponents, which can number up to 8,000. Five parts, however, are vital and can behave nonlinearly: soil, foundation, and blades. This hub, tower presentation depicts how wind turbine engineers confront these structural nonlinearities.







#### **TOWERS**

The tower is constructed to hold the rotor blades off the ground and at an ideal wind speed. Towers are usually between 50-100 m above the surface of the ground or water. Offshore towers are generally fixed to the bottom of the water body, although research is ongoing to develop a tower that floats on the surface.









#### **TOWERS**

Taller towers for wind turbines make sense. For instance, an 80-m tower can let 2 to 3-MW wind turbines produce more power, and enough to justify the additional cost of 20-m more, than if installed at 60 m. Taller towers will also let larger turbines enter the market. Taller towers allow putting turbines in less turbulent winds, thereby decreasing wear and fatigue. Haliade-X, the most powerful offshore wind turbine in the world (14 MW capacity) has a 260 m high tower.









## **TOWERS: Steel, Concrete or both of them?**

tower

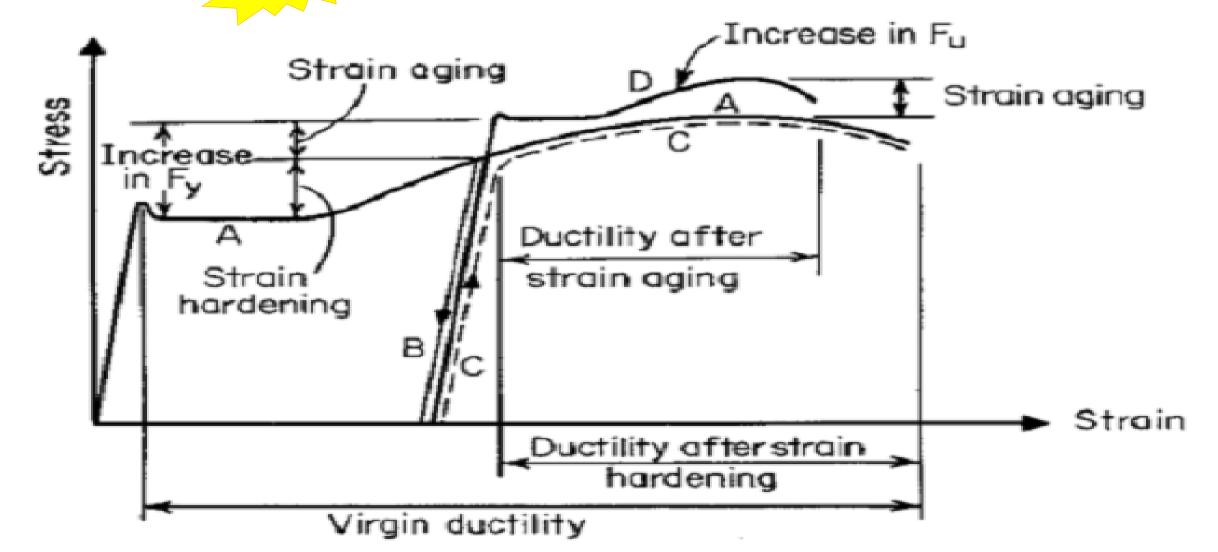


- □ Wind turbine tower design is a key parameter in contemporary wind generator design in general.
- Tower cost is about 15% of total cost of a wind turbine.
- The prevailing structural configuration of the total installed wind capacity is the **steel tubular tower**, providing the advantage of robust structural design, prefabrication of large wind tower parts, limited onsite labor and easier mounting between parts. Cylindrical shells are traditionally preferred by designers in order to minimize material use in structures, because due to their geometry they are capable of carrying great loads with small shell thicknesses.















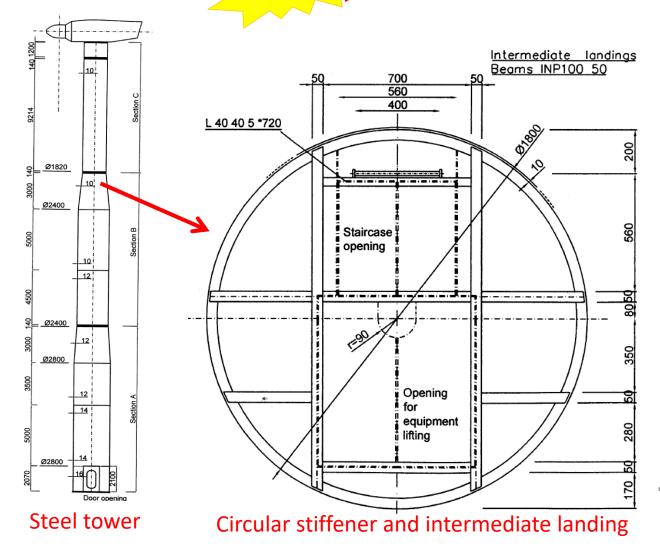


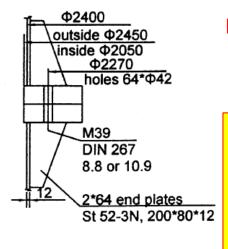






## TOWERS: Steel, Concrete or both of them?





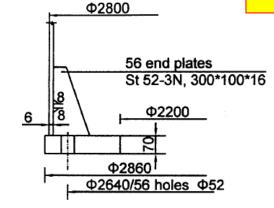
#### Flange connections

Intermediate
L - flange ring connection

# What can we do to avoid buckling

L - flange ring connection at tower base

(a)



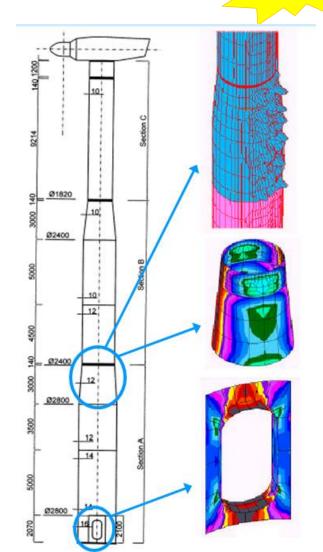




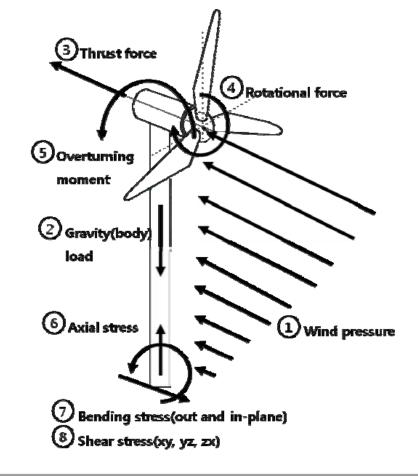
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# TOWERS: Steel, Concrete or both of them?



of nonlinear The purpose buckling analysis is to find the load at which the buckling occurs. The buckling speed is calculated backwards by matching the sum of the main horizontal forces, i.e. the wind pressure force and rotor thrust, to the buckling limit load.

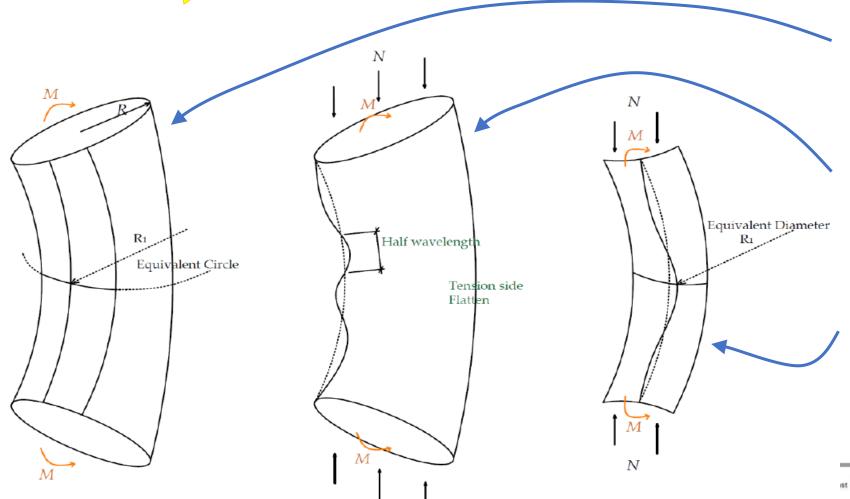








## TOWERS: Steel, Concrete or both of them?



Section ovalisation of a bending cylindrical shell;

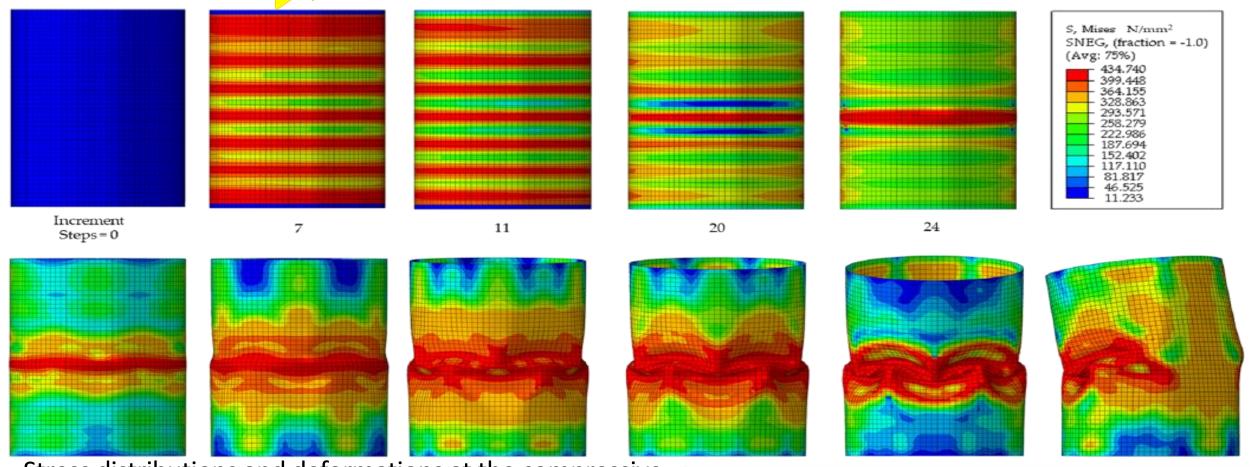
The resulted effect of combined loads;

The strip of a highly stressed shell segment.





# TOWERS: Steel, Concrete or both of them?



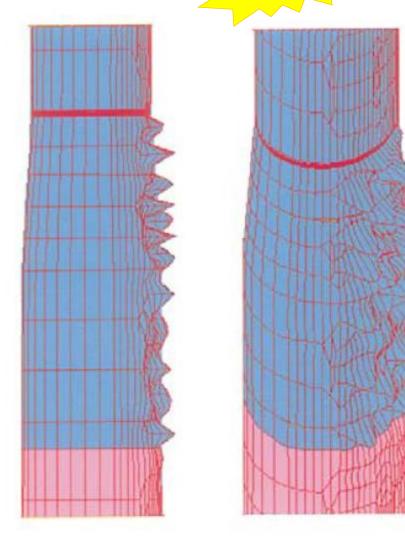
Stress distributions and deformations at the compressive side of a cylindrical shell at 10 increment steps.







## TOWERS: Steel, Concrete or both of them?



$$[K]\{\phi_i\} = \lambda_i[S]\{\phi_i\}$$

[K] = Structural stiffness matrix

 $\{\phi_i\}$  = Eigen vector

 $\lambda_i$  = Eigen value

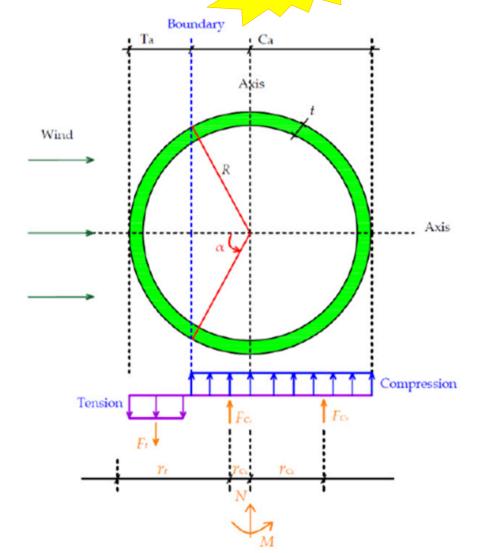
 ${S} = Stress stiffness matrix$ 

Note: It is well known that it is very difficult or sometimes even impossible to manufacture thin cylindrical shells without imperfections. Hence to obtain accurate result from numerical analysis is necessary to know about the exact shape and size of the imperfections which in turn depends on manufacturing process. As the amplitude of imperfections increases the buckling pressure decreases.









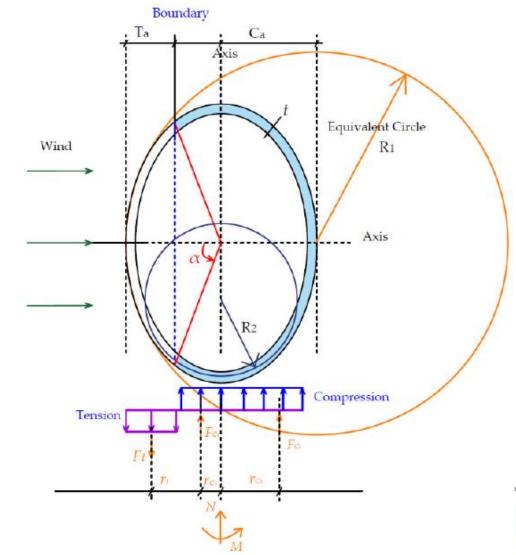
$$N = F_{C_1} + F_{C_2} - F_t = F_y \left( AreaC_A - AreaT_A \right)$$
$$= F_y \left( 2(\pi - \alpha)rt - 2\alpha rt \right)$$
$$= F_y \left[ 2rt(\pi - 2\alpha) \right]$$

$$\begin{split} M &= F_{t} \left( r_{t} + r_{C_{1}} \right) - F_{C_{1}} r_{C_{1}} + F_{C_{2}} r_{C_{2}} \\ &= F_{y} \left\{ 2 \alpha r t \left( r_{t} + r_{C_{1}} \right) - \left[ 2 \left( \frac{\pi}{2} - \alpha \right) r_{t} r_{C_{1}} \right] + \left( \pi r t \cdot r_{C_{2}} \right) \right\} \end{split}$$

$$\begin{split} N &= F_{C_1} + F_{C_2} - F_t = F_y \left( AreaC_A - AreaT_A \right) \\ &= F_y \Big[ 2\pi R_2 \left( \pi - 2\alpha \right) t + 2\alpha R_1 t - 2\alpha R_1 t \Big] \\ &= F_y \Big[ 2\pi R_2 t \left( \pi - 2\alpha \right) \Big] \end{split}$$

$$M = F_{t}(r_{t} + r_{c_{1}}) - F_{c_{1}}r_{c_{1}} + F_{c_{2}}r_{c_{2}}$$

$$= F_{y}\left\{2\alpha R_{1}t(r_{t} + r_{c_{1}}) - \left[R_{2}(\pi - \alpha)t(r_{c_{1}})\right] + (\pi rt \cdot r_{c_{2}})\right\}$$



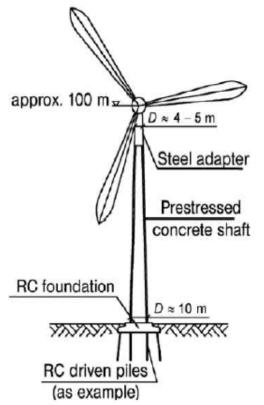
## TOWERS: Steel, Concrete or both of them?

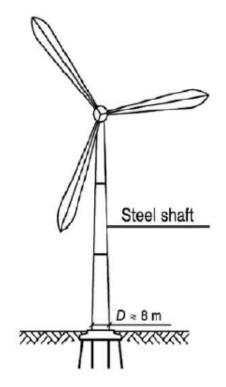
The consequence of taller wind towers is the need to increase the structural strength and stiffness required to carry both increased turbine weight and bending forces under wind action on the rotors and the tower, and to avoid damaging resonance from excitation by forcing frequencies associated with the rotor and blades passing the tower. In turn this will require larger cross sectional diameters, which may introduce significant transportation problems, bearing in mind that 4.5m is the practical limit for the diameter of complete ring sections that can be transported along the public highway.

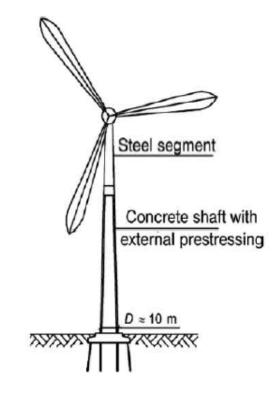
Concrete towers can accommodate these requirements and also offer a range of associated benefits.

# TOWERS: Steel, Concrete or both of them?









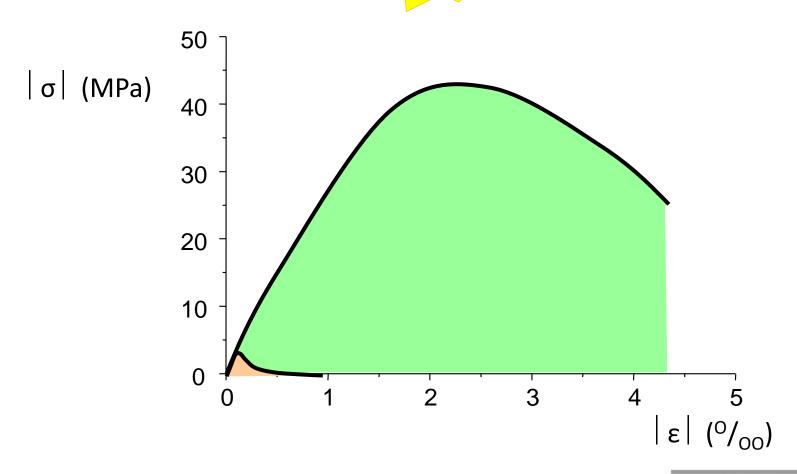
Transportation of wind turbine tower's segments







## TOWERS: Steel, Concrete or both of them?



- Tension and compression –
   Different behavior
- Softening

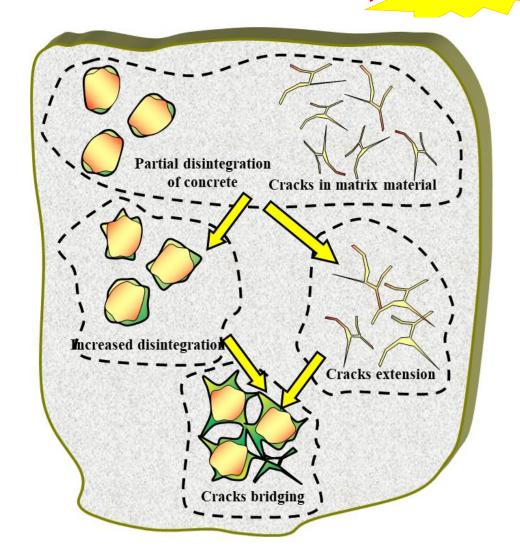
Nonlinear behavior of concrete

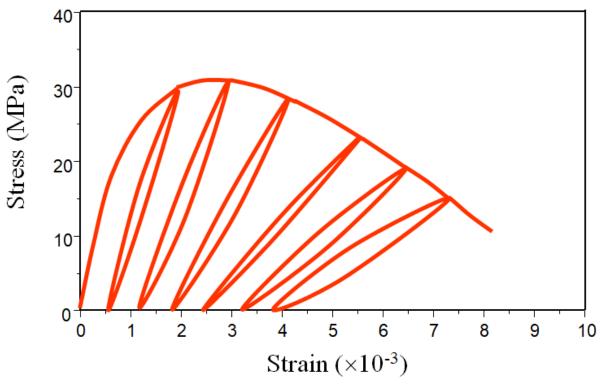






## TOWERS: Steel, Concrete or both of them?





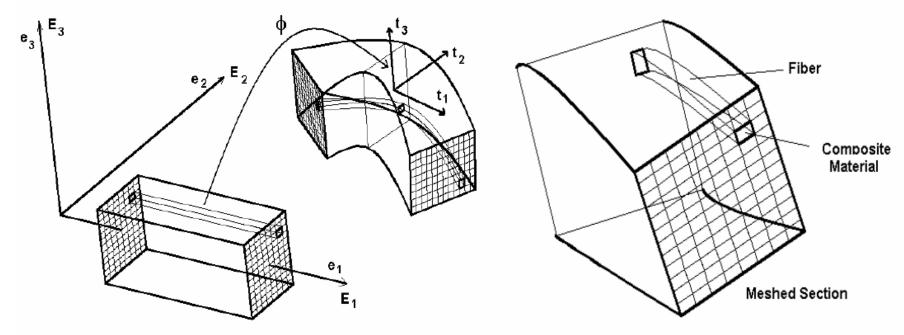
## Nonlinear behavior of concrete







## TOWERS: Steel, Concrete or both of them?



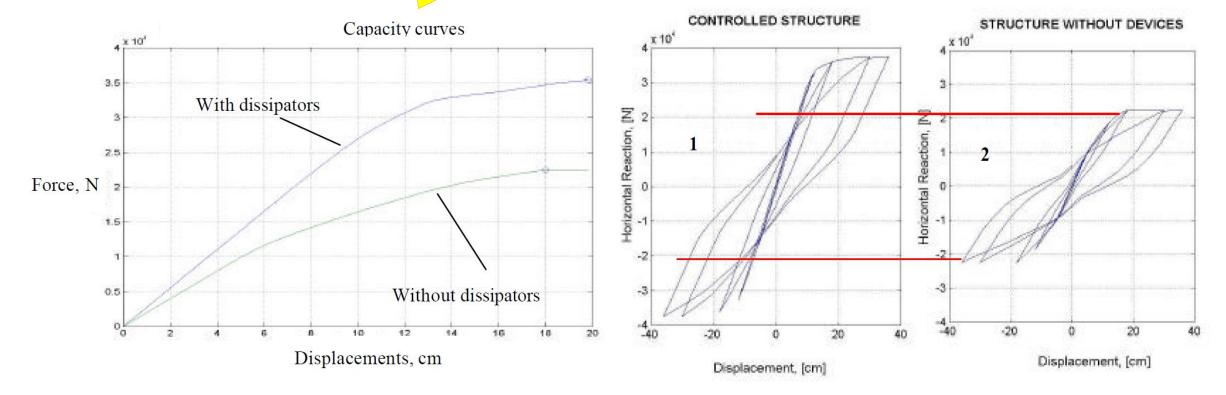
## Nonlinear behavior of concrete: fiber modeling







## TOWERS: Steel, Concrete or both of them?



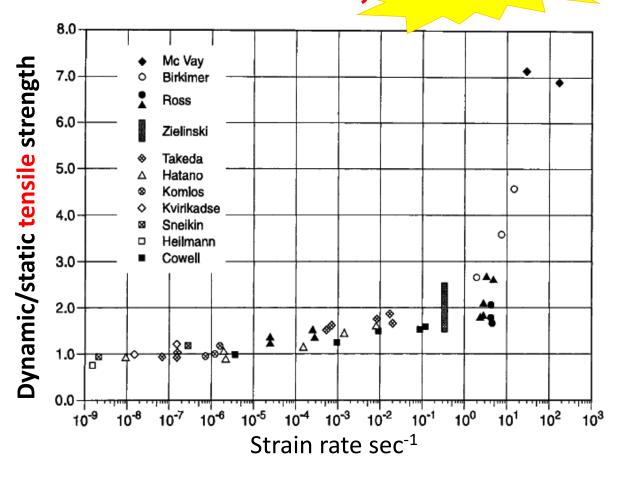
## Concrete towers under monotonic and cyclic loading

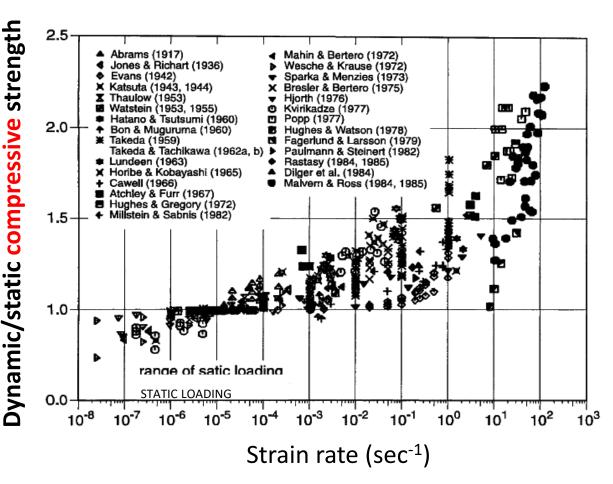






## TOWERS: Steel, Concrete or both of them?





**Strain rate effects** 







## TOWERS: static loads (self-weight, wind loads, e.t.c.)

Two sets of static loads can be considered:

- (a) The pseudostatic aerodynamic loads under survival and operational conditions. The concentrated aerodynamic loads at the elevation of the power transmission axis, are due to the wind resistance and/or operation of the runner. In addition, aerodynamic loads distributed along the body of the tower itself should be computed and be accounted for in the analysis under survival conditions. Aerodynamic loads under survival, shut down, conditions have a recurrence period of 50 years.
- (b) The second set of static loads due to gravity, consists of a concentrated load at the top of the tower representing the weight of the nacelle, runner, generator, gear box, etc. the tower itself distributed along its height (78,500 N/m³, specific weight of steel, or, 25,000 N/m³, specific weight of concrete).

The safety factors for the static loads are specified as [1]:

Favorable gravity loads: 1.00

Unfavorable gravity loads: 1.35

Aerodynamic loads: 1.50







#### **TOWERS:** seismic loads

**EUROPÄISCHE NORM** 

June 2005

ICS 91.120.25

Supersedes ENV 1998-3:1996

English version

Eurocode 8: Design of structures for earthquake resistance - Part 6: Towers, masts and chimneys

Eurocode 8: Calcul des structures pour leur résistance aux séismes - Partie 6 : Tours, mâts et cheminées Eurocode 8: Auslegung von Bauwerken gegen Erdbeben -Teil 6: Türme, Maste und Schornsteine

Teil 6: Türme, Maste und Schornsteil

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#### **TOWERS:** seismic loads

The seismic loads are in accordance with the specifications of a Seismic Code where the design seismic motion has a 10% likelihood of being exceeded during a period of 50 years. For the case of EC8, the corresponding elastic design spectrum of horiz. acceleration  $S_d(T)$  is defined as:

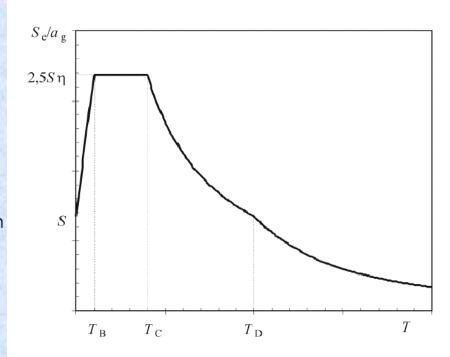
$$0 \le T \le T_{\rm B}: S_{\rm d}(T) = a_{\rm g} \cdot S \cdot \left[ \frac{2}{3} + \frac{T}{T_{\rm B}} \cdot \left( \frac{2.5}{q} - \frac{2}{3} \right) \right]$$

$$T_{\rm B} \le T \le T_{\rm C}$$
:  $S_{\rm d}(T) = a_{\rm g} \cdot S \cdot \frac{2.5}{q}$ 

$$T_{\rm C} \le T \le T_{\rm D} : S_{\rm d}(T) \begin{cases} = a_{\rm g} \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_{\rm C}}{T}\right] \\ \ge \beta \cdot a_{\rm g} \end{cases}$$

$$T_{\rm D} \le T : S_{\rm d}(T) \begin{cases} = a_{\rm g} \cdot S \cdot \frac{2.5}{q} \cdot \left[ \frac{T_{\rm C} T_{\rm D}}{T^2} \right] \\ \ge \beta \cdot a_{\rm g} \end{cases}$$

S <sub>d</sub> (T)	design spectrum
T	vibration period of a linear SDOF system
a <sub>g</sub>	design ground acceleration on type A ground $(a_g = \gamma_I.a_{gR})$
T <sub>B</sub> ,T <sub>C</sub>	limits of the constant spectral acceleration branch
T <sub>D</sub>	value defining the beginning of the constant displacement response range of the spectrum
S	soil factor
q	behaviour factor
η	damping correction factor with reference value $\eta = 1$ for 5% viscous damping
β	lower bound factor for the horizontal design spectrum. Recommended value: $\beta$ =0,2



The EC8 response spectrum

#### **TOWERS:** seismic loads

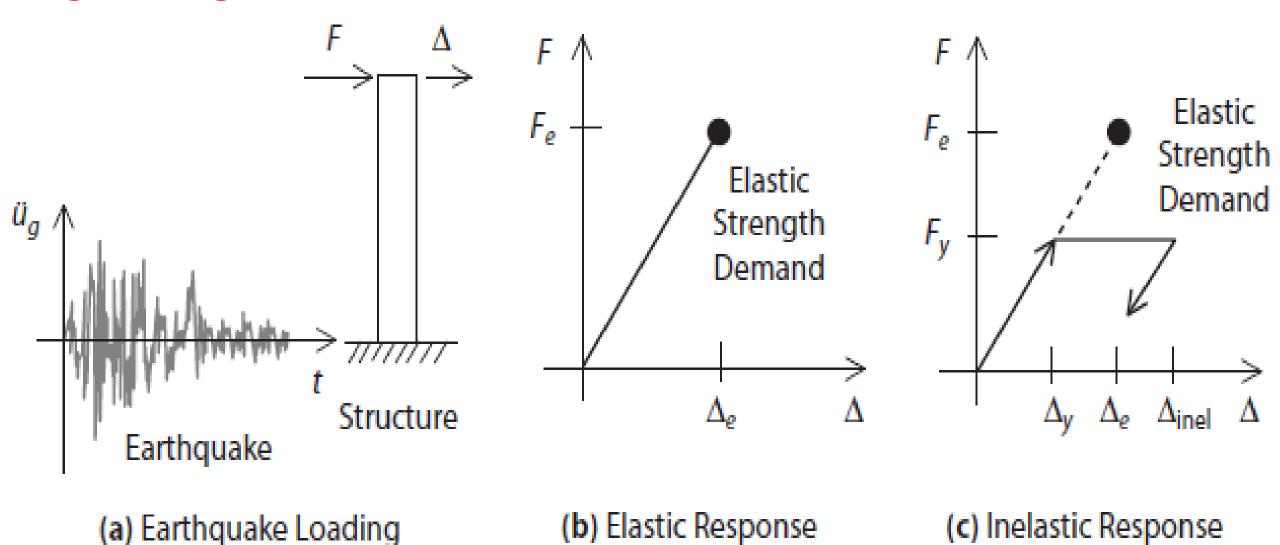
Table 4.1 Importance classes for towers, masts and chimneys

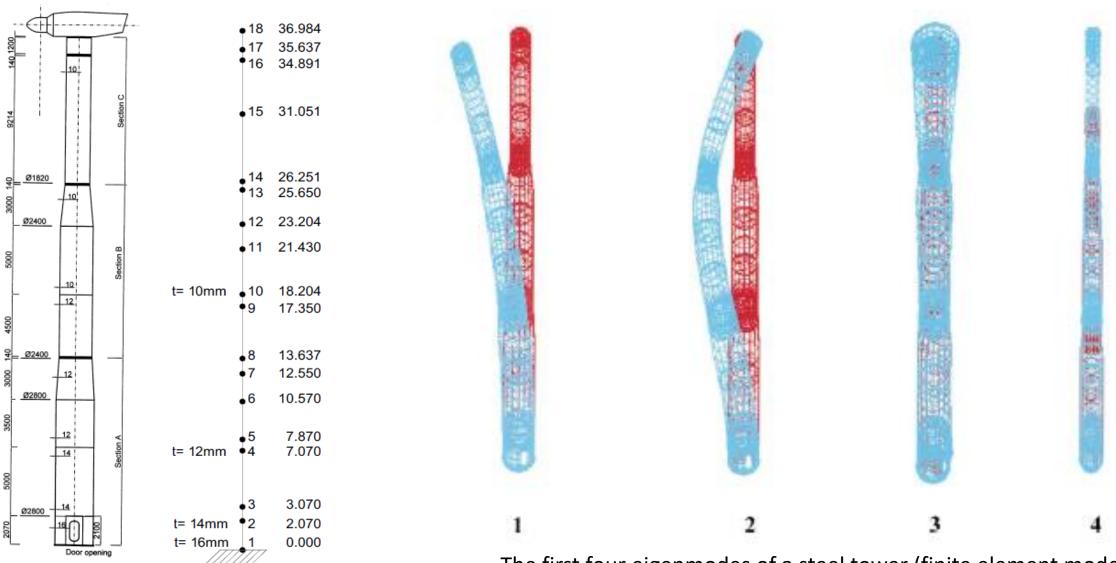
Importance class		
I	Tower, mast or chimney of minor importance for public safety	
II	Tower, mast or chimney not belonging in classes I, III or IV	
III	Tower, mast or chimney whose collapse may affect surrounding buildings or areas likely to be crowded with people.	
IV	Towers, masts or chimneys whose integrity is of vital importance	
	to maintain operational civil protection services (water supply systems, an electrical power plants, telecommunications, hospitals).	



Importance factor  $\gamma_{IV}$ =1.40 (i.e. seismic forces are 40% than that of 'regular' structures.

#### **TOWERS**





Tower

Simplified concentrated masses model for seismic analysis

The first four eigenmodes of a steel tower (finite element model)

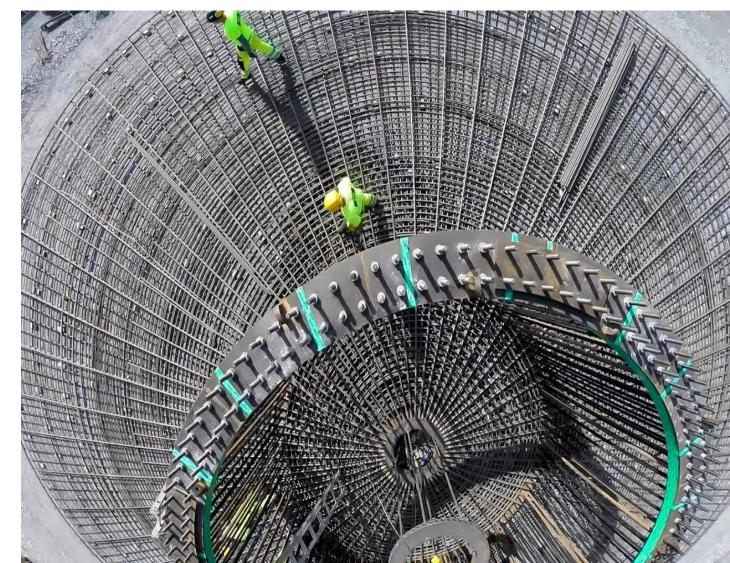




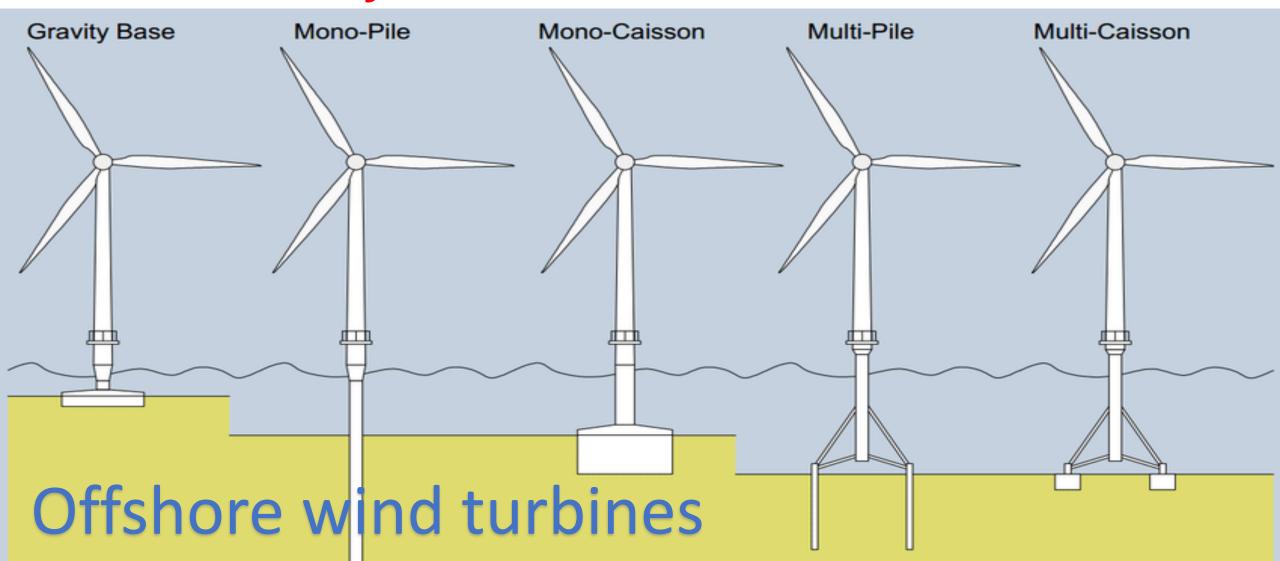


**Inelastic Analysis of Foundation Structures** 

The earthquake, wind or wave (for offshore) wind turbines response of shallow and deep foundations is having a significant importance due to its complex behavior because of the semi-infinite soil media. The nonlinear response of wind turbines resting on this improved foundation model can be analyzed by assuming that the foundation resists compression and tension. In reality, soil is weak in tension and its tension capacity needs to be neglected, which leads to liftoff regions at different locations. This phenomenon becomes much complicated by considering the inelastic soil structure behavior, which leads to a highly nonlinear problem.

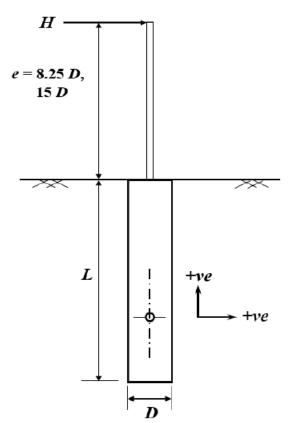


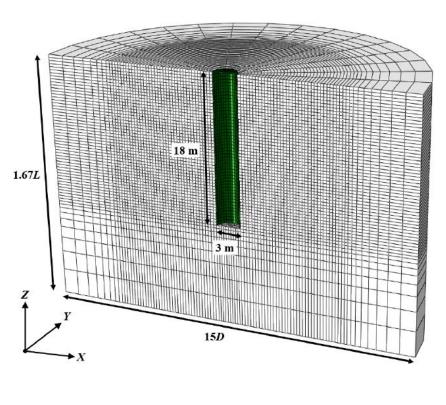
## **Inelastic Analysis of Foundation Structures**



**Inelastic Analysis of Foundation Structures** 

Case study





The schematic of the

Finite element mesh

Offshore wind turbines base

Mono-Pile







## **Inelastic Analysis of Foundation Structures**

Description	Constitutive Equation	Soil Parameters
Relative density index	$I_R = I_D(Q - \ln p') - R$	$I_D = \frac{D_r(\%)}{100}, R = 1 [24], Q = 7.4 + 0.6 \ln(\sigma'_c) \&$ $7.4 \le Q \le 10 [25], \sigma'_c = p' \left(1 - \frac{2\sin\phi'_c}{3 - \sin\phi'_c}\right)$
Peak friction angle	$\phi_p' = \phi_c' + A_\psi I_R$	$\phi_c', A_\psi$
Peak dilation angle	$\psi_p = \left(rac{\phi_p' - \phi_c'}{k_\psi} ight)$	$k_{\psi}$
Strain softening parameter	$\gamma_c^p = C_1 + C_2 I_D$	$C_1, C_2$
Plastic strain at $\phi_p'$	$\gamma_p^p = \gamma_c^p \left(\frac{p'}{p'_a}\right)^m$	$p_a', m$







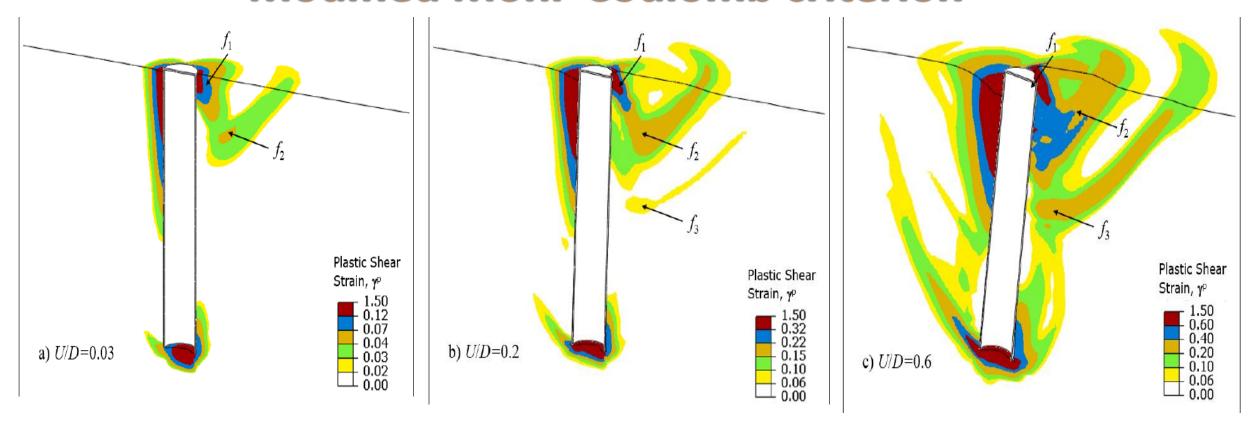
## **Inelastic Analysis of Foundation Structures**

Description	Constitutive Equation	Soil Parameters
Mobilized friction angle at zone-II	$\phi' = \phi'_{in} + \sin^{-1} \left[ \left( \frac{2\sqrt{\gamma^p \gamma_p^p}}{\gamma^p + \gamma_p^p} \right) \sin(\phi'_p - \phi'_{in}) \right]$	50 A
Mobilized dilation angle at Zone-II	$\psi = \sin^{-1} \left[ \left( \frac{2\sqrt{\gamma^p \gamma_p^p}}{\gamma^p + \gamma_p^p} \right) \sin(\psi_p) \right]$	Zone II Zone II Pre-yield Post-peak softening  C  Zone II Zone II Pre-yield Post-peak softening  C
Mobilized friction angle at zone-III	$\phi' = \phi'_c + \exp\left[-\left(\frac{\gamma^p - \gamma_p^p}{\gamma_c^p}\right)^2\right](\phi'_p - \phi'_c)$	Pre-yield Post-peak softening  C $ \begin{array}{cccccccccccccccccccccccccccccccccc$
Mobilized dilation angle at Zone-III	$\psi = \exp\left[-\left(rac{\gamma^p - \gamma_p^p}{\gamma_c^p} ight)^2 ight]\psi_p$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

**Symbols**:  $A_{\psi}$ : slope of  $(\phi'_p - \phi'_c)$  vs.  $I_R$ ;  $m, C_1, C_2$ : soil parameters;  $I_R$ : relative density index;  $k_{\psi}$ : slope of  $(\phi'_p - \phi'_c)$  vs.  $\psi_p$ ;  $\phi'_{in}$ :  $\phi'$  at the start of plastic deformation;  $\phi'_c$ : critical state friction angle;  $\gamma^p$ : engineering plastic shear strain

## **Inelastic Analysis of Foundation Structures**

Modified Mohr-Coulomb criterion



U/D: Normalized maximum displacement

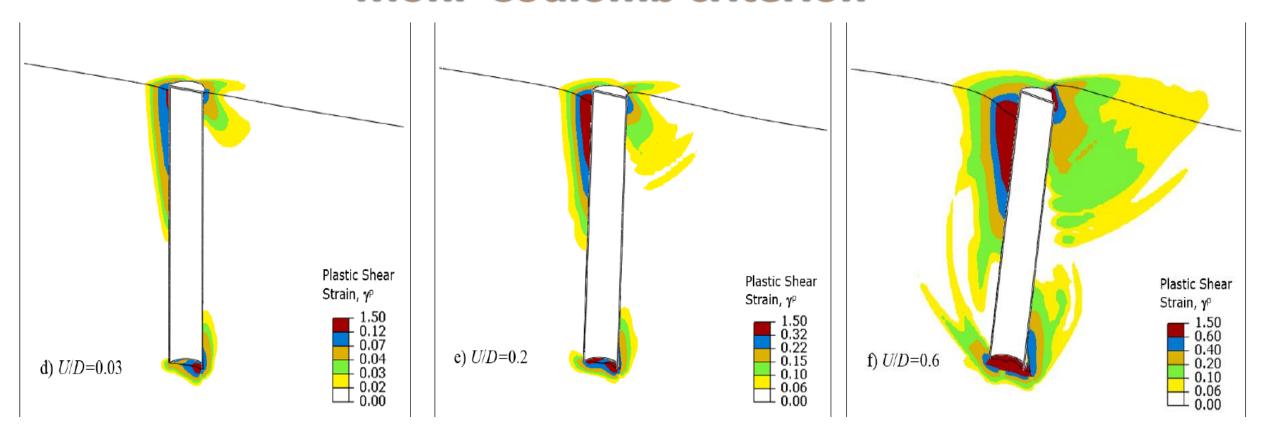






## Inelastic Analysis of Foundation Structures

Mohr-Coulomb criterion



U/D: Normalized maximum displacement







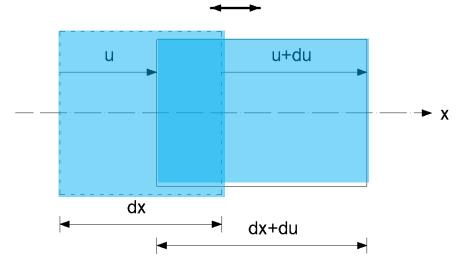
## Brief description of critical parts behave nonlinearly

## Seismic Inelastic Behavior of Soil

#### **Wave propagation**

- Principal or volume waves P
- Motion parallel to wave propagation.
- Compression or decompression:  $\varepsilon = \frac{\partial u}{\partial x}$

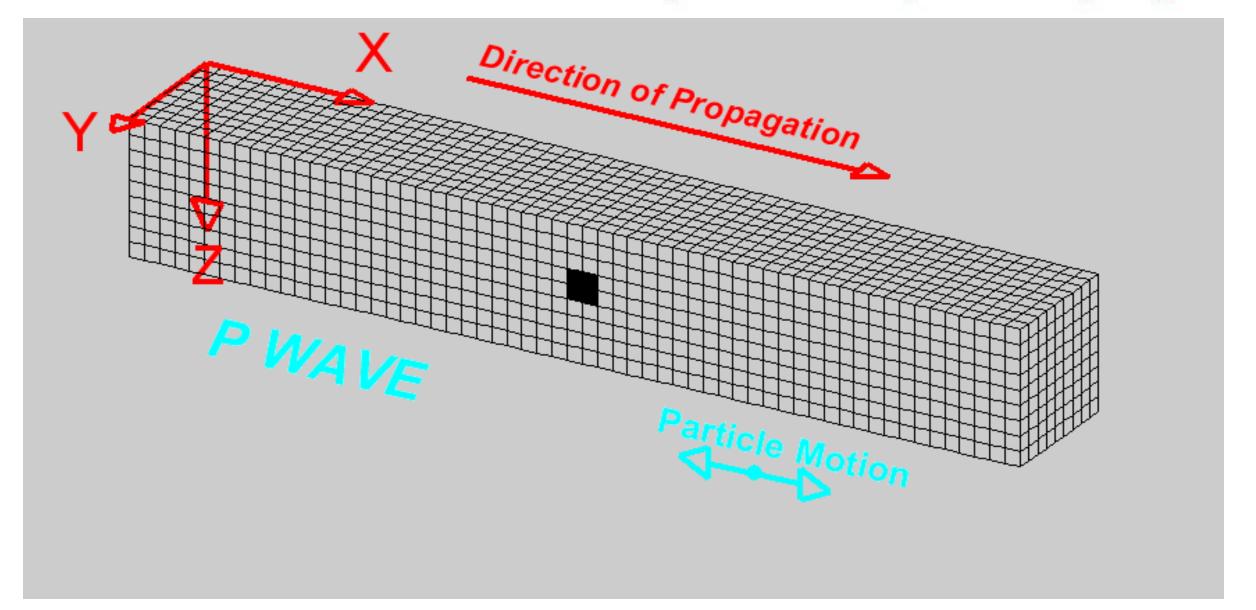
• Velocity of propagation: 
$$C = C_p = V_p = \alpha = \sqrt{\frac{\lambda + 2G}{\rho}}$$















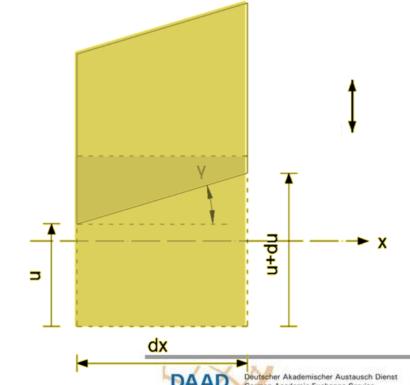
Transverse or shear waves S

Transverse motion vs. wave propagation.

Shear stresses and strains: 
$$\gamma = \frac{\partial u}{\partial x}$$

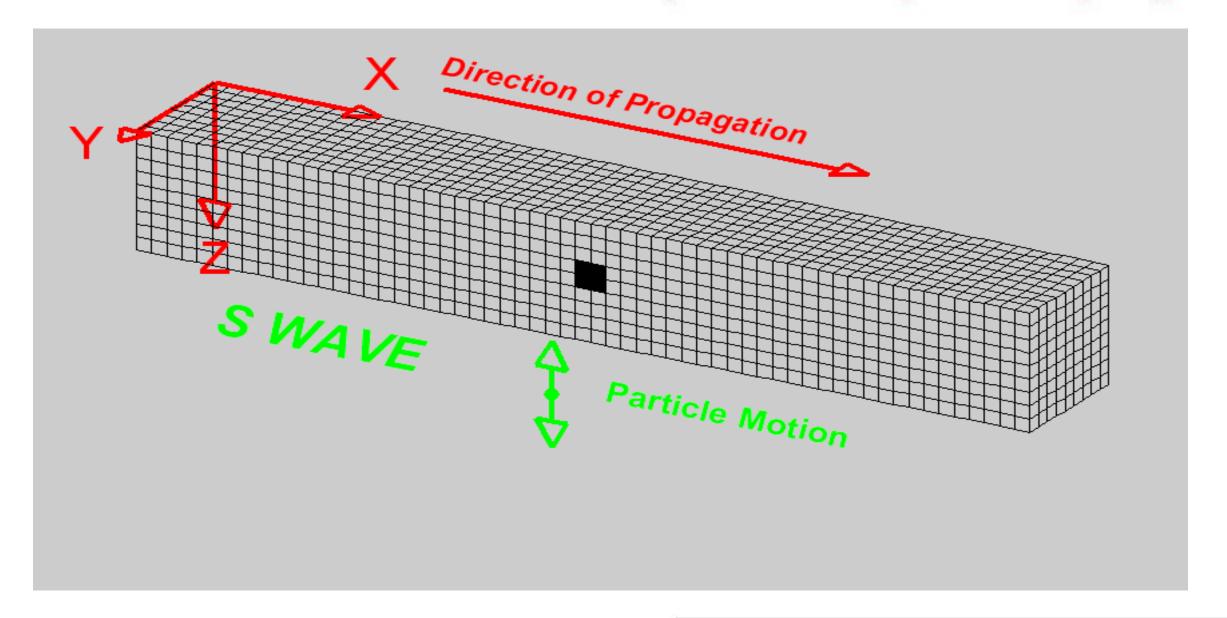
Propagation velocity: 
$$C = C_s = V_s = \beta = \sqrt{\frac{G}{\rho}}$$

#### **Wave propagation**















#### One has:

$$C = V = \sqrt{\frac{N}{\rho}}$$

where:

$$N = D = \lambda + 2G$$

P-wave

$$N = G$$

S-wave

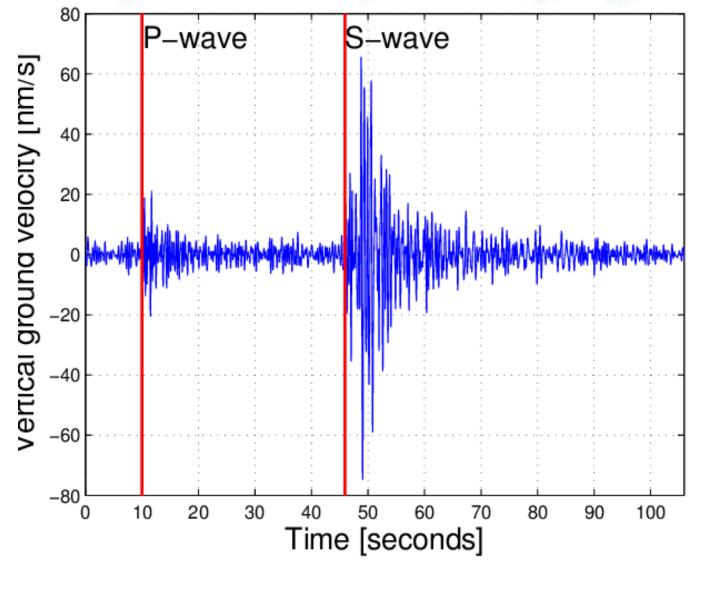
#### Furthermore:

$$\sigma = (\lambda + 2G) \cdot \varepsilon$$
 P-wave

$$\tau = G \cdot \gamma$$

S-wave

Earthquake with P-wave and S-wave arrivals record







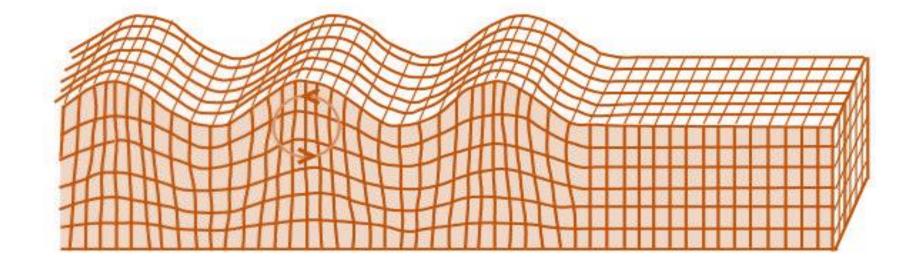


#### **Rayleigh Surface Waves**

They are created and spread on the surface of the earth. Wave propagation

The motion of the particles is elliptical in the direction of propagation.

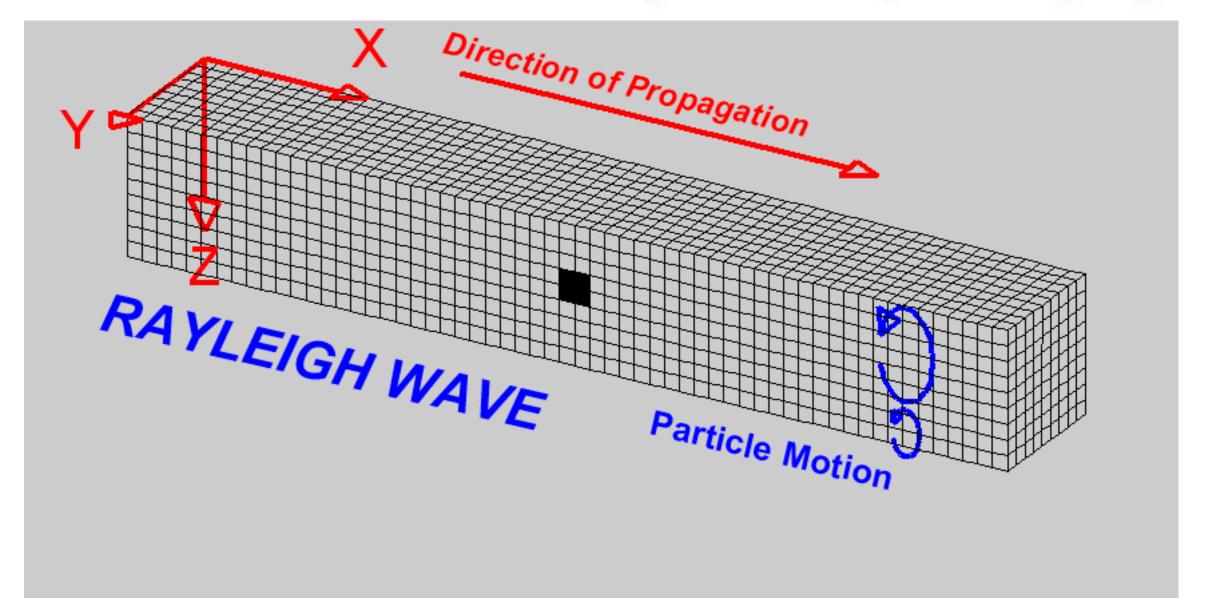
The motion of the particles decreases with depth.

















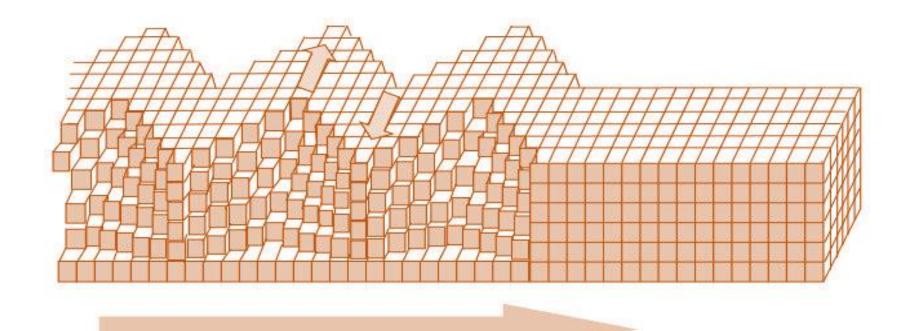
**Love Waves** 

They are created and spread on the surface of the earth.

The motion of the particles is horizontal/shear.

The motion of the particles decreases with depth.

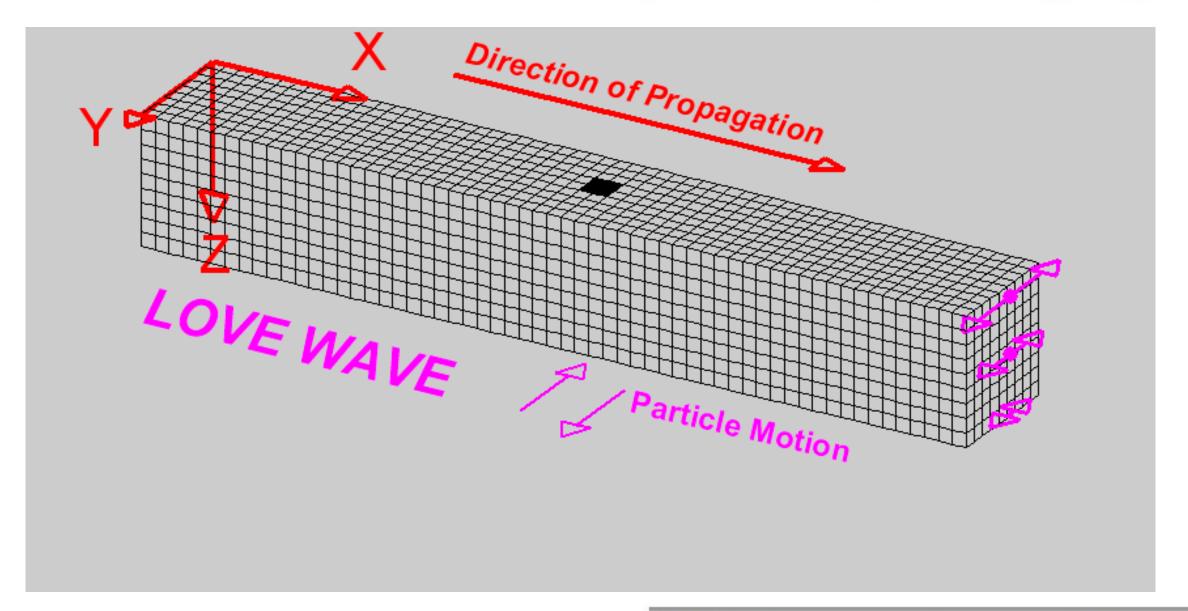
**Wave propagation** 









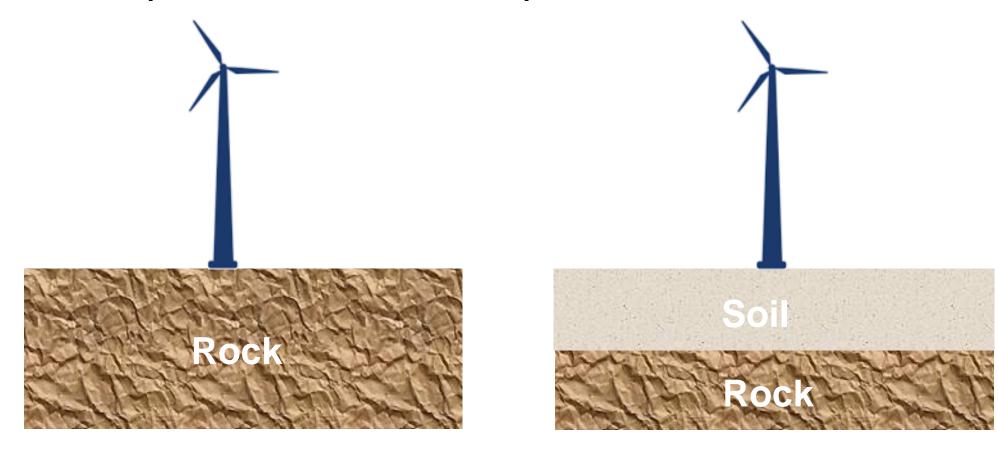








#### How does the presence of soil affect the response of the tower?

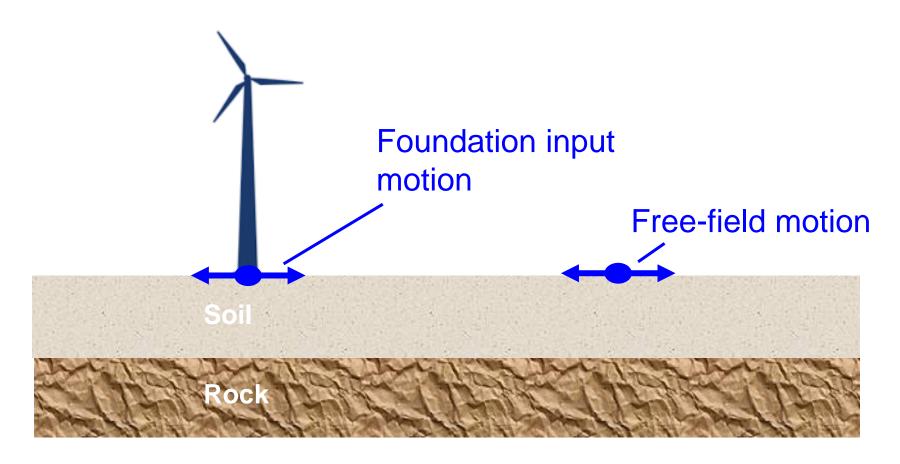


Does the structure founded on rock respond differently than when founded on soil?









How does the motion at the base of the tower differ from the free-field motion?







In reality, the response of the soil affects the response of the tower, and the response of the tower affects the response of the soil



#### Kinematic interaction

Presence of stiff foundation elements on or in soil cause foundation motions to deviate from free-field motions.

#### Inertial interaction

Inertial response of tower causes base shear and moments which cause displacements of foundation relative to free-field.







#### **Soil-Tower Interaction (STI)**

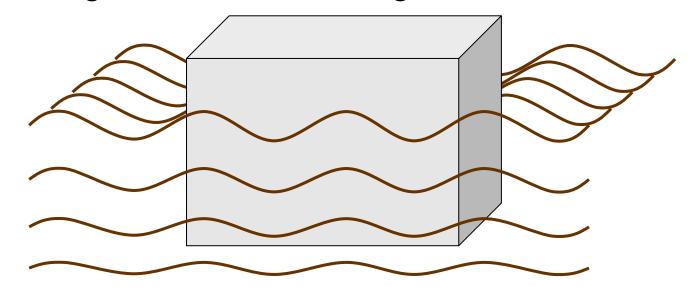
#### **Kinematic STI has three primary causes:**

Base slab averaging – results from stiffness of foundation

Embedment – reduction of ground motion with depth

Wave scattering – scattering off corners and edges

Ground motion amplitude decreases with depth







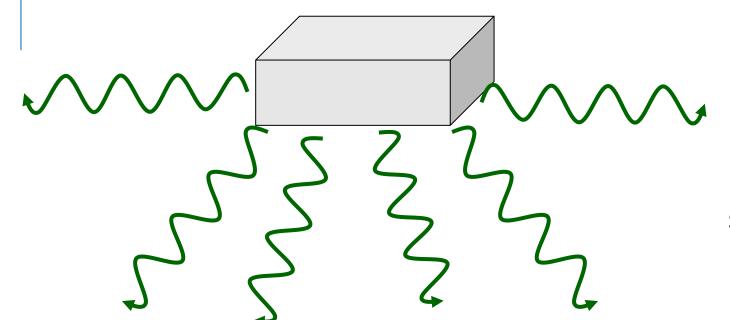


#### Soil-Tower Interaction (STI)

#### Inertial STI results from compliance of soil

Soil is not rigid – will deform due to loads from structure

Deformations resulting from structural forces will propagate away from structure



Energy "removed" from structure – radiation damping





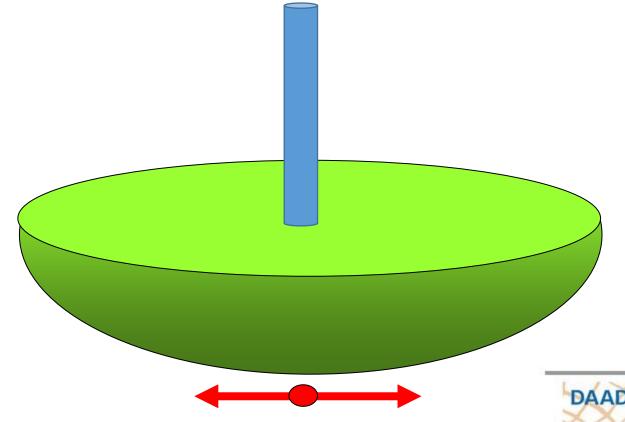


#### Soil-Tower Interaction (STI)

#### **Analysis of soil-tower interaction**

#### Two approaches

Direct approach – model soil and tower together



Requires detailed model of tower and soil in one computer program

Can handle nonlinear soil and tower responses







Can use different

codes for soil and

tower response

#### **Soil-Tower Interaction**

Analysis of soil-tower interaction

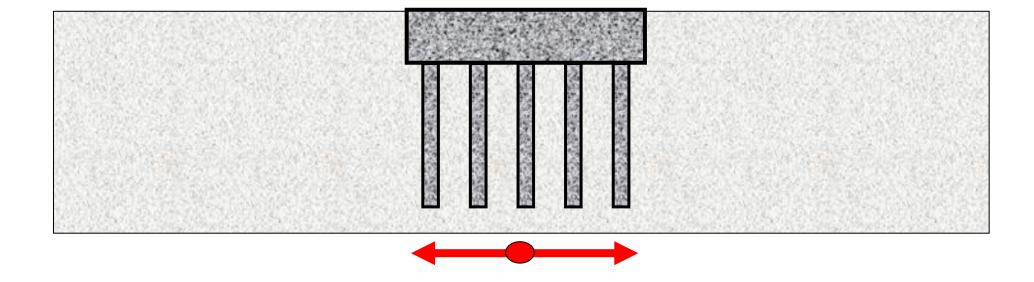
#### Two approaches

Direct approach – model soil and tower together

Substructure approach – model separately and combine

# 

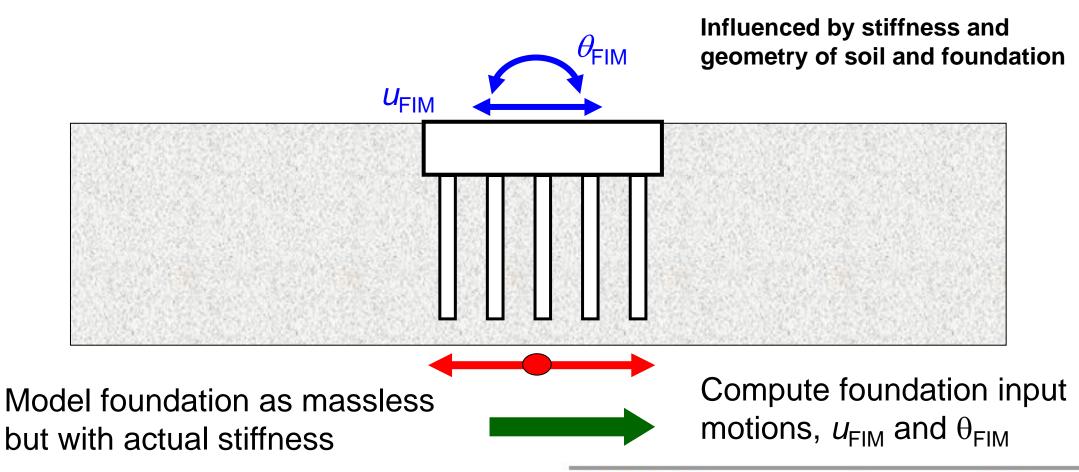
#### Analysis of kinematic soil-tower interaction







#### Analysis of kinematic soil-tower interaction

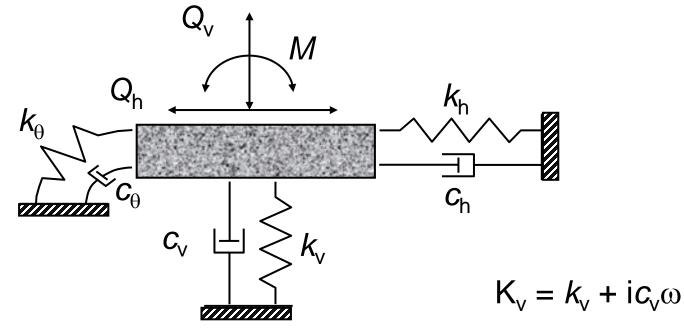








#### Impedance function – foundation stiffness and damping



#### 6 x 6 matrix of complex impedance coefficients

- 3 translational coefficients
- 3 rotational coefficients
- **Cross-coupling (off-diagonal) coefficients**



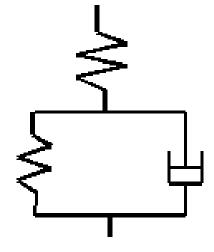




## Nonlinear soil behavior

#### Three elements viscoelastic model

(Voigt model + linear elastic spring)



- Kondner, R. L., & Ho, M. M. (1965). Viscoelastic response of a cohesive soil in the frequency domain. Transactions of the Society of Rheology, 9(2), 329-342.
- Cox, W. R., Reese, L. C., & Grubbs, B. R. (1974, January). Field testing of laterally loaded piles in sand. In Offshore Technology Conference. Offshore Technology Conference.
- Oka, F., Kodaka, T., & Kim, Y. S. (2004). A cyclic viscoelastic—viscoplastic constitutive model for clay and liquefaction analysis of multi-layered ground. International journal for numerical and analytical methods in geomechanics, 28(2), 131-179.

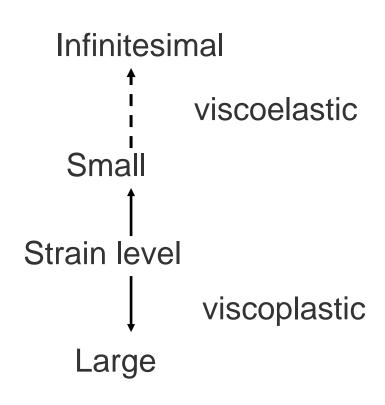


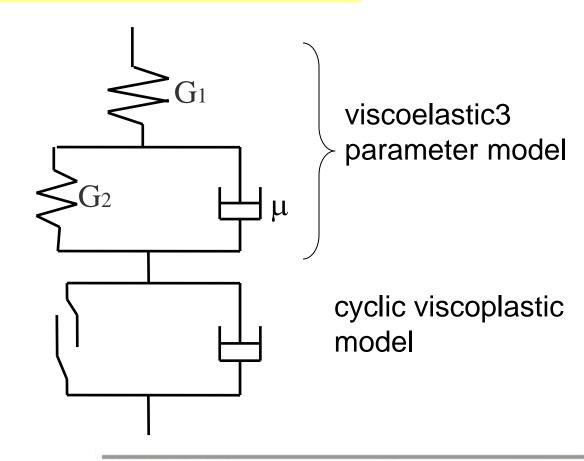




## 3. Cyclic Viscoelastic-Viscoplstic Model

### **Viscoelastic - Viscoplastic Model**



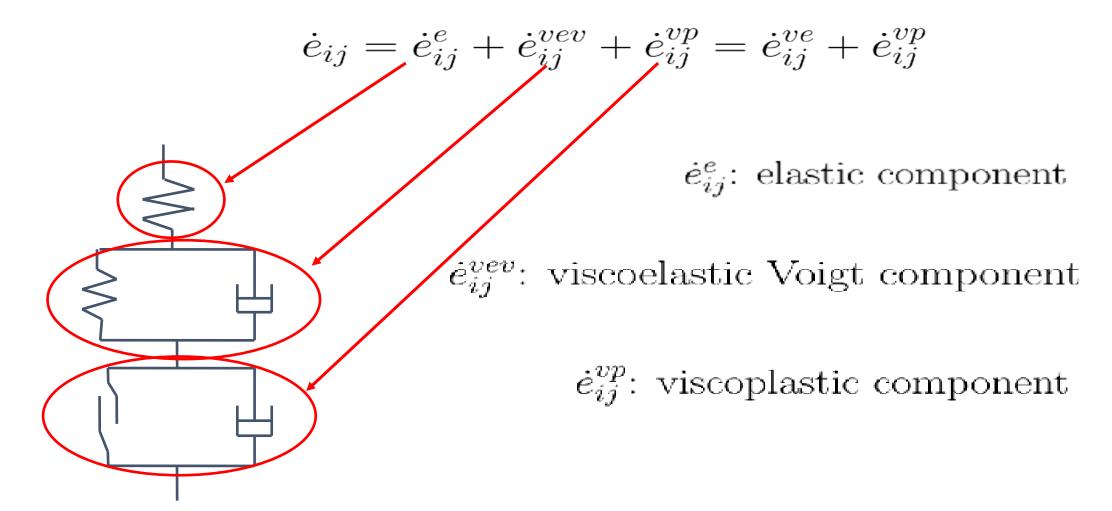








## Deviatric strain rate tensor: $\dot{e}_{ij}$







### **Elastic Component**

$$\dot{e}_{ij}^e = \frac{1}{2G_1} \dot{S}_{ij}$$

where

 $G_1$ : first elastic shear modulus

 $\dot{S}_{ij}$ : deviatoric stress rate tensor

## **Voigt Viscoelastic Component**

$$\dot{e}_{ij}^{vev} = \frac{1}{\mu} (S_{ij} - 2G_2 \cdot e_{ij}^{vev})$$
 where  $\mu$ : viscosity coefficient

where

 $G_2$ : second elastic shear modulus of Voigt element

 $S_{ij}$ : deviatoric stress tensor

## 3 Elements Viscoelastic Component

$$\dot{e}_{ij}^{ve} = \frac{1}{2G_1} \dot{S}_{ij} + \frac{1}{u} (S_{ij} - 2G_2 \cdot e_{ij}^{vev})$$







## A Proposed Cyclic Viscoelastic and Viscoplastic Constitutive Model

$$\dot{\varepsilon}_{ij} = \frac{1}{2G_1} \dot{S}_{ij} + \frac{1}{\mu} \left( S_{ij} - 2G_2 \cdot e_{ij}^{vev} \right) + \frac{\kappa}{3(1+e)} \frac{\dot{\sigma}_m'}{\sigma_m'} \delta_{ij} 
+ C_{01} \frac{\langle \Phi_1'(F) \rangle}{\sigma_m'} \frac{\left( \eta_{ij}^* - \chi_{ij}^* \right)}{\bar{\eta}_x^*} 
+ C_{02} \frac{\langle \Phi_1'(F) \rangle}{\sigma_m'} \{ \tilde{M}^* - \frac{\eta_{mn}^* \left( \eta_{mn}^* - \chi_{mn}^* \right)}{\bar{\eta}_x^*} \} \frac{1}{3} \delta_{ij}$$

Three elements viscoelastic model

+

Cyclic viscoplastic model







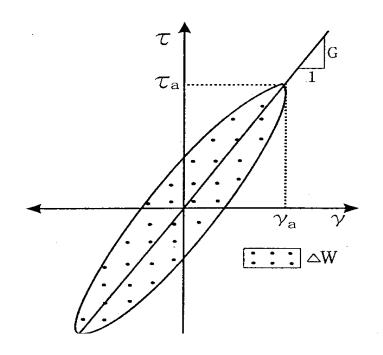
#### **Cyclic Triaxial Deformation Tests**

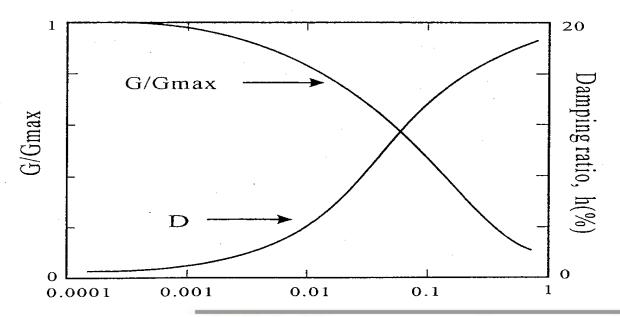
The equivalent shear modulus, G;

$$G = \frac{\tau_a}{\gamma_a}$$

The hysteretic or equivalent viscous damping ratio, D

$$D = \frac{1}{2\pi} \frac{\Delta W}{W} = \frac{1}{2\pi} \frac{\Delta W}{G\gamma_a^2}$$



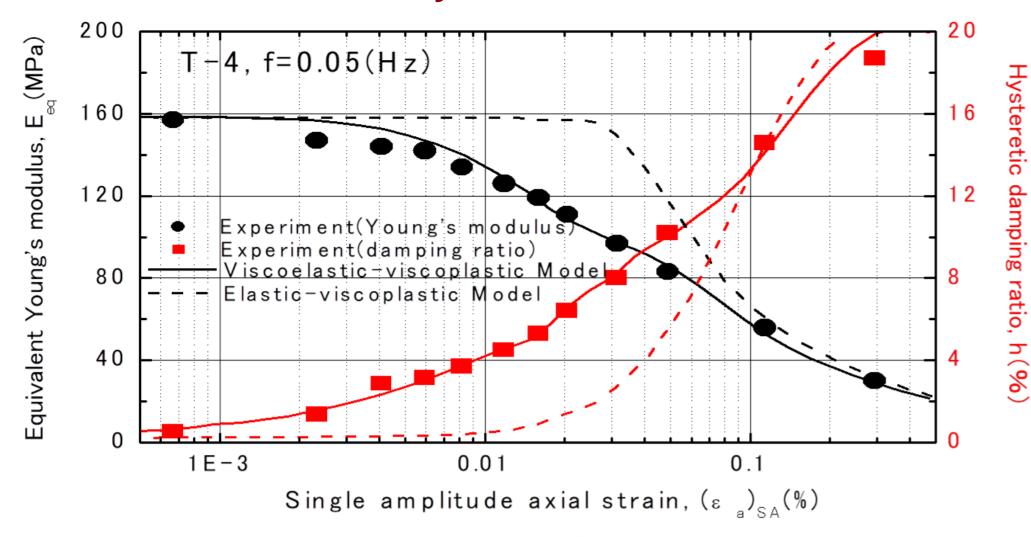








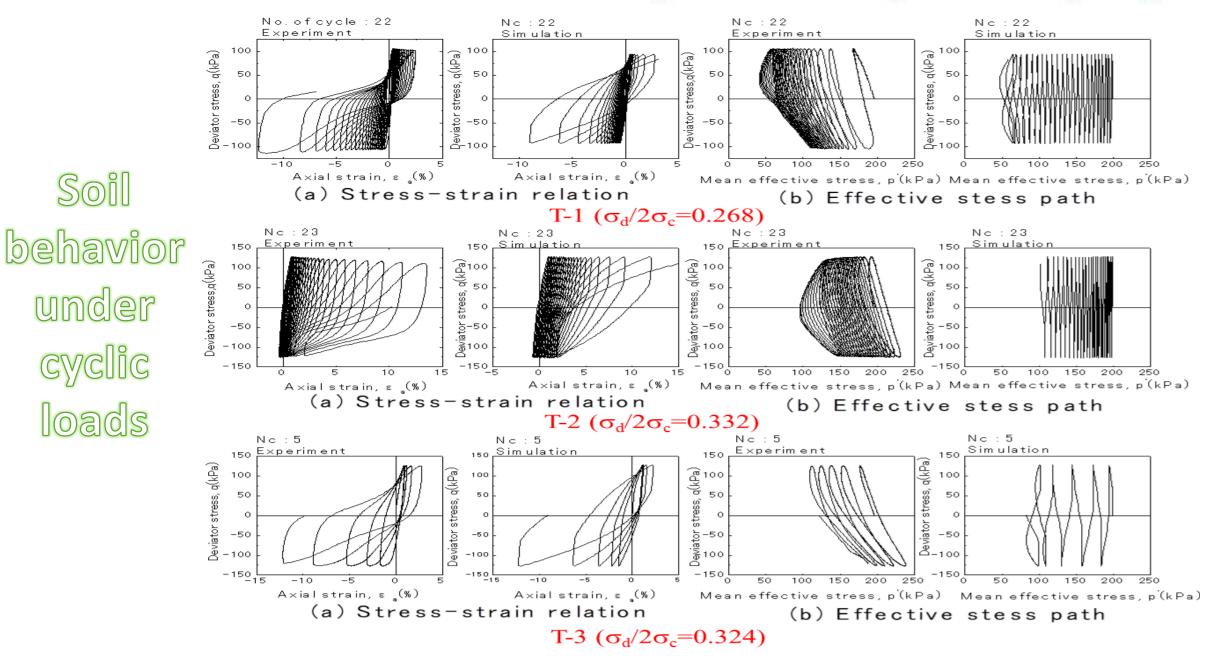
## The results of the Cyclic Triaxial Deformation Test









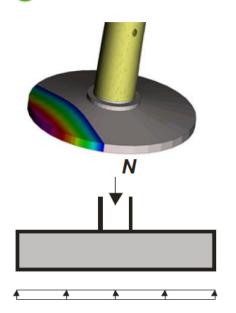


Soil

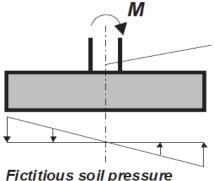
under

cyclic loads

## Soil behavior under large deformations



Soil pressure from symmetric load case



Fictitious soil pressure from asymmetric load case



#### **NONLINEAR BEHAVIOR OF BOLTED L-FLANGED CONNECTIONS**













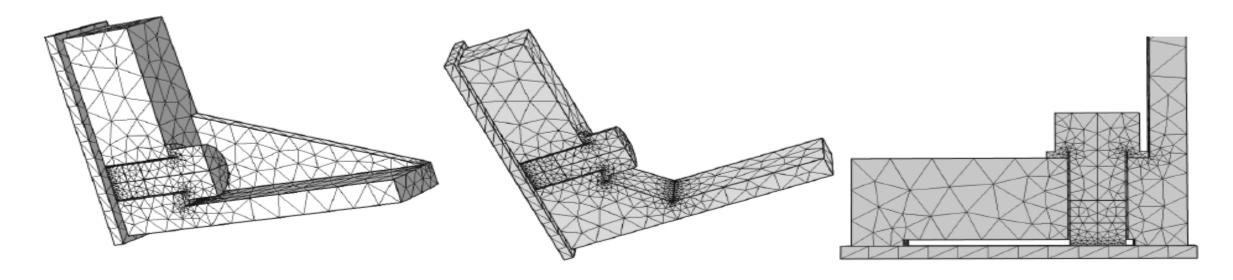


#### NONLINEAR BEHAVIOR OF BOLTED L-FLANGED CONNECTIONS

In wind turbine towers the preferred design is circular tubes that are connected to each other by a bolted flange joint.

The design is typically that of an L-flange resulting in an eccentrically loaded bolted connection.

The eccentricity results in a non-linear relationship between external load on the tower and the tensile force in the bolt.

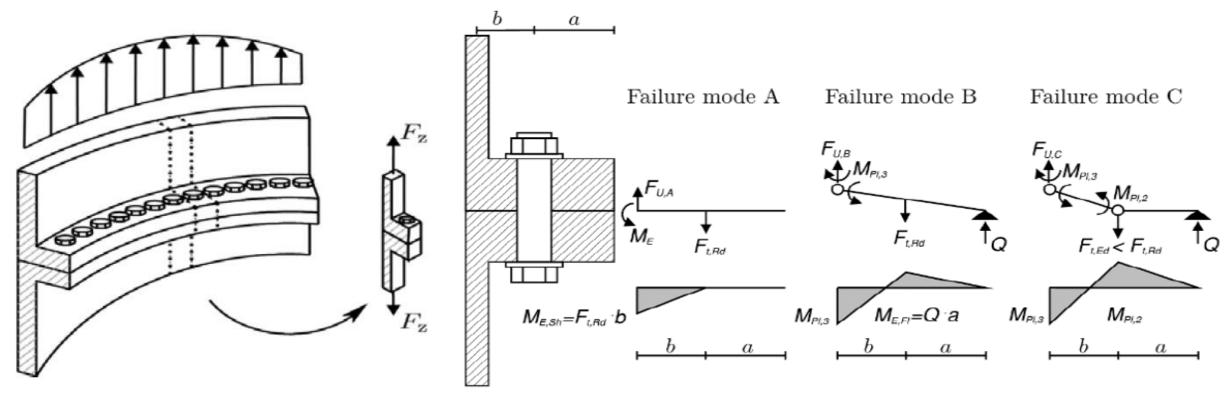








#### NONLINEAR BEHAVIOR OF BOLTED L-FLANGED CONNECTIONS



$$F_z = \frac{4 \cdot M_{Ed}}{n_b \cdot D_{sh,m}} + \frac{N_{Ed}}{n_b}$$

$$F_{U,A} = F_{t,Rd}$$

$$F_{z} = \frac{4 \cdot M_{Ed}}{n_b \cdot D_{sh,m}} + \frac{N_{Ed}}{n_b} \qquad F_{U,A} = F_{t,Rd} \qquad F_{U,A} = F_{t,Rd} \qquad F_{U,B} = \frac{F_{t,Rd} \cdot a + M_{pl,3}}{a + b} \\ F_{t,Rd} = \min \left( \frac{f_{y,bolt} \cdot A_s}{\gamma_{M2}}; \frac{0.9 \cdot f_{u,bolt} \cdot A_s}{\gamma_{M7}} \right) \qquad M_{E,Fl} \leq W_{E,Flange} \cdot f_{yd,Flange}$$

$$F_{U,B} = \frac{F_{t,Rd} \cdot a + M_{pl,3}}{a+b} \qquad F_{U,C} = \frac{M_{pl,2} + M_{pl,3}}{b}$$

$$M_{E,Fl} \leq W_{E,Flange} \cdot f_{yd,Flange}$$

$$F_{U,C} = \frac{M_{pl,2} + M_{Pl,3}}{h}$$



## Nonlinear analysis of the wind turbine blade dynamics

- Due to economical reasons, the new generation of wind turbine structures with larger and more flexible blades is observed. So, the significant large deformation of such flexible structures makes the application of nonlinear models more crucial.
- Wind turbine blades often have the specific geometry with high slenderness ratios. Therefore, the nonlinear beam theory for modeling purposes can be used.



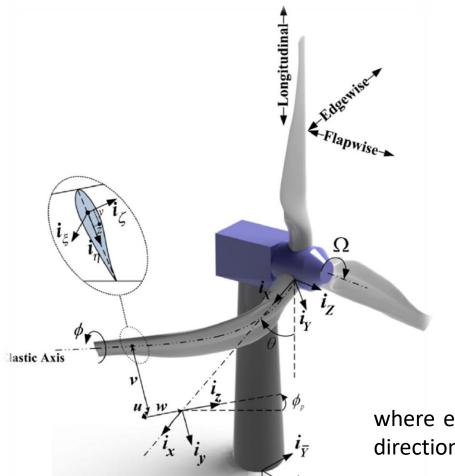






## Nonlinear analysis of the wind turbine blade dynamics

#### Large deformation kinematics of the blade



$$\{i_{\xi\eta\zeta}\} = T\{i_{xyz}\},\$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix}$$

$$\times \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ -B_{12} & B_{11} + B_{13}^2 / (1+B_{11}) & -B_{12}B_{13}/(1+B_{11}) \\ -B_{13} & B_{23} & B_{11} + B_{12}^2 / (1+B_{11}) \end{pmatrix}$$

$$B_{11} = \frac{1+u'}{1+e}, \quad B_{12} = \frac{v'}{1+e}, \quad B_{13} = \frac{w'}{1+e},$$
  
 $e = \sqrt{(1+u')^2 + v'^2 + w'^2} - 1$ 

where e is the elongation of the elastic axis and u is the longitudinal deflection in  $i_x$  direction, v and w are the lateral deflections in  $i_y$  and  $i_z$  directions, respectively.

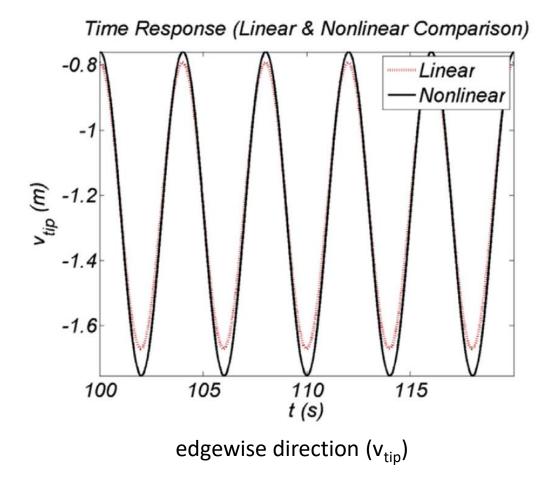






## Nonlinear analysis of the wind turbine blade dynamics

#### Large deformation kinematics of the blade: results



Time Response (Linear & Nonlinear Comparison) Linear 7.4 Nonlinear 7.2 6.8 6.6 105 115 100 110 t(s)

flapwise direction (w<sub>tip</sub>)







