The Boundary Element Method for Acoustics and Fluid-Structure interaction problems

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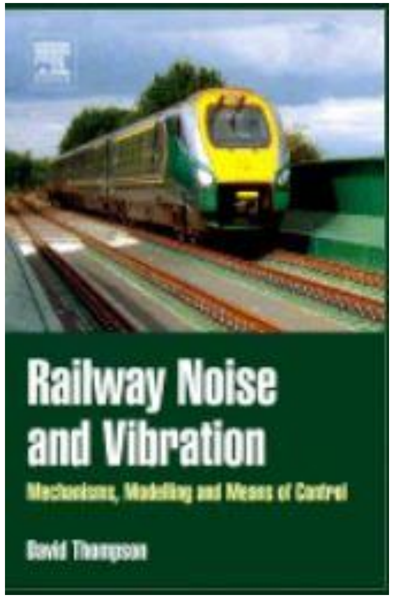
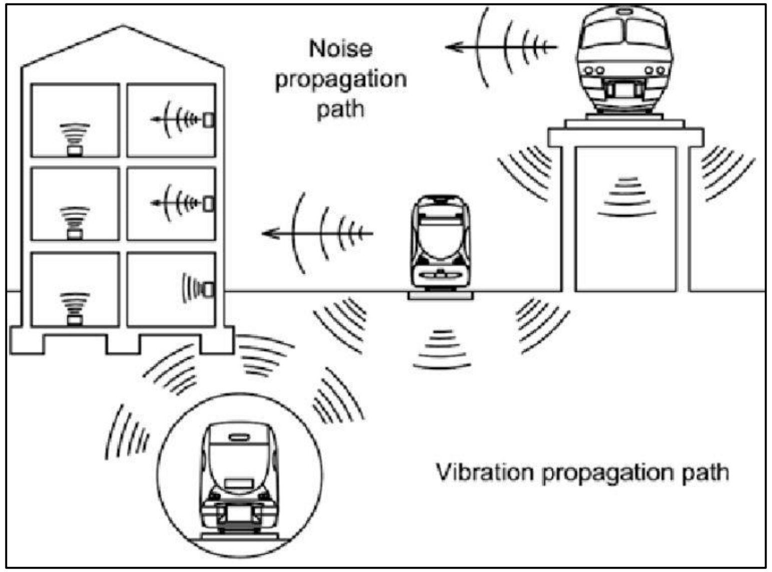
1) Motivation: Engineering problems and numerical tools

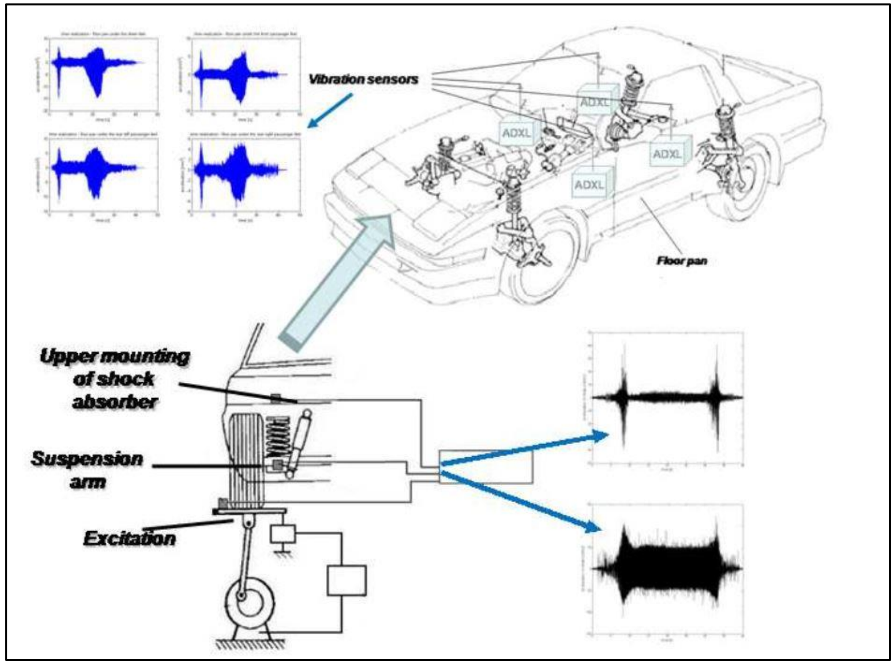
2) Physics and mathematical modelling

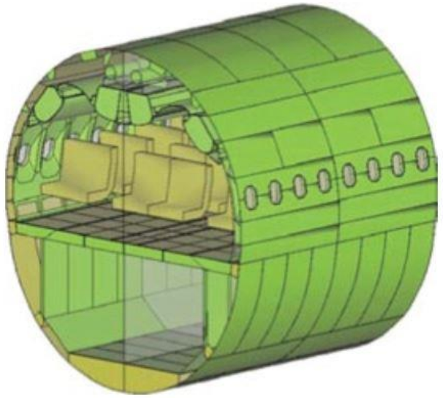
3) Boundary Element Method

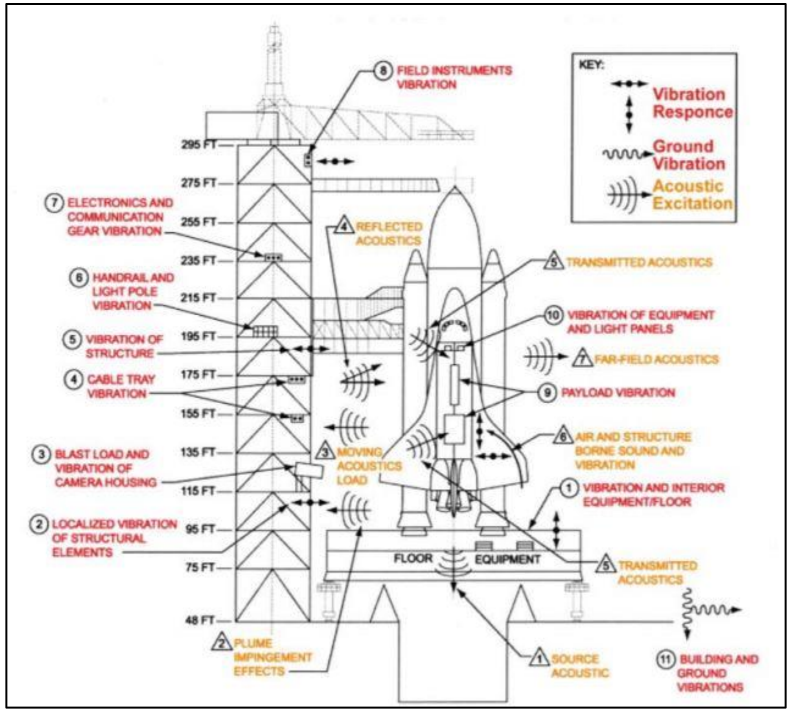
Engineering Vibroacoustic problems











Biomechanics

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Effect of receptors on the resonant and transient harmonic vibrations of Coronavirus

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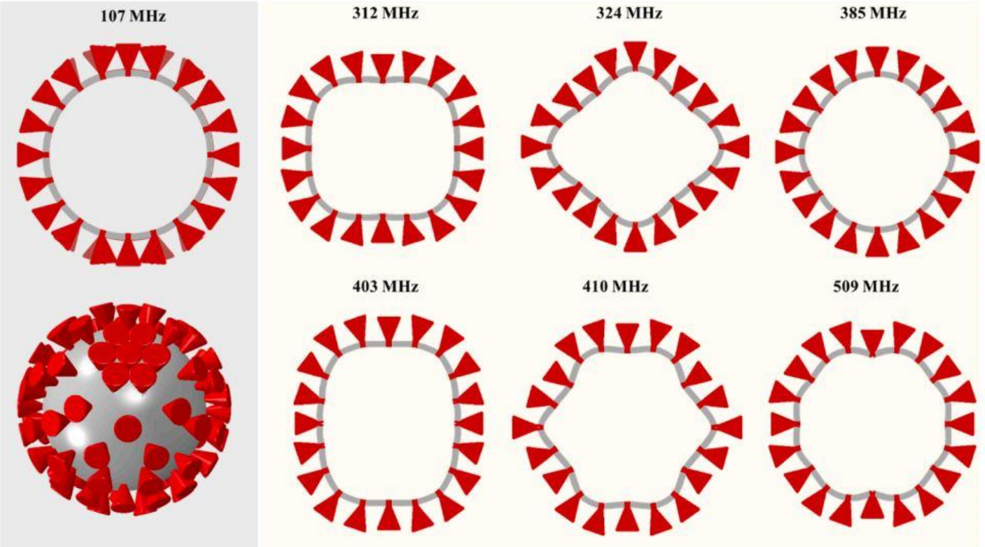
Department of Mechanical Engineering, MIT, United States

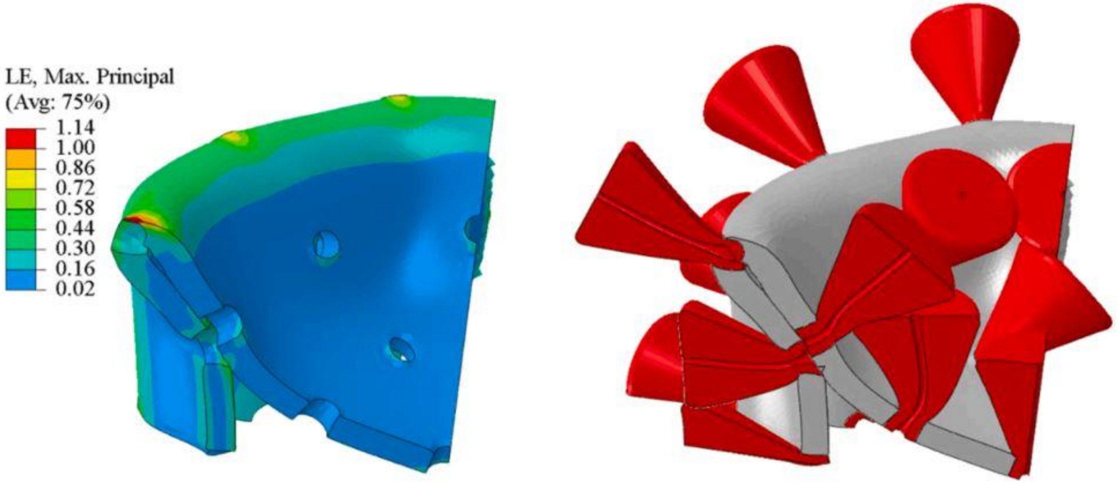
A R T I C L E I N F O

Keywords:
Ultrasound excitations
Coronavirus family
Collapse of lipid bi-layer envelope
Resonant and transient vibrations of spikes
Large tensile strains
Disruption of life-cycle of coronaviruses

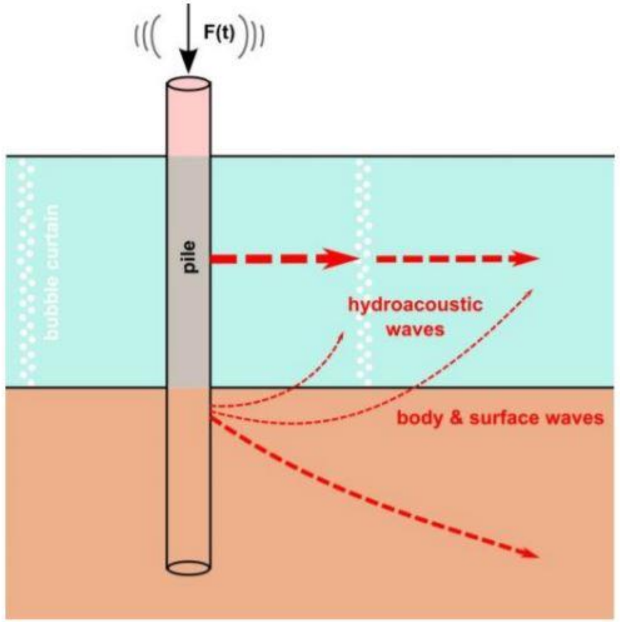
A B S T R A C T

The paper is concerned with the vibration characteristics of the Coronavirus family. There are some 25–100 receptors, commonly called spikes protruding from the envelope shell of the virus. Spikes, resembling the shape of a hot air balloon, may have a total mass similar to the mass of the lipid bi-layer shell. The lipid proteins of the virus are treated as homogeneous elastic material and the problem is formulated as the interaction of thin elastic shell with discrete masses, modeled as short conical cross-sectional beams. The system is subjected to ultrasonic excitation. Using the methods of structural acoustics, it is shown that the scattered pressure is very small and the pressure on the viral shell is simply the incident pressure. The modal analysis is performed for a bare shell, a single spike, and the spike-decorated shell. The predicted vibration frequencies and modes are shown to compare well with the newly derived closed-form solutions for a single spike and existing analytical solutions for thin shells. The fully nonlinear dynamic simulation of the transient response revealed the true character of the complex interaction between local vibration of spikes and global vibration of the multi-degree-of-freedom system. It was shown that harmonic vibration at or below the lowest resonant modes can excite large amplitude vibration of spikes. The associated maximum principal strain in a spike can reach large values in a fraction of a millisecond. Implications for possible tearing off spikes from the shell are discussed. Another important result is that after a finite number of cycles, the shell buckles and collapses, developing internal contacts and folds with large curvatures and strains exceeding 10%. For the geometry and elastic properties of the SARS-CoV-2 virus, these effects are present in the range of frequencies close to the ones used for medical ultrasound diagnostics.

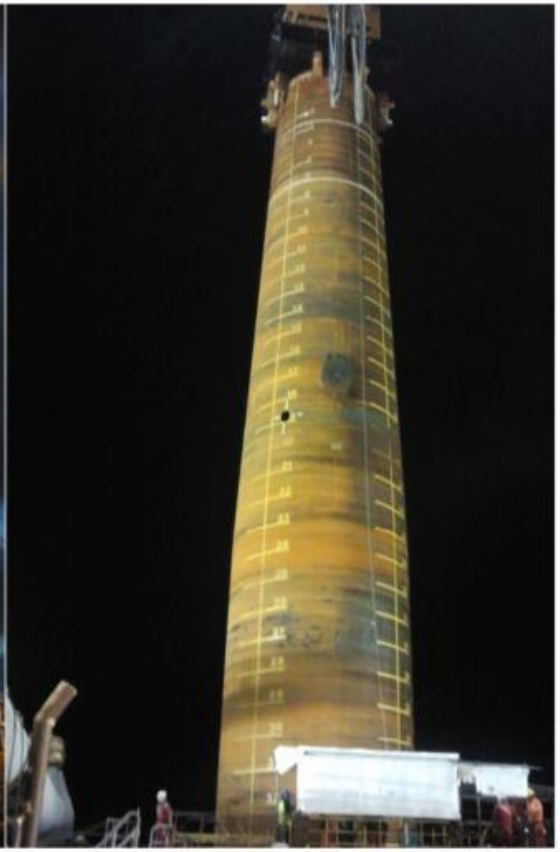




Wind turbines

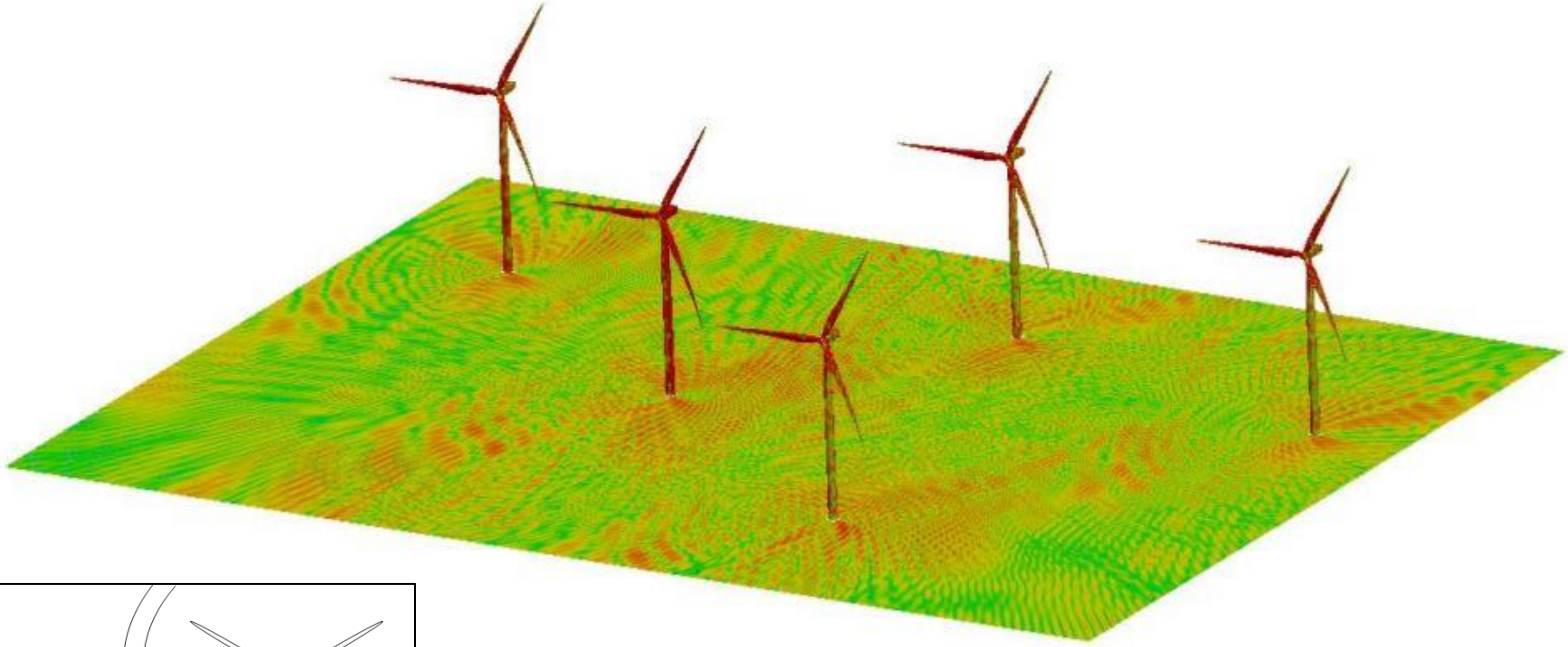


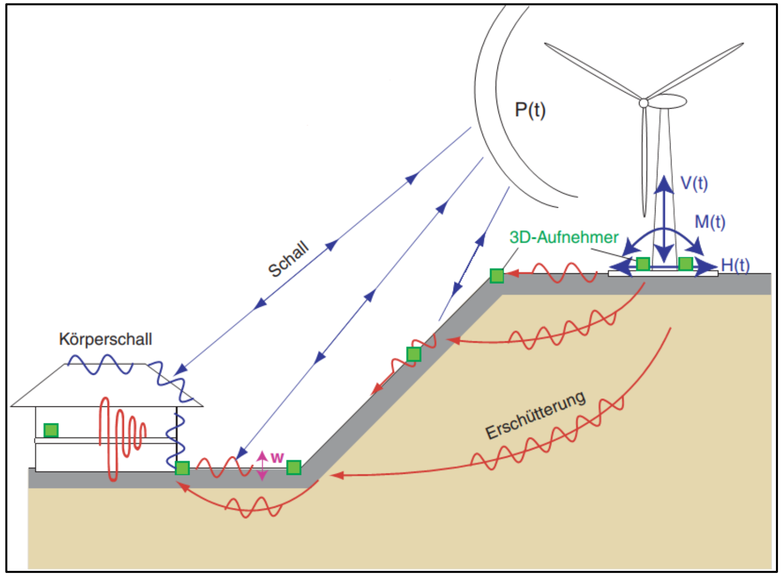
Impact hammer



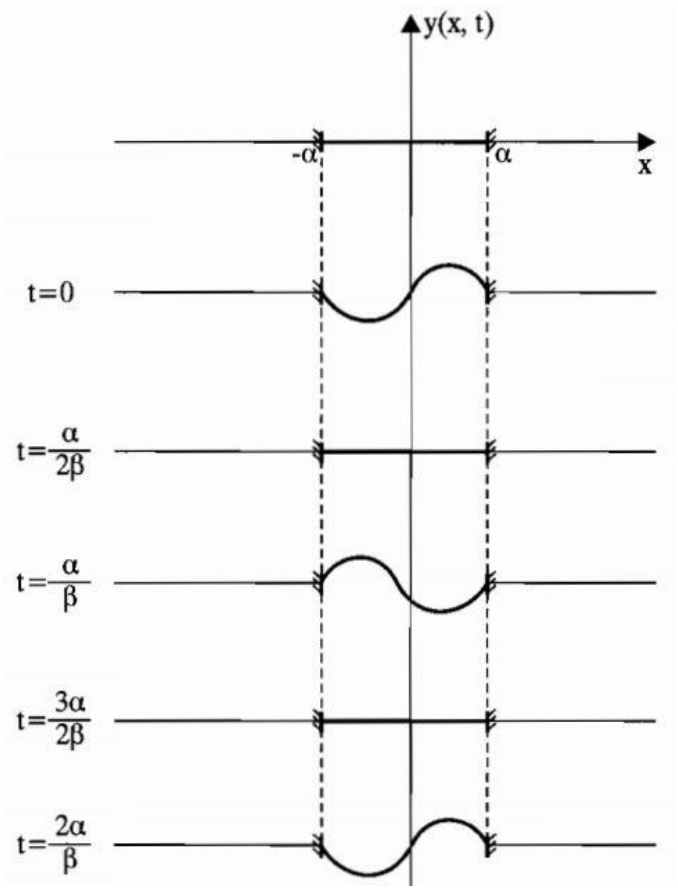
Vibratory device

Wind turbines acoustic and elastic wave propagation





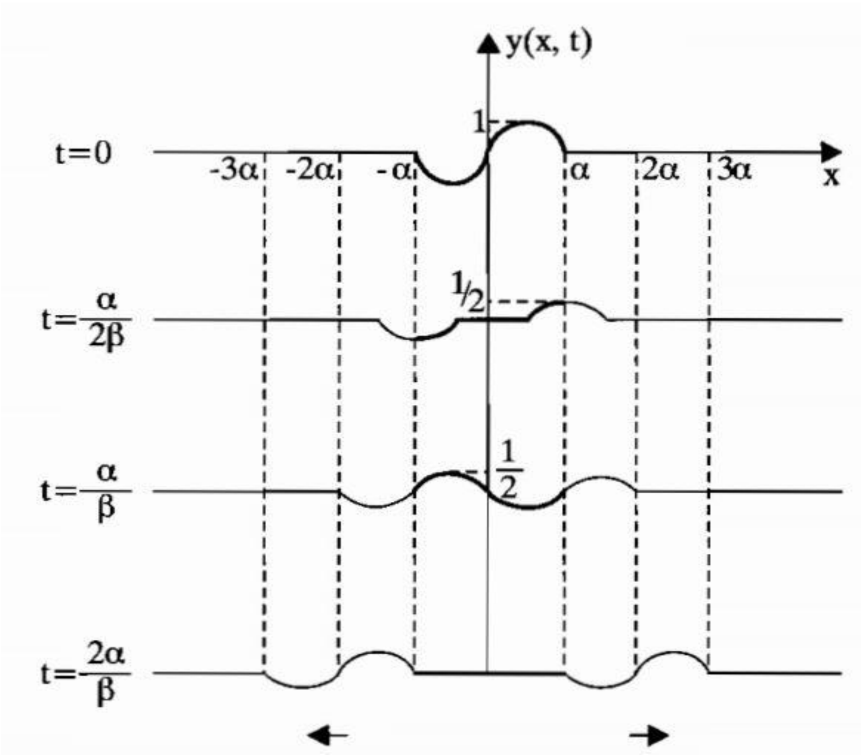
Vibration



Discrete systems

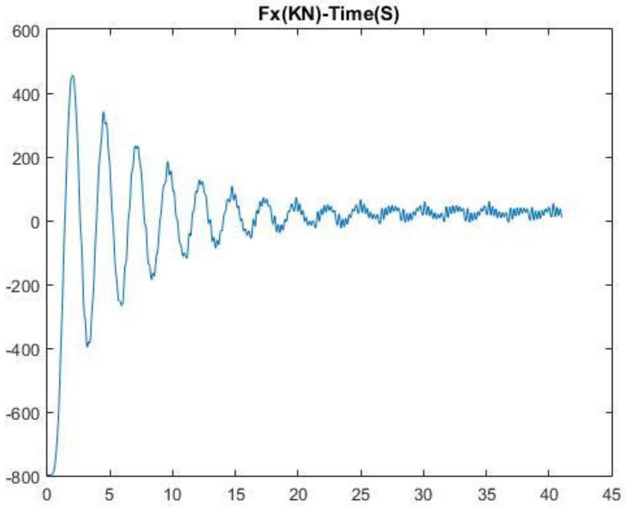
$$[\mathbf{M}]\ddot{\mathbf{x}}(t)+[\mathbf{C}]\dot{\mathbf{x}}(t)+[\mathbf{K}]\mathbf{x}(t)=\mathbf{f}(t)$$

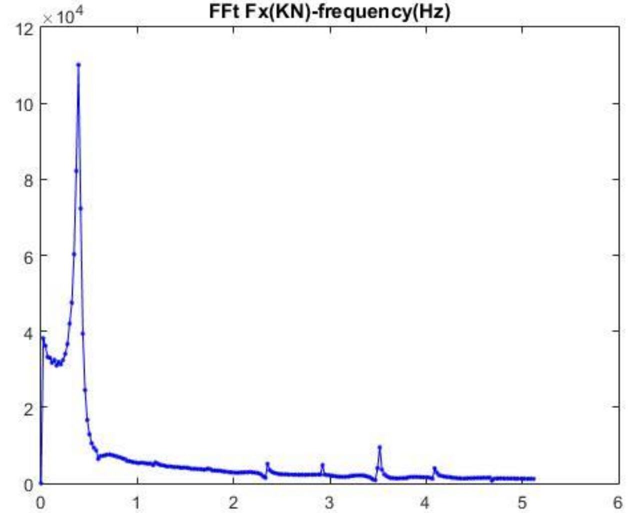
Wave propagation



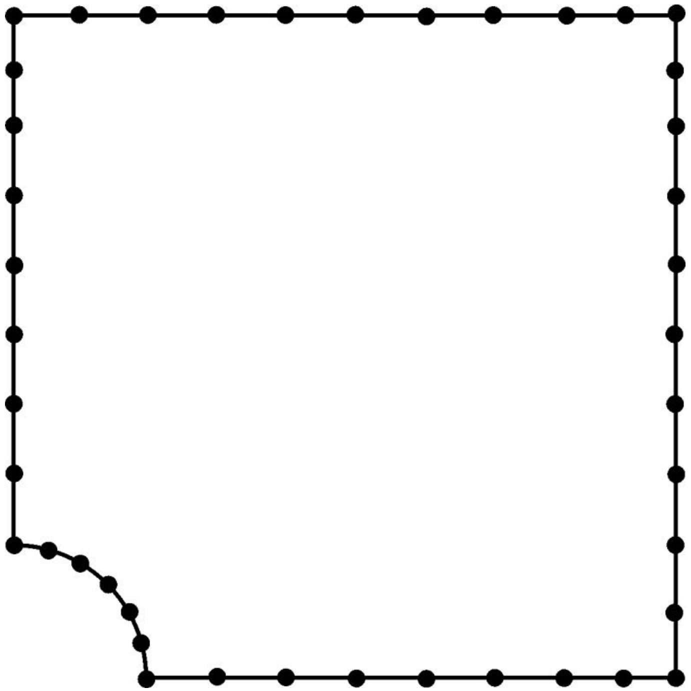
Infinite or semi - infinite domains

Time Domain – Frequency domain analysis

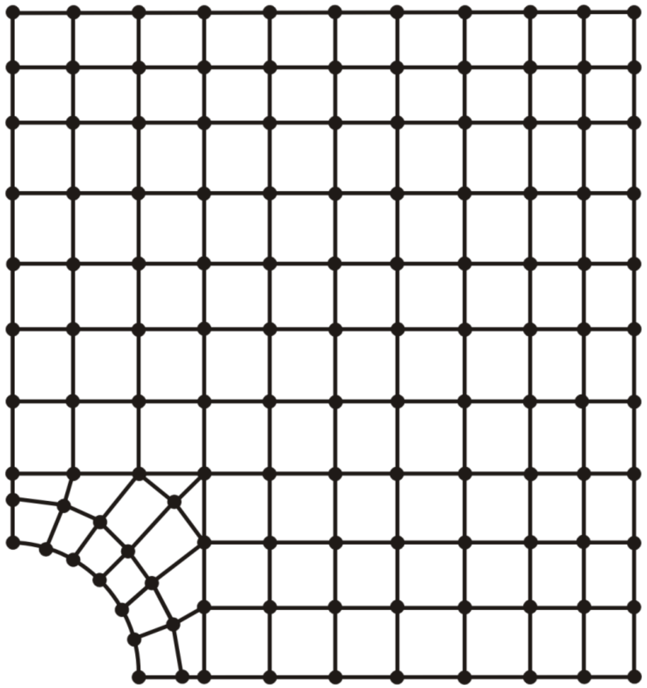




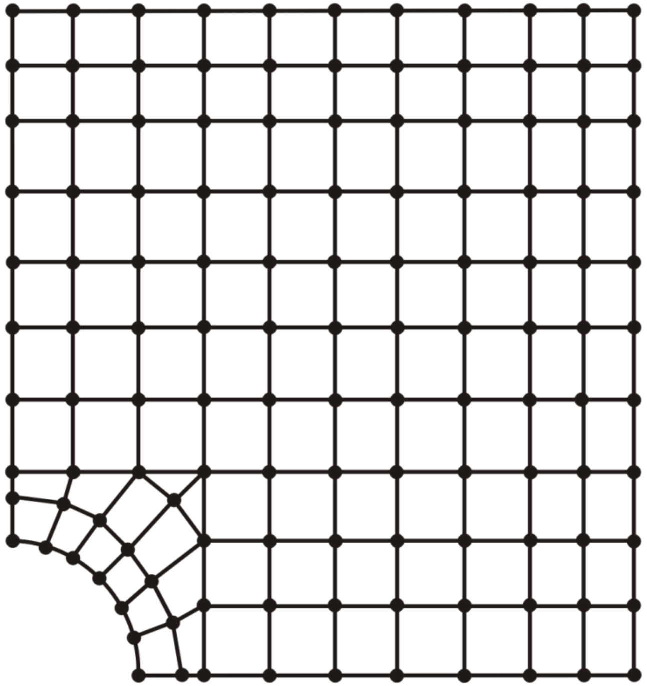
Numerical Methods - Vibroacoustics



BEM

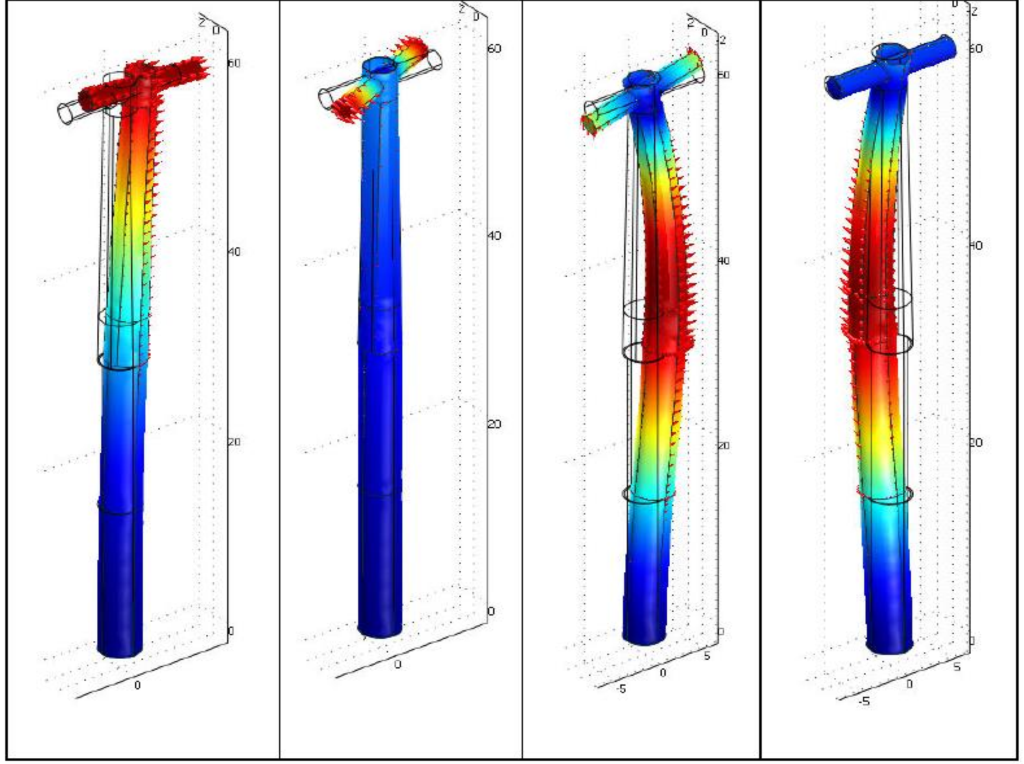


FEM



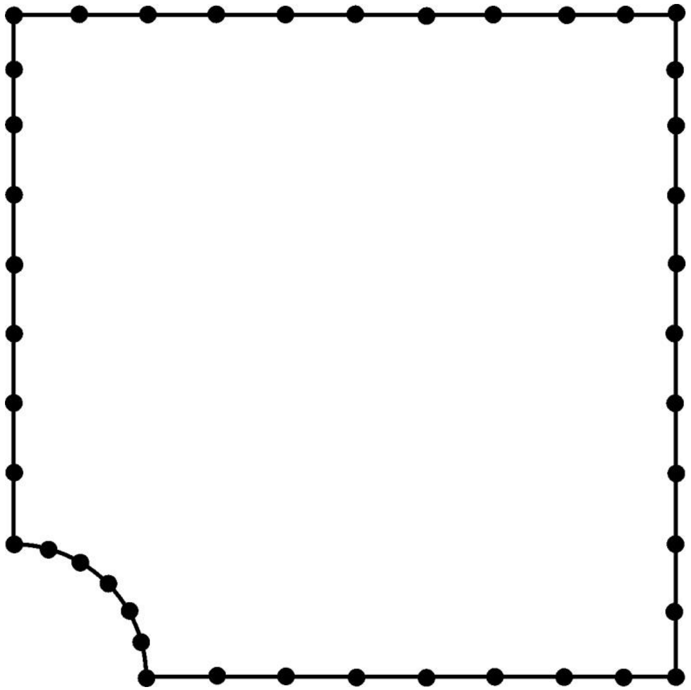
FEM

A. 1st Bend	B. Torsion	C. 2nd Bending A	D. 2nd Bending B
0.45 Hz	3.0 Hz	4.1 Hz	4.6 Hz

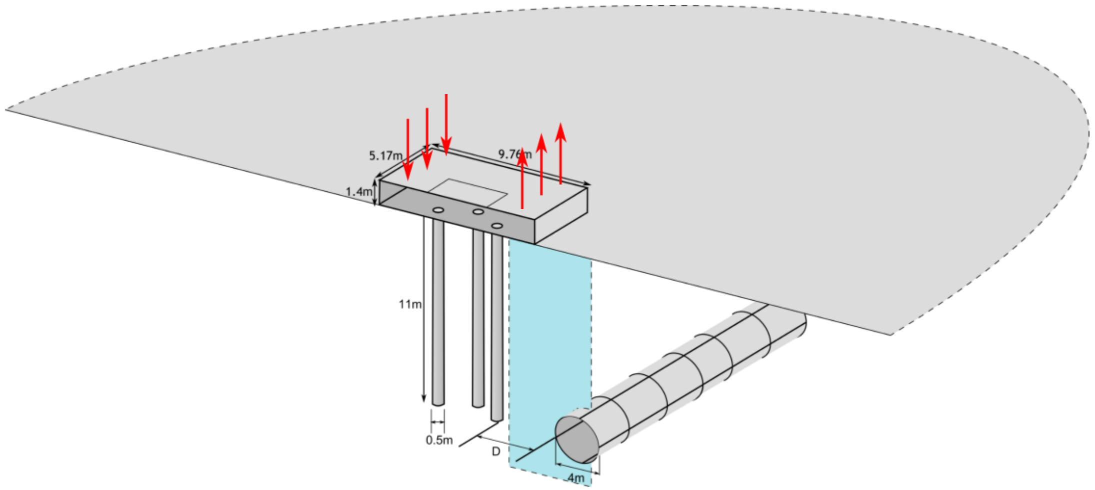


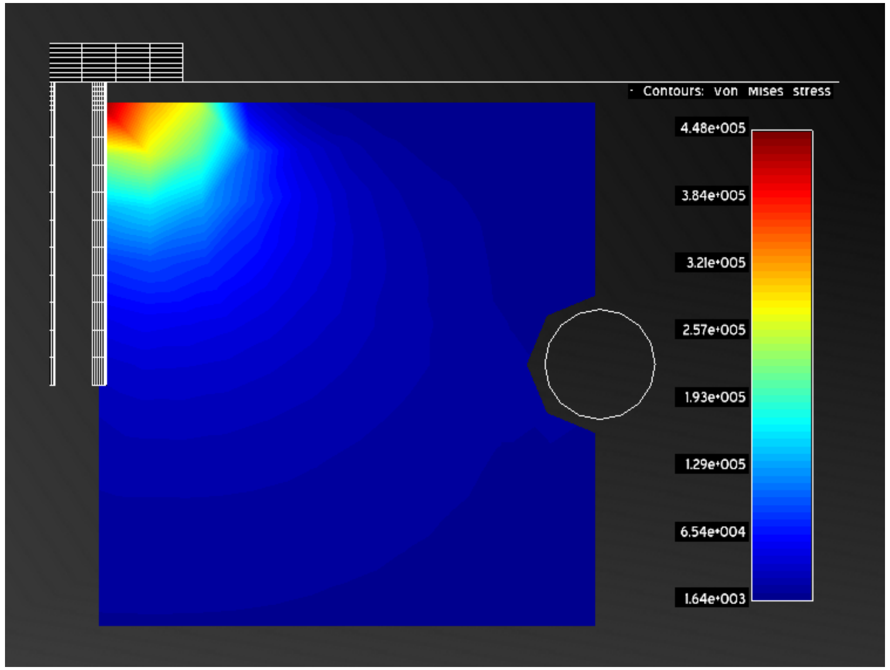
$$-\omega^2 [\mathbf{M}] \cdot \mathbf{A} + [\mathbf{K}] \cdot \mathbf{A} = 0$$

$$\mathbf{x}(t) = \mathbf{A} e^{-i\omega t}$$



BEM


5.17m
9.76m
1.4m
11m
0.5m
D
4m

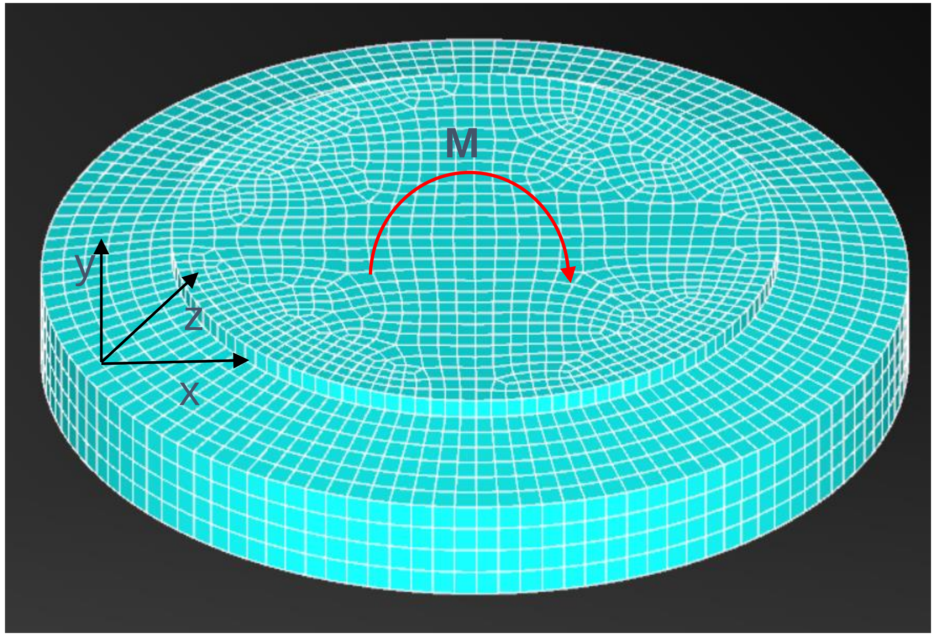

Contours: von Mises stress
4.48e+005
3.84e+005
3.21e+005
2.57e+005
1.93e+005
1.29e+005
6.54e+004
1.64e+003

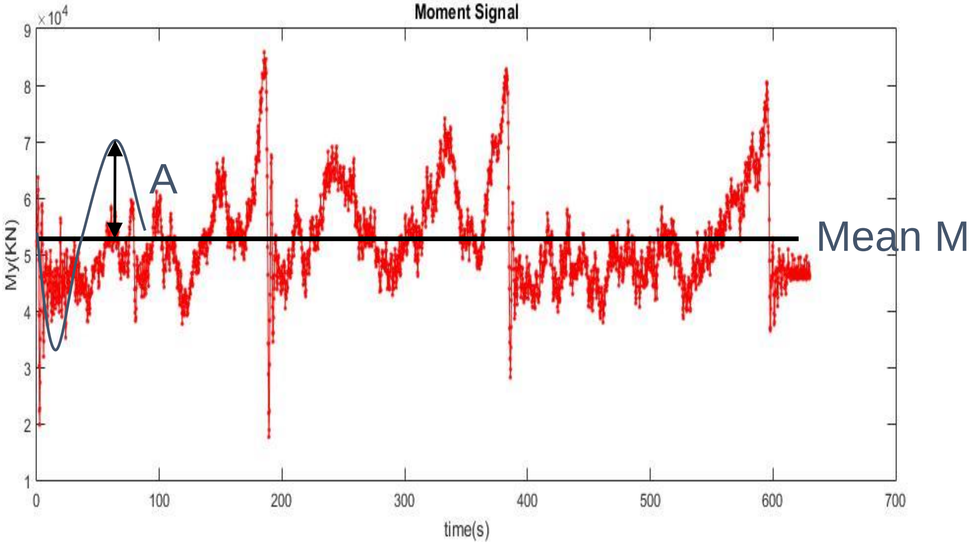
FEM or BEM?

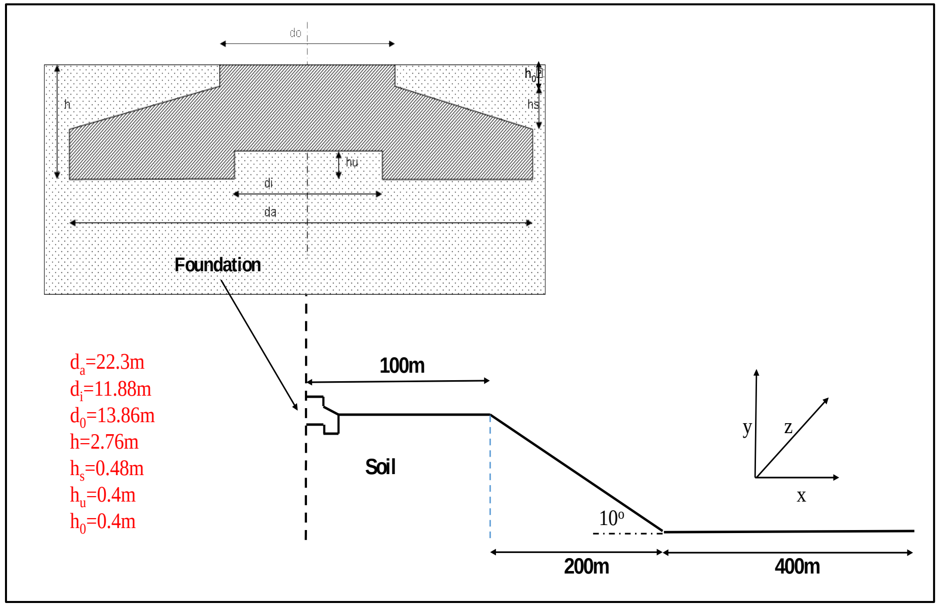
- Knowing the advantages and disadvantages of each method makes it possible to choose the most appropriate tool for each task.

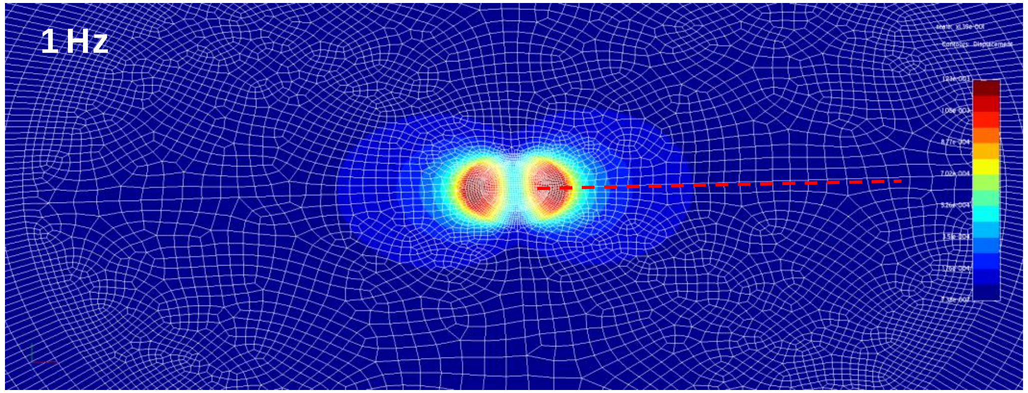
- **P. K. Banerjee**: "Changes in the geometric configurations are very easy to incorporate on a surface mesh. Additionally, most of today's analysis problems are so complex that engineers would love to have two vastly different analysis tools (FEM and BEM) to produce similar answers."

WT foundation moment loading

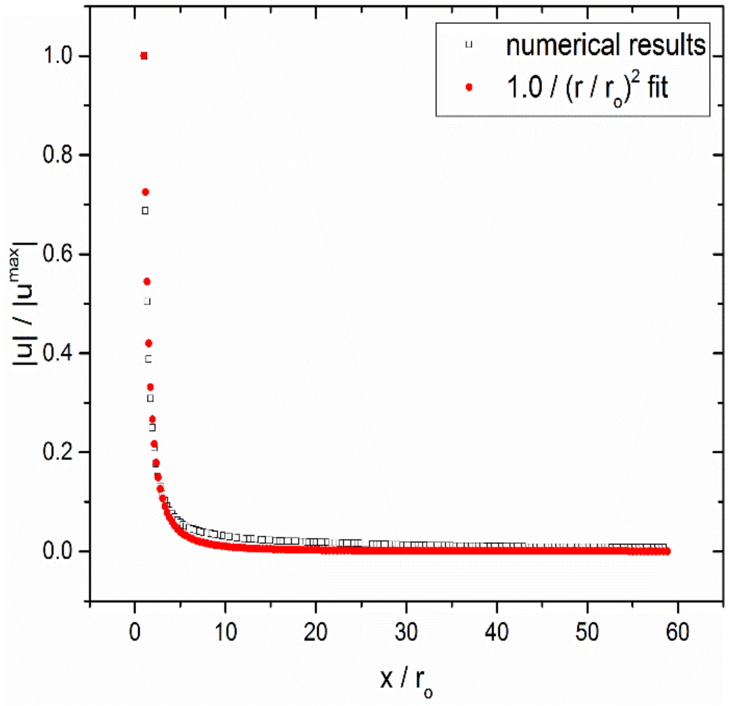




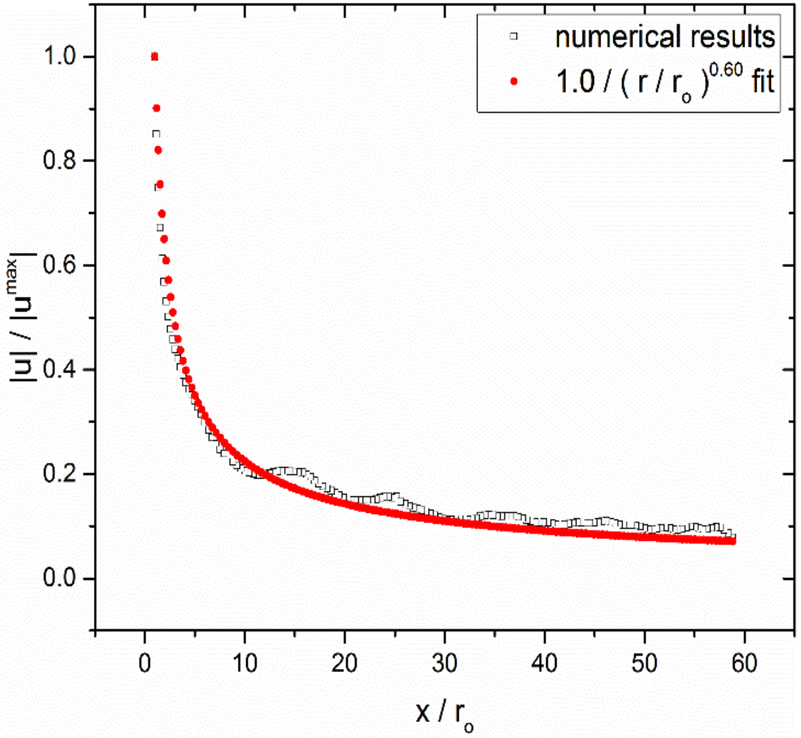




1Hz--- [1.0 / (r / r_o)$^{2.0}$] fit



5Hz--- [1 / (r / r_o) $^{0.60}$] fit

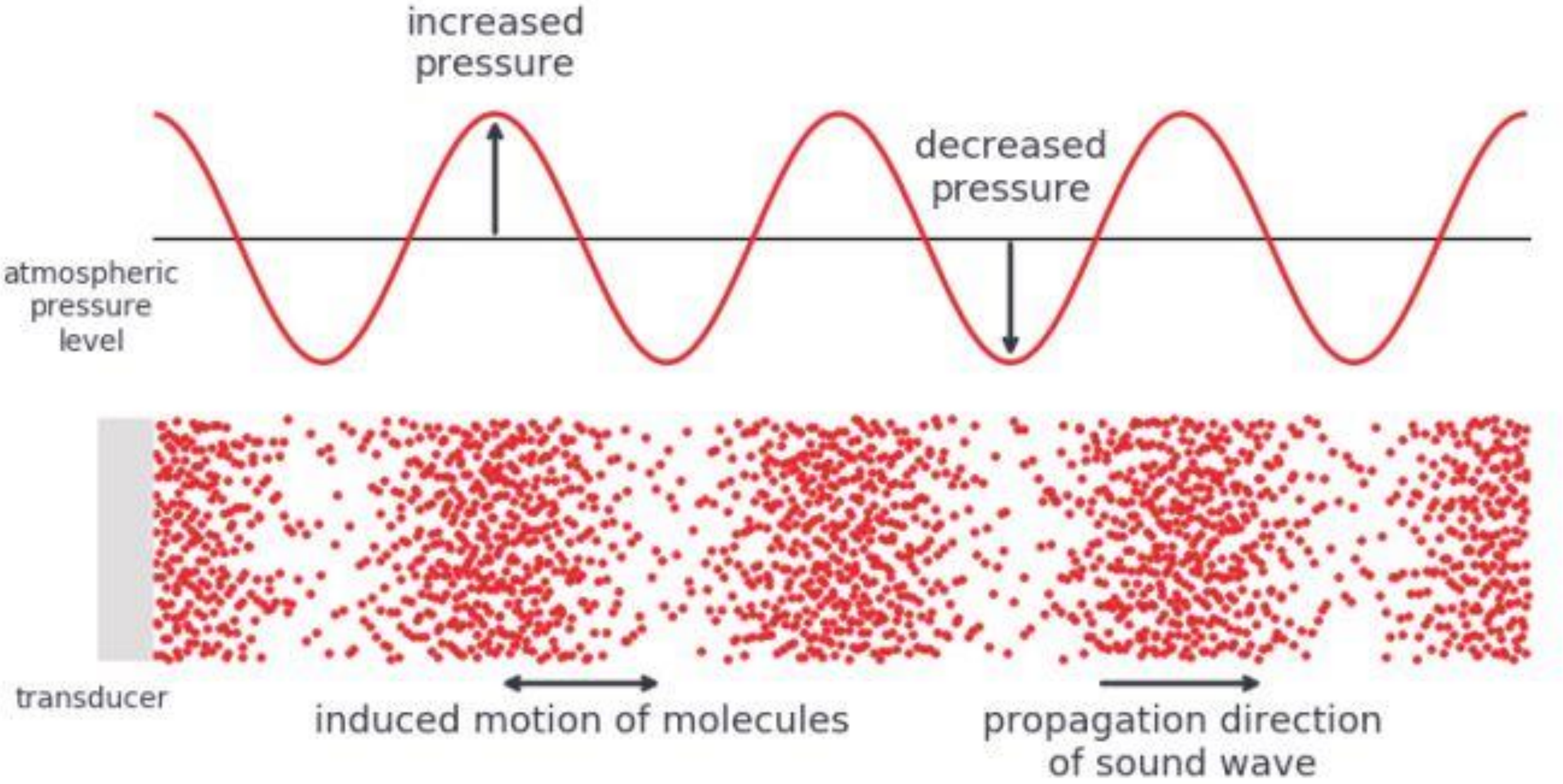


1) Motivation: Engineering problems and numerical tools

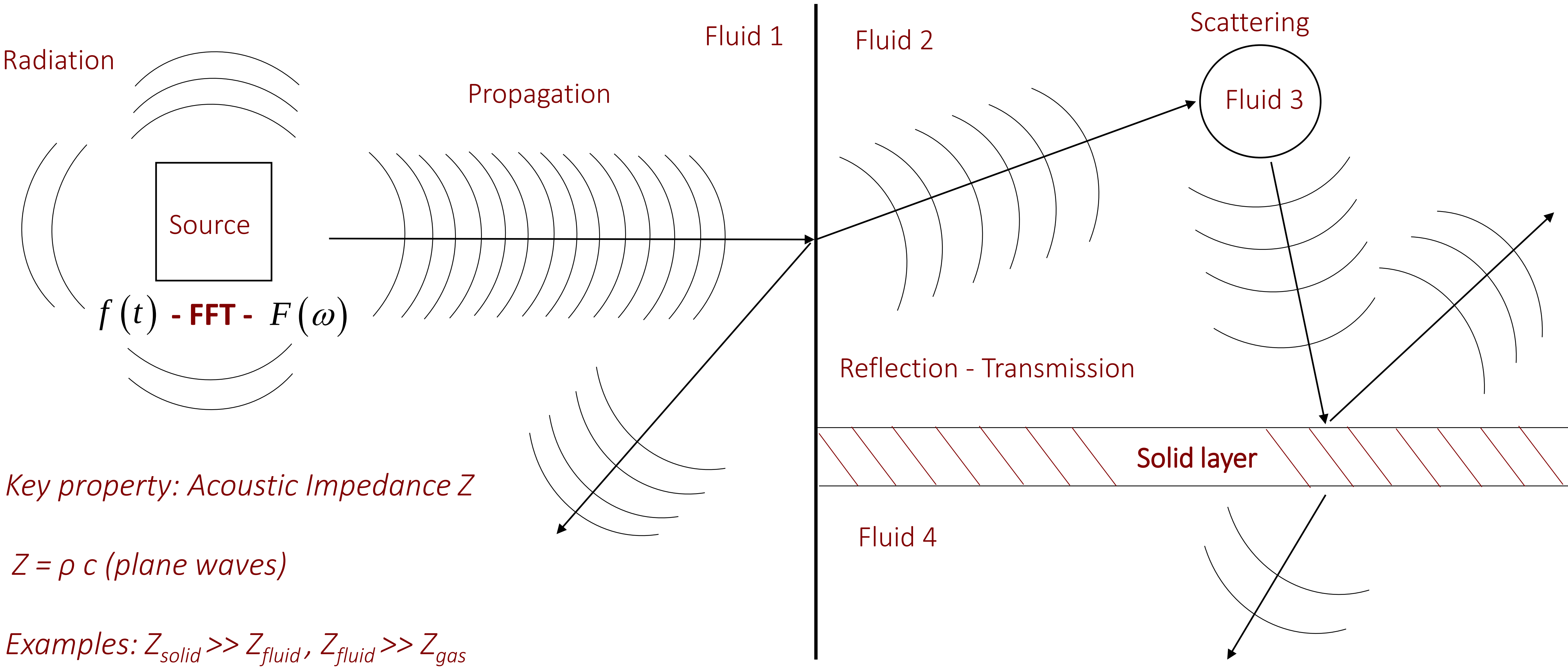
2) Physics and mathematical modelling

3) Boundary Element Method

Acoustic waves



ACOUSTIC WAVE PHENOMENA

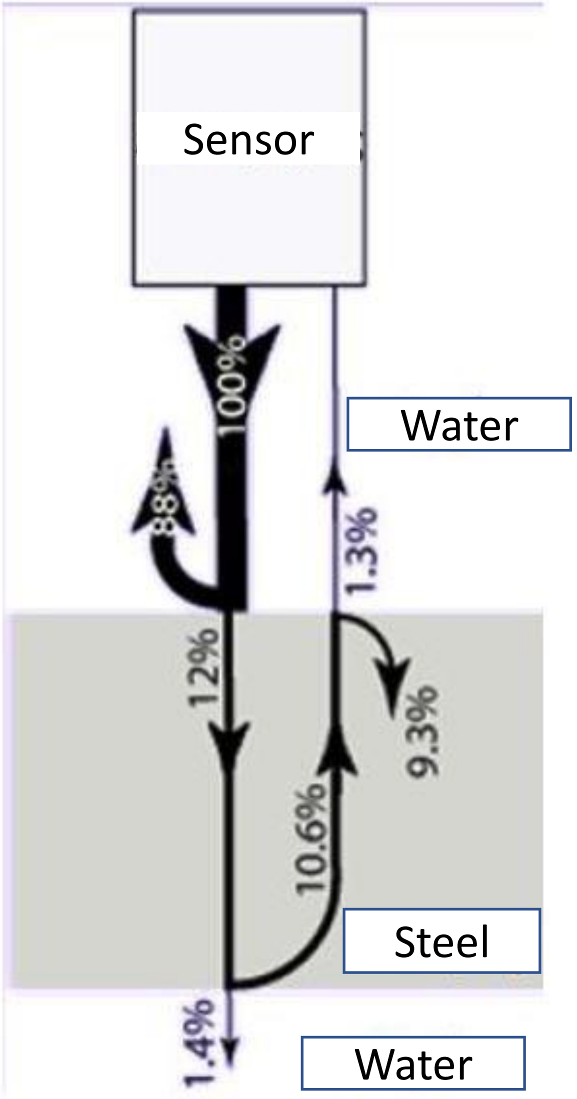


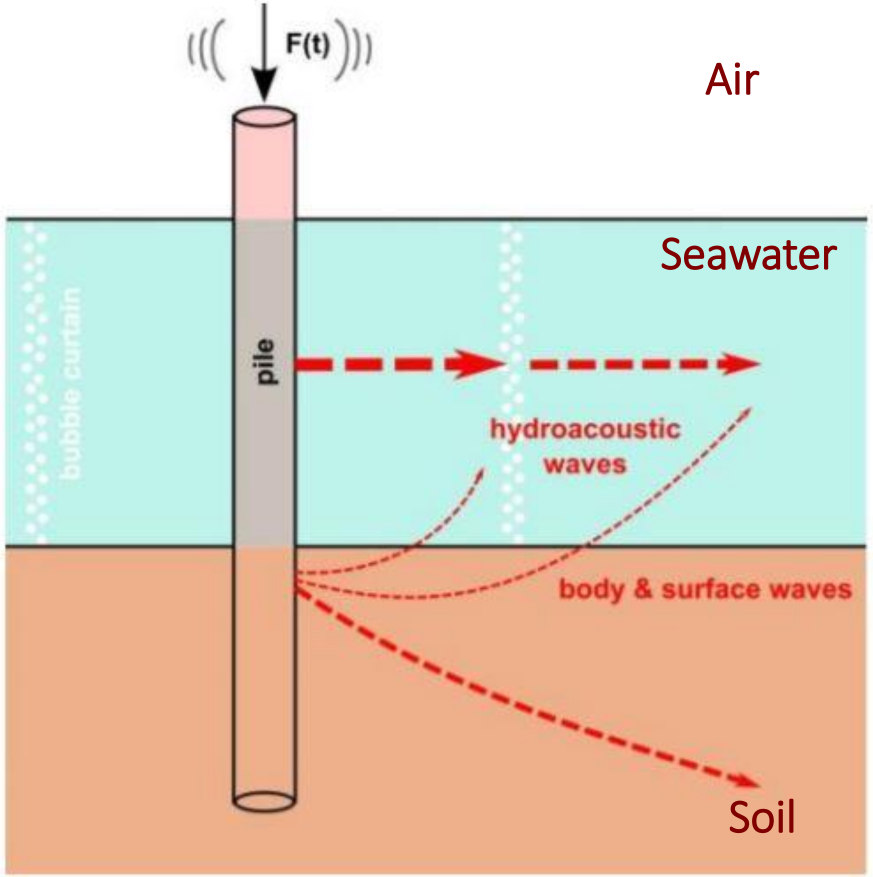
Key property: Acoustic Impedance Z

$Z = \rho c$ (plane waves)

Examples: $Z_{solid} >> Z_{fluid}$, $Z_{fluid} >> Z_{gas}$

The impact of impedance difference

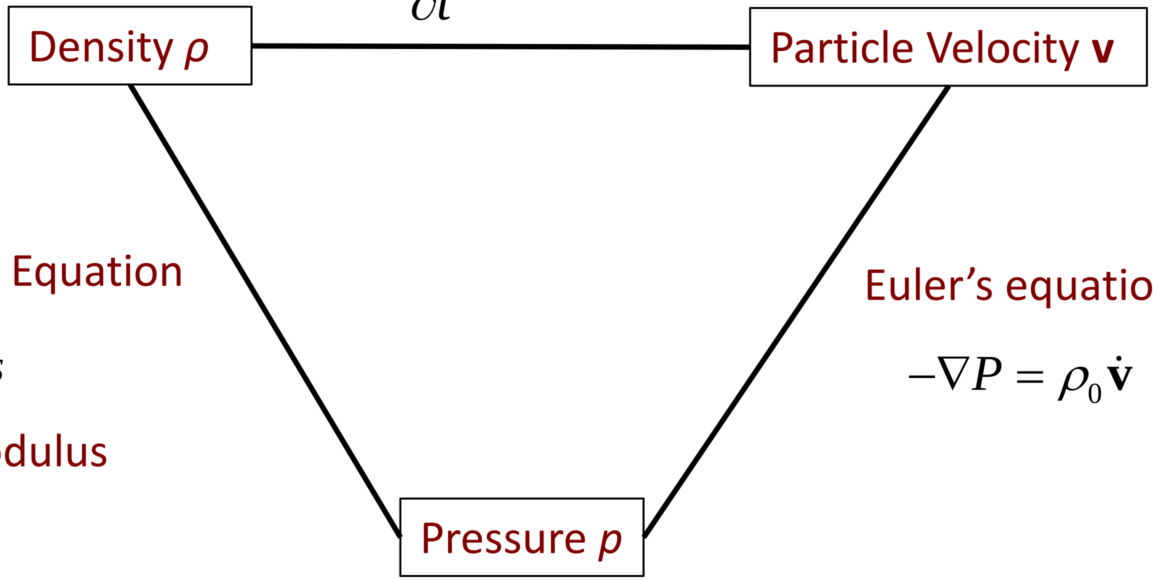




Mathematical modelling

Continuity equation

$$\frac{\partial s}{\partial t} + \nabla \cdot \mathbf{v} = 0$$



Constitutive Equation

$$P = Bs$$

B: bulk modulus

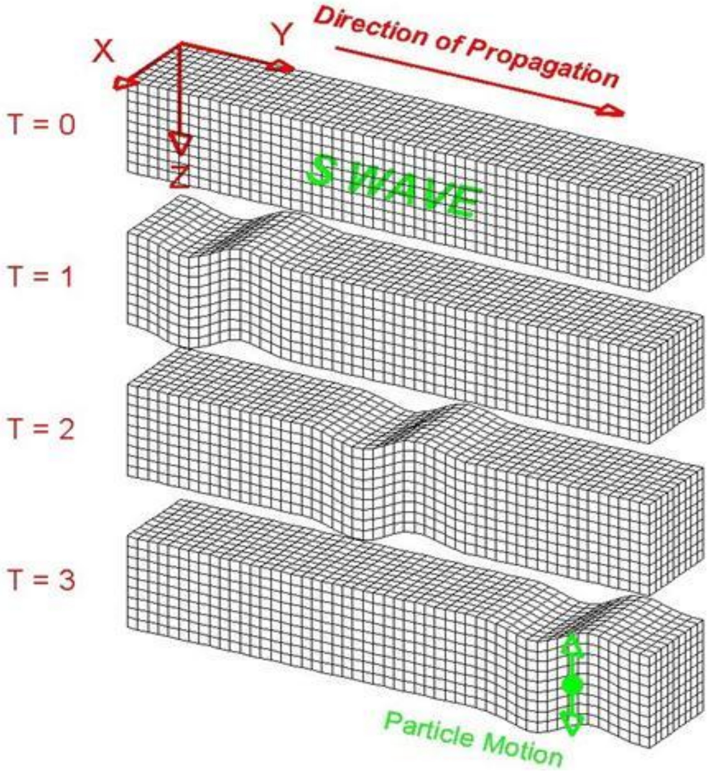
Euler's equation

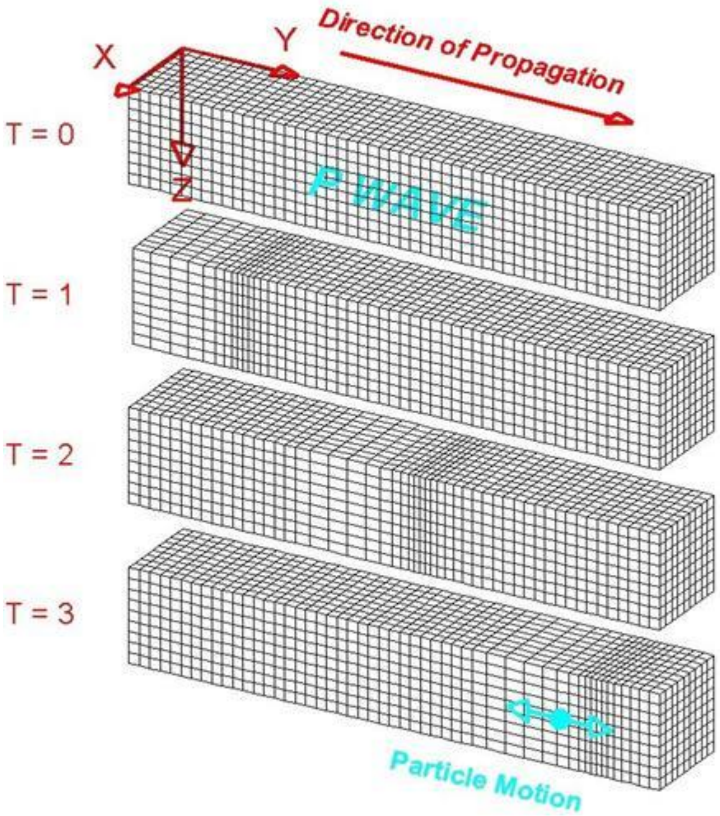
$$-\nabla P = \rho_0 \dot{\mathbf{v}}$$

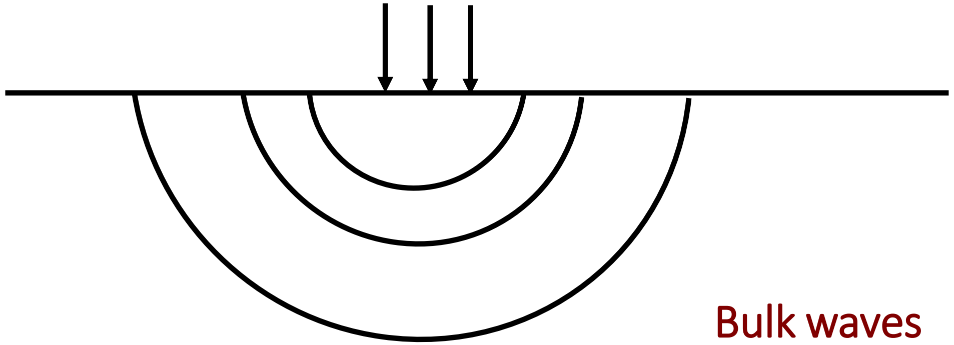
Linear acoustics assumptions

- Small amplitude disturbances
- **v** << c
- s << 1
- Adiabatic conditions

ELASTIC WAVES

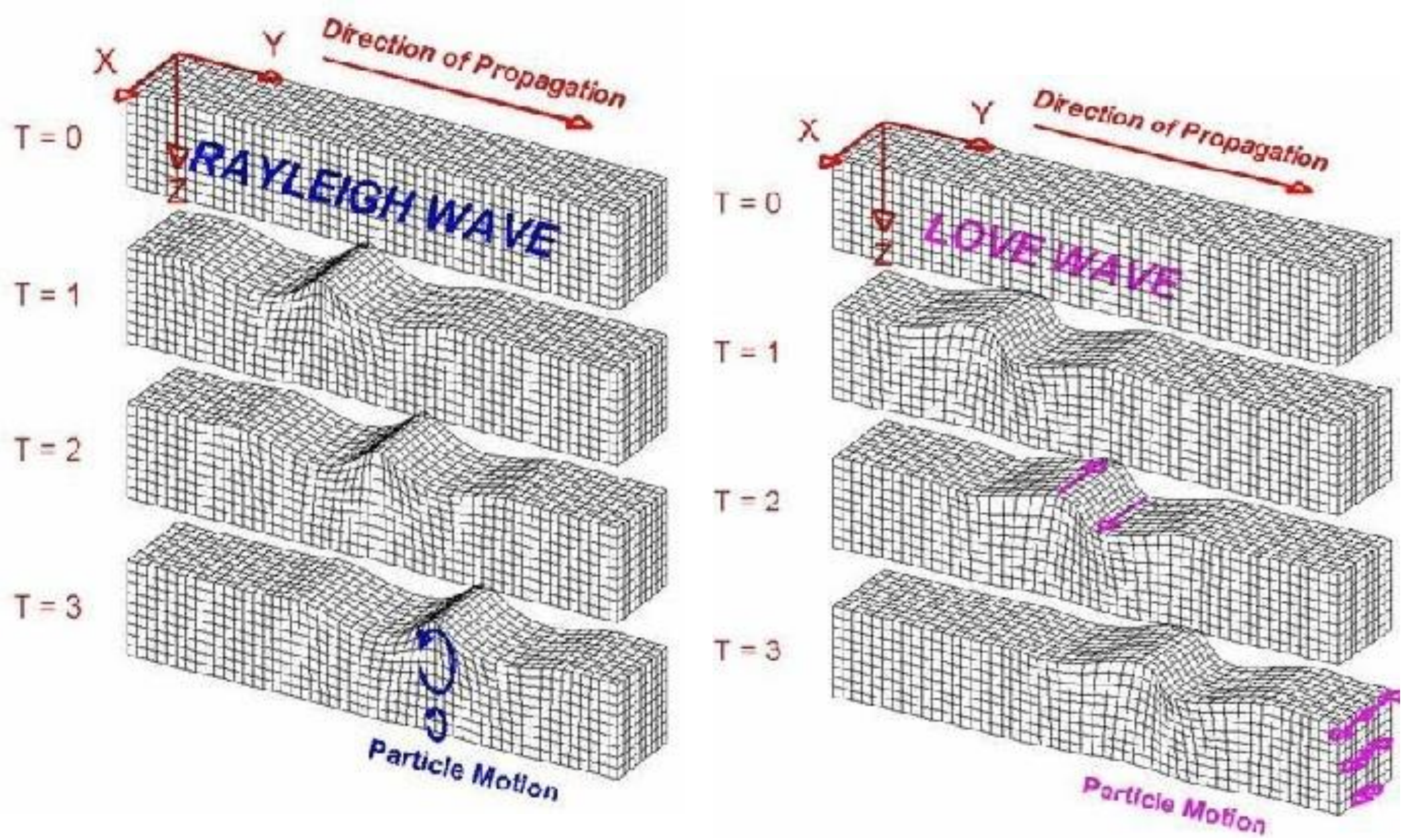




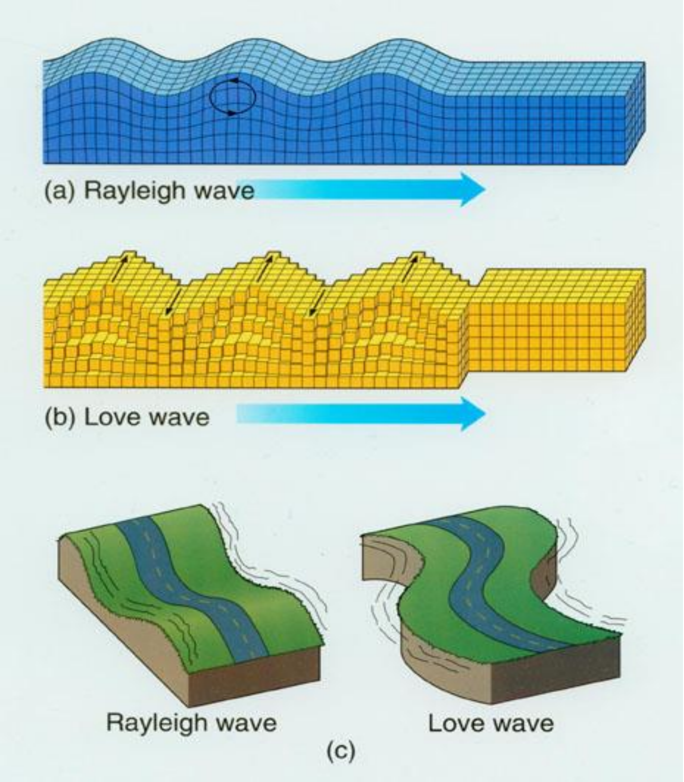


Bulk waves

ELASTIC WAVES



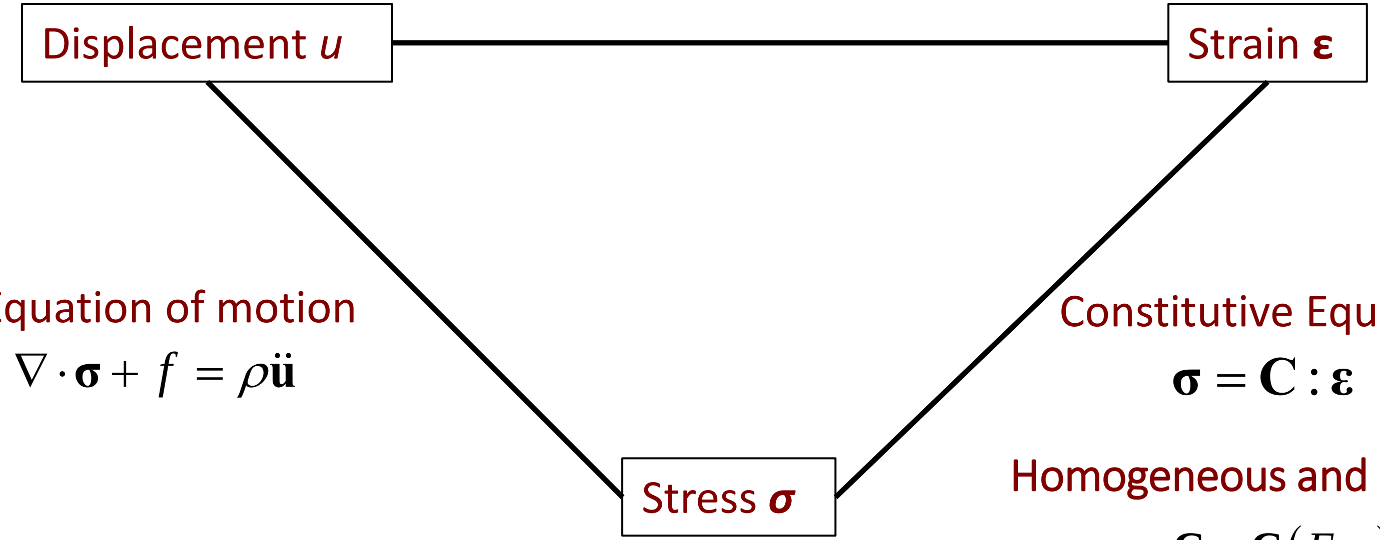
Surface waves



Mathematical modelling

Kinematic (Strain-displacement)

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla \mathbf{u} + \left(\nabla \mathbf{u}\right)^T\right]$$



Equation of motion

$$\nabla \cdot \boldsymbol{\sigma} + f = \rho \ddot{\mathbf{u}}$$

Constitutive Equation

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon}$$

Homogeneous and isotropic

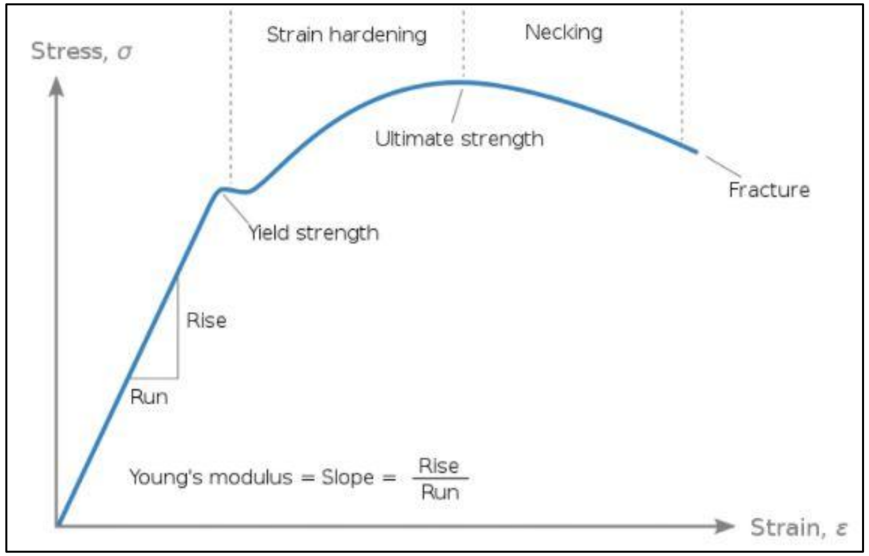
$$\mathbf{C} = \mathbf{C}(E, v)$$

2D

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

Linear elasticity assumptions

- Linear material behavior
- Small strains



Equations in the time domain

Elastodynamics

$$\mu\nabla^2\mathbf{u}(\mathbf{x},t)+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}(\mathbf{x},t)=\rho\ddot{\mathbf{u}}(\mathbf{x},t)$$

$$\mu=\frac{E}{2(1+v)},\lambda=\frac{Ev}{(1+v)(1-2v)}$$

λ, μ Lame parameters

Boundary Conditions

$$\left.\begin{array}{ll}\mathbf{u}(\mathbf{x},t)=\mathbf{u}^b(\mathbf{x},t) & \mathbf{x}\in S_u\\ \mathbf{t}(\mathbf{x},t)=\mathbf{t}^b(\mathbf{x},t) & \mathbf{x}\in S_t\end{array}\right\} S=S_u\cup S_t$$

Initial Conditions

$$\mathbf{u}(\mathbf{x},0)=\mathbf{u}^V(\mathbf{x})\quad \mathbf{x}\in V$$

Acoustics

$$\nabla^2 p(\mathbf{x},t)=\frac{1}{c^2}\ddot{p}(\mathbf{x},t),\ c^2=\frac{\gamma P_0}{\rho_0}$$

Boundary Conditions

$$\left.\begin{array}{ll}p(\mathbf{x},t)=p^b(\mathbf{x},t) & \mathbf{x}\in S_p\\ \dfrac{\partial p}{\partial n}(\mathbf{x},t)=\dfrac{\partial p^b}{\partial n}(\mathbf{x},t) & \mathbf{x}\in S_q\end{array}\right\} S=S_p\cup S_q$$

Initial Conditions

$$p(\mathbf{x},0)=p^V(\mathbf{x})\quad \mathbf{x}\in V$$

Frequency domain solutions

$$\mu\nabla^2\mathbf{u}(\mathbf{x},\mathrm{t})+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}(\mathbf{x},\mathrm{t})=\rho\ddot{\mathbf{u}}(\mathbf{x},\mathrm{t})$$

$$\mathbf{u}(\mathbf{x},\mathrm{t})=\bar{\mathbf{u}}(\mathbf{x},\omega)e^{i\omega t}$$

$$\mu\nabla\bar{\mathbf{u}}(\mathbf{x},\omega)+(\lambda+\mu)\nabla\nabla\cdot\bar{\mathbf{u}}(\mathbf{x},\omega)+\rho\omega^2\bar{\mathbf{u}}(\mathbf{x},\omega)=0$$

Equations in the frequency domain

Elastodynamics

$$\mu\nabla^2\mathbf{u}(\mathbf{x},\omega)+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}(\mathbf{x},\omega)+\rho\omega^2\mathbf{u}(\mathbf{x},\omega)=\mathbf{0}$$

Boundary Conditions

$$\left.\begin{aligned}\mathbf{u}(\mathbf{x},\omega)&=\mathbf{u}^b(\mathbf{x},\omega) \quad \mathbf{x}\in S_u\\ \mathbf{t}(\mathbf{x},\omega)&=\mathbf{t}^b(\mathbf{x},\omega) \quad \mathbf{x}\in S_t\end{aligned}\right\} S=S_u\cup S_t$$

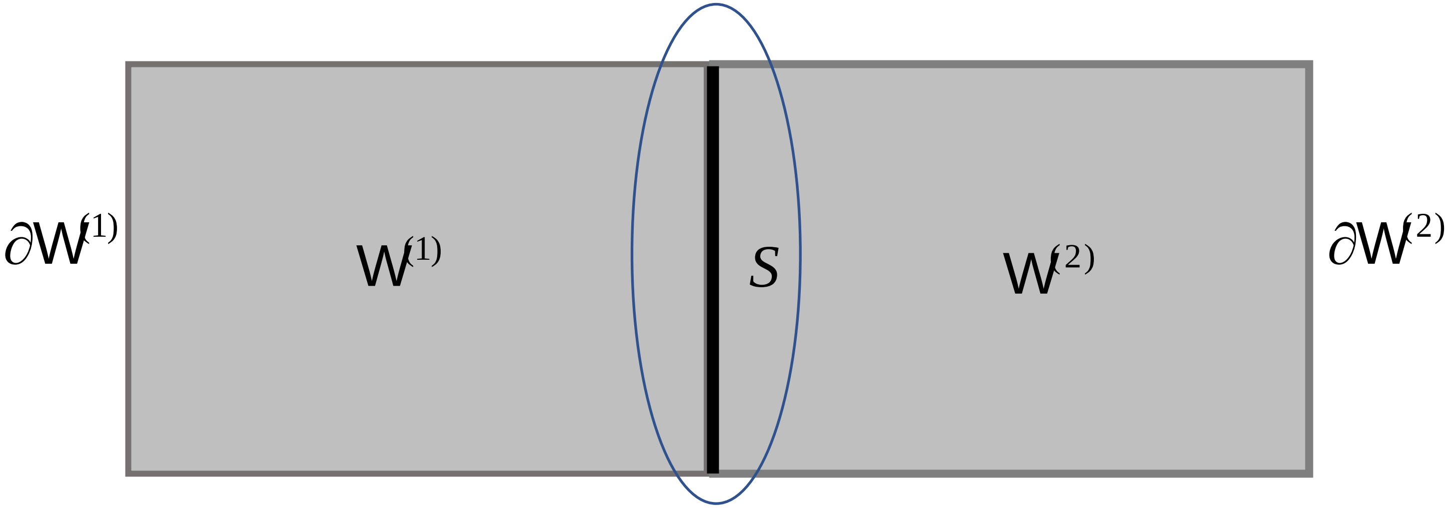
Acoustics

$$\nabla^2 p(\mathbf{x},\omega)+\left(\frac{\omega}{c}\right)^2 p(\mathbf{x},\omega)=0,\ c^2=\frac{\gamma P_0}{\rho_0}$$

Boundary Conditions

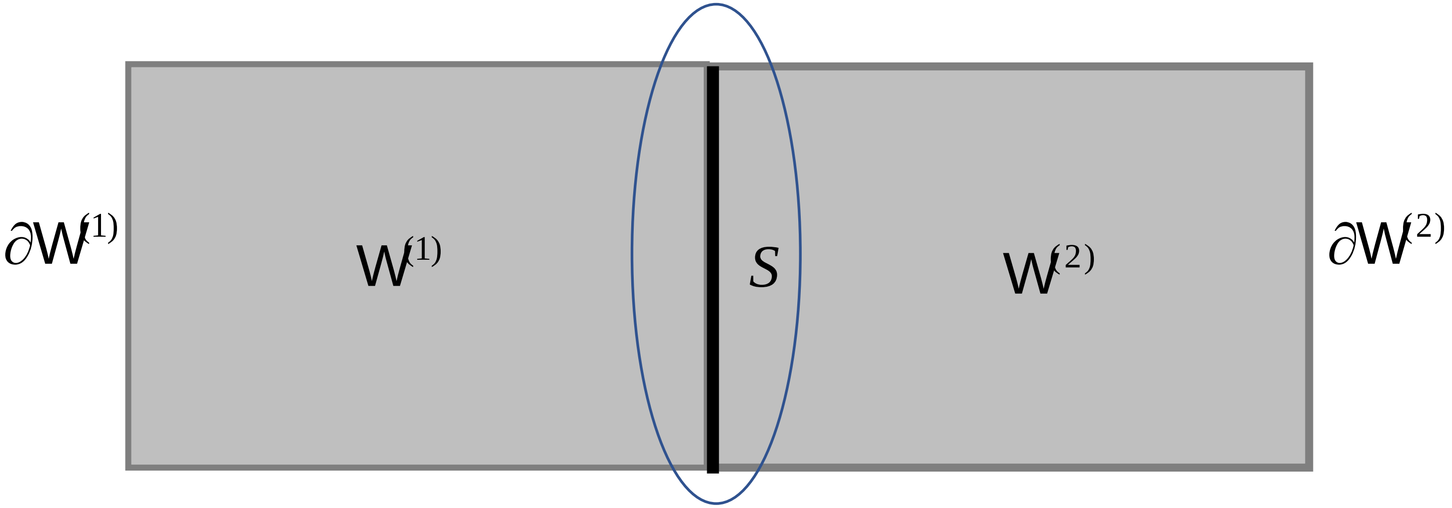
$$\left.\begin{aligned}p(\mathbf{x},\omega)&=p^b(\mathbf{x},\omega) \qquad \mathbf{x}\in S_p\\ \frac{\partial p}{\partial n}(\mathbf{x},\omega)&=\frac{\partial p^b}{\partial n}(\mathbf{x},\omega) \quad \mathbf{x}\in S_q\end{aligned}\right\} S=S_p\cup S_q$$

Coupling acoustic domains



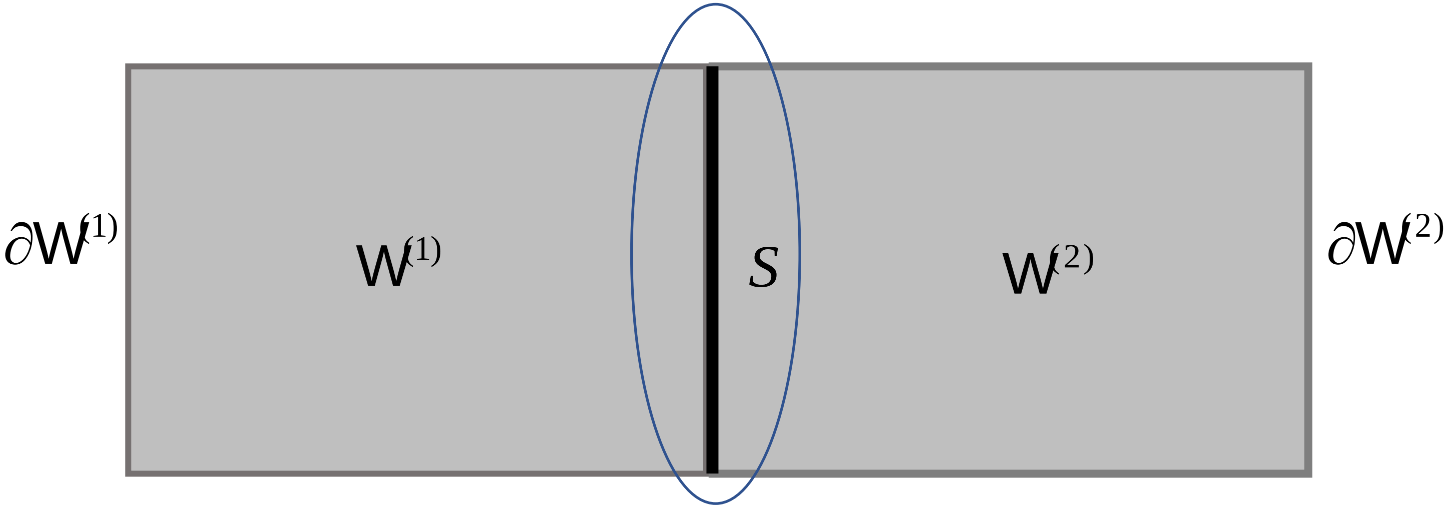
$$\mathbf{x} \in S \Rightarrow \begin{cases} p^{(1)} = p^{(2)} \\ \dfrac{1}{p^{(1)}} \partial_n p^{(1)} = -\dfrac{1}{p^{(2)}} \partial_n p^{(2)} \end{cases}$$

Coupling elastic domains



$$\mathbf{x} \in S \Rightarrow \begin{cases} \mathbf{u}^{(1)} = \mathbf{u}^{(2)} \\ \mathbf{t}^{(1)} = -\mathbf{t}^{(2)} \end{cases}$$

Coupling acoustic elastic domains



$$\mathbf{x} \in S \Rightarrow \begin{cases} \dfrac{1}{\omega^2 \rho^{(1)}} \partial_n p^{(1)} = -\hat{\mathbf{n}} \cdot \mathbf{u}^{(2)} \\ p^{(1)} = \hat{\mathbf{n}} \cdot \mathbf{t}^{(2)} \end{cases}$$

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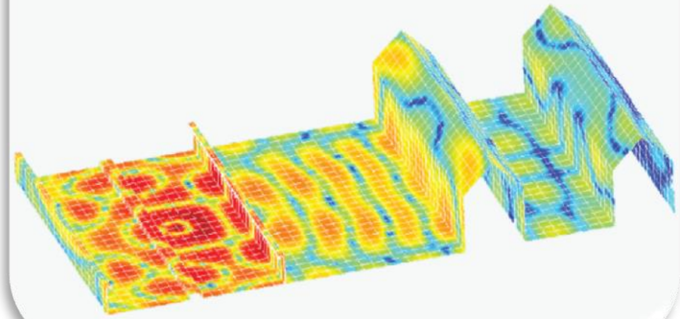
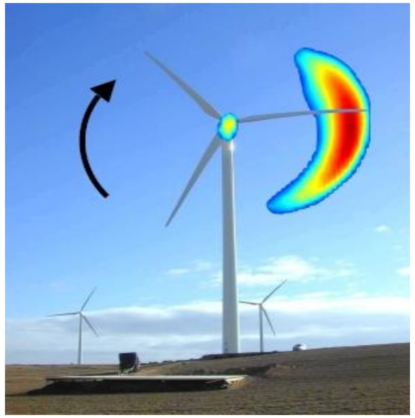
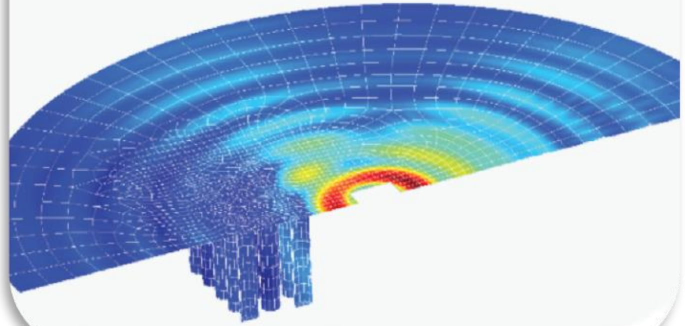
BEM in acoustics and fluid structure interaction

NOISE BARRIER DESIGN

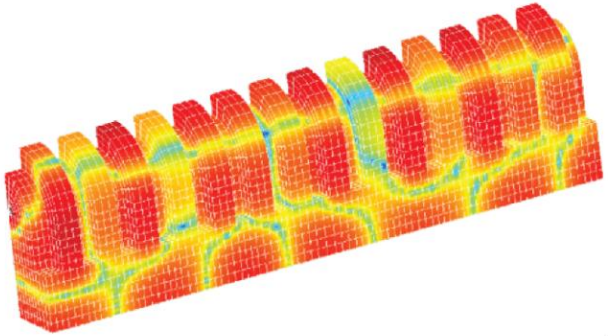




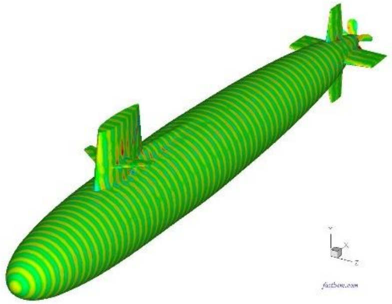
VIBRATION ISOLATION



INTERIOR ACOUSTICS



BEM



MULTIPLE SCATTERING