Advanced nonlinear structural analysis

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Outline

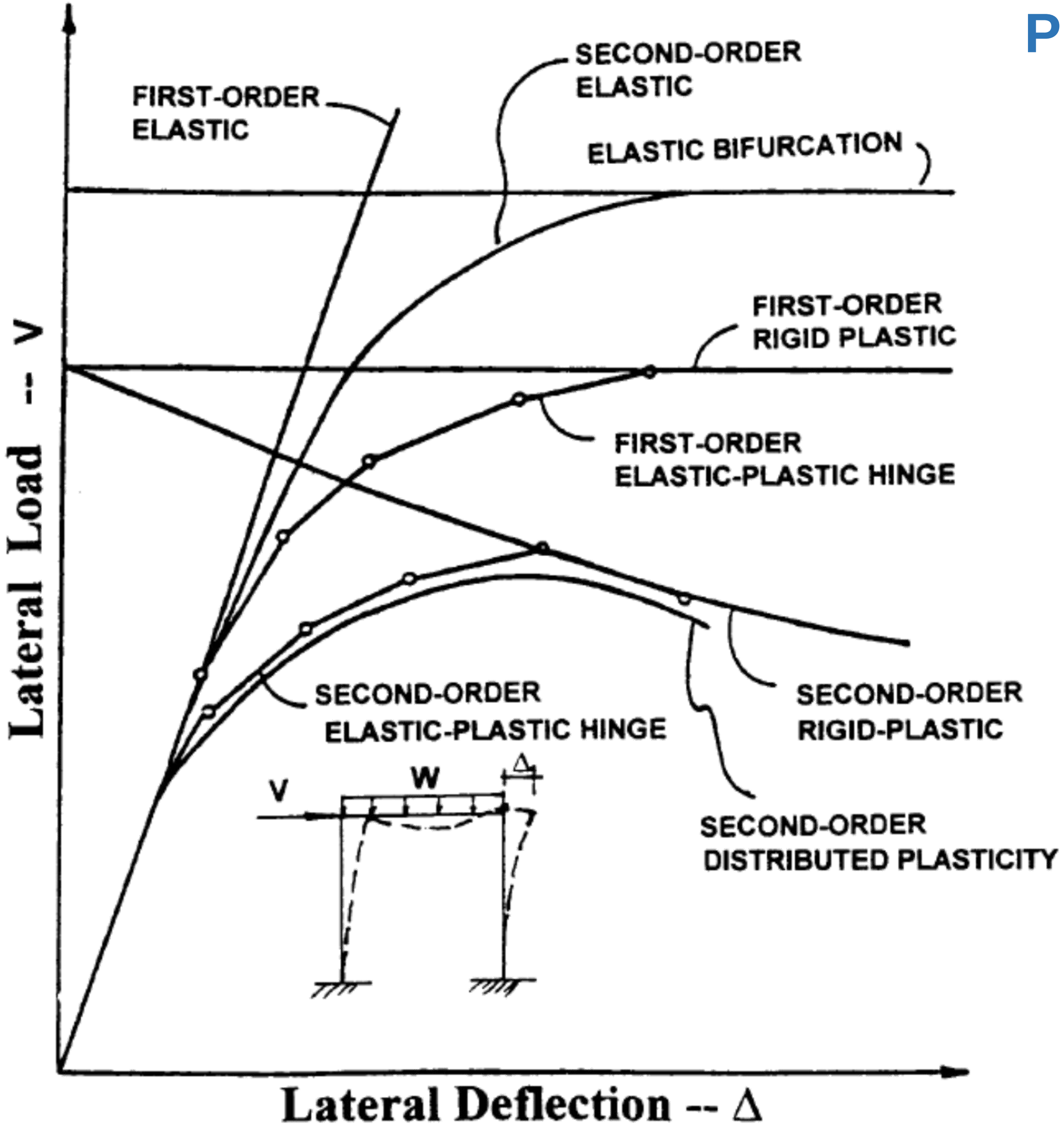
- Sources of Nonlinearities
- Introduction to Nonlinear Structural Analysis
- Finite element formulation for nonlinear dynamic analysis
- Advanced topics

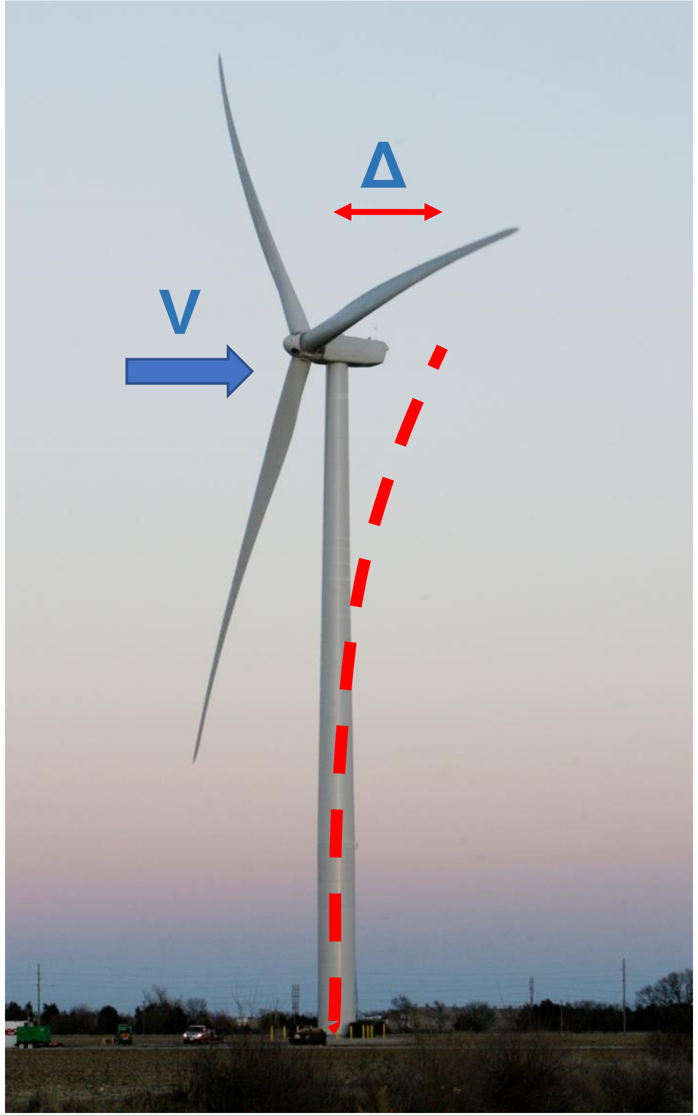

Linear
Nonlinear

Sources of nonlinearity

- Geometric
- Material
- Force Boundary Conditions
- Displacement Boundary Conditions

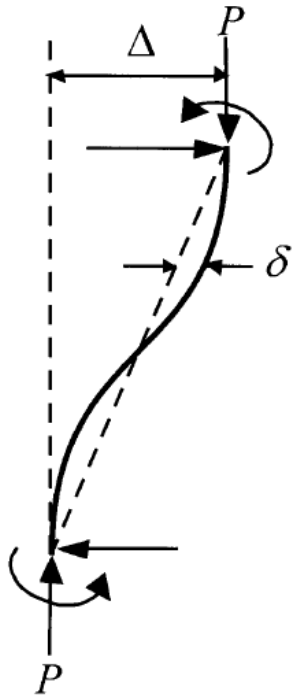
Pushover analysis of wind turbine towers





Methods		Features		
		Constitutive Relationship	Equilibrium Formulation	Geometric Compatibility
First-order	Elastic Rigid–plastic Elastic–plastic hinge Distributed plasticity	Elastic Rigid plastic Elastic perfectly plastic Inelastic	Original undeformed geometry	Small strain and small displacement
Second-order	Elastic Rigid–plastic Elastic–plastic hinge Distributed plasticity	Elastic Rigid–plastic Elastic perfectly plastic Inelastic	Deformed structural geometry (P-Δ and P-δ)	Small strain and moderate rotation (displacement may be large)
True large displacement	Elastic Inelastic	Elastic Inelastic	Deformed structural geometry	Large strain and large deformation

Structural Analysis Methods



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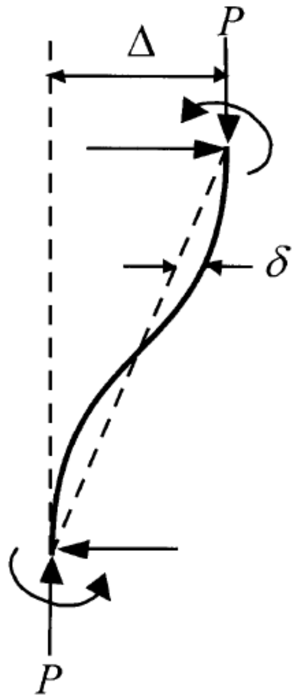
Structural Analysis Methods



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Structural Analysis Methods

General Guidelines

First-order analysis VS. second-order analysis

True large displacement analysis

Elastic analysis VS. Inelastic analysis

Bowing effect / Wagner effect / Other effects

Steel nonlinearity

Concrete nonlinearity

Composite materials nonlinearity

Other nonlinearities

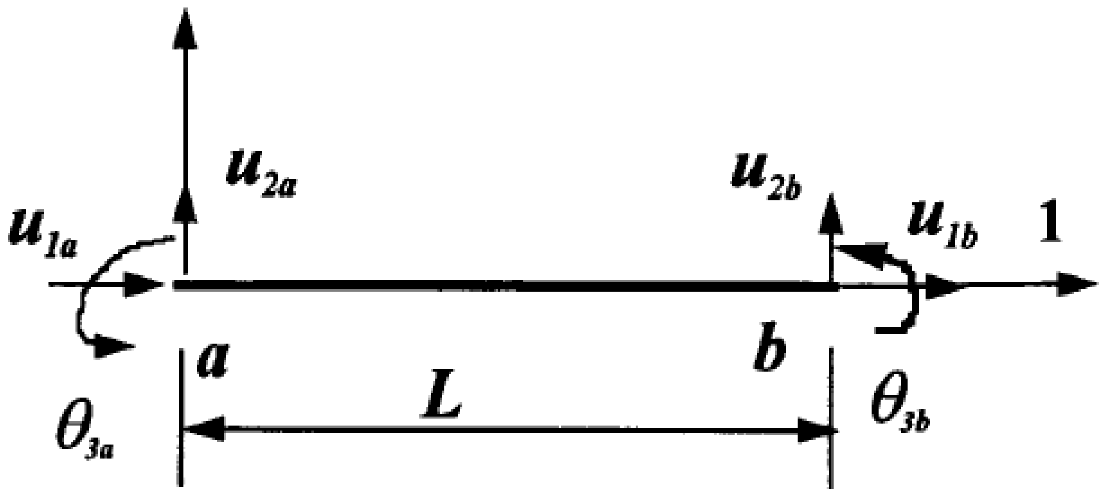
Geometric Nonlinearity Formulation

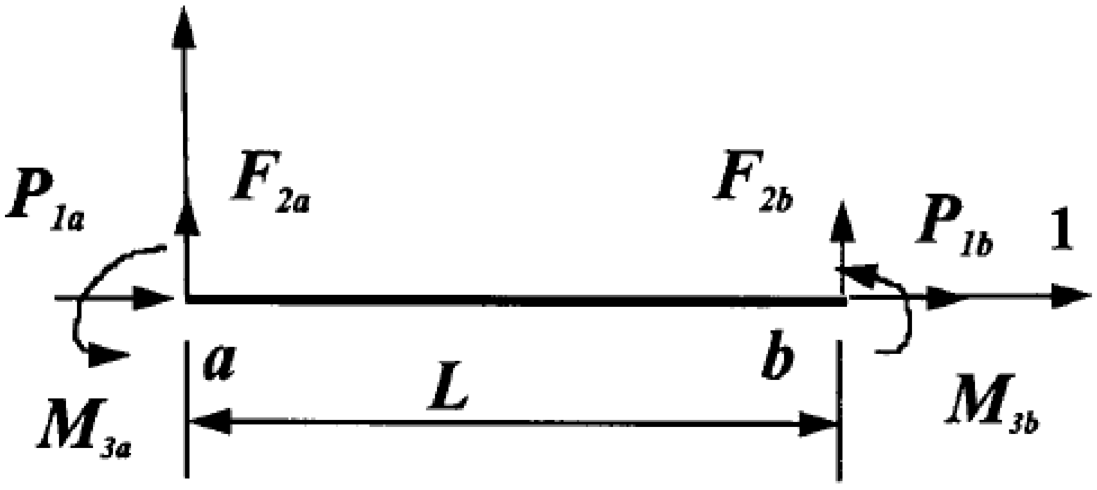
2-D

$$\{F\} = [K]\{D\}$$

$$\{F\} = \{P_{1a},\ F_{2a},\ M_{3a},\ P_{1b}, F_{2b}, M_{3b}\}^T$$

$$\{D\} = \{u_{1a},\ u_{2a},\ \theta_{3a},\ u_{1b}, u_{2b}, \theta_{3b}\}^T$$





Geometric Nonlinearity Formulation

$$\{F\} = [K]\{D\}$$

$$[K] = [K_e] + [K_g]$$

$$[K] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ & \frac{12EI}{L^3}\phi_1 & \frac{-6EI}{L^2}\phi_2 & 0 & \frac{-12EI}{L^3}\phi & \frac{-6EI}{L^2}\phi_2 \\ & & 4\phi_3 & 0 & \frac{6EI}{L^2}\phi_2 & 2\phi_4 \\ & & & \frac{AE}{L} & 0 & 0 \\ & & & & \frac{12EI}{L^3}\phi & \frac{6EI}{L^2}\phi_2 \\ & & & & & 4\phi_3 \end{bmatrix}$$

$$\phi_1 \quad \frac{1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)!}\left[\mp(kL)^2\right]^n}{12\phi}$$

$$\phi_2 \quad \frac{\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{(2n+2)!}\left[\mp(kL)^2\right]^n}{6\phi}$$

$$\phi_3 \quad \frac{\frac{1}{3} + \sum_{n=1}^{\infty} \frac{2(n+1)}{(2n+3)!}\left[\mp(kL)^2\right]^n}{4\phi}$$

$$\phi_4 \quad \frac{\frac{1}{6} + \sum_{n=1}^{\infty} \frac{1}{(2n+3)!}\left[\mp(kL)^2\right]^n}{2\phi}$$

$$\phi \quad \frac{1}{12} + \sum_{n=1}^{\infty} \frac{2(n+1)}{(2n+4)!}\left[\mp(kL)^2\right]^n$$

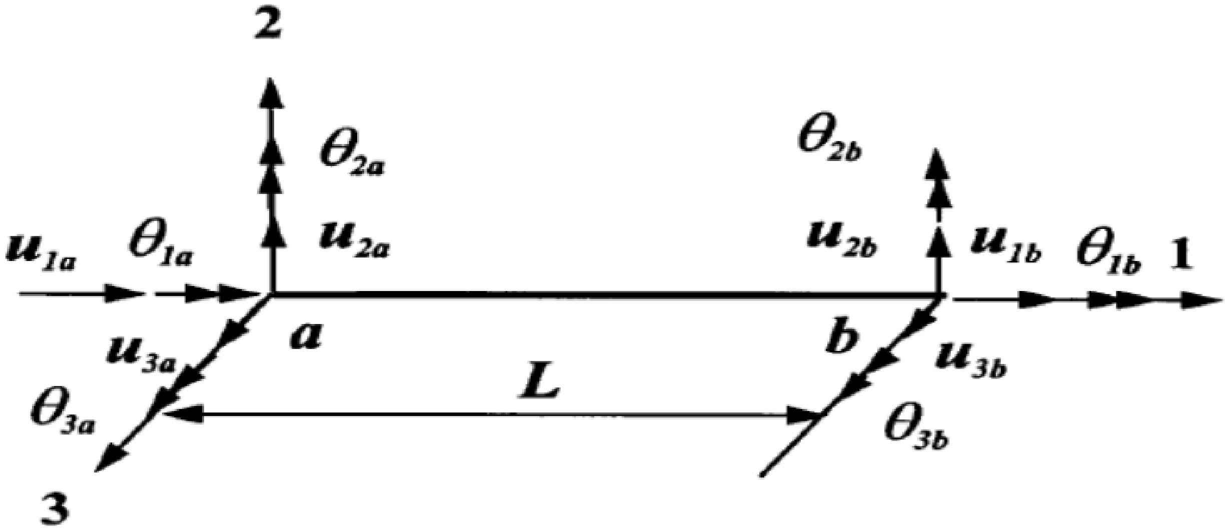
Geometric Nonlinearity Formulation

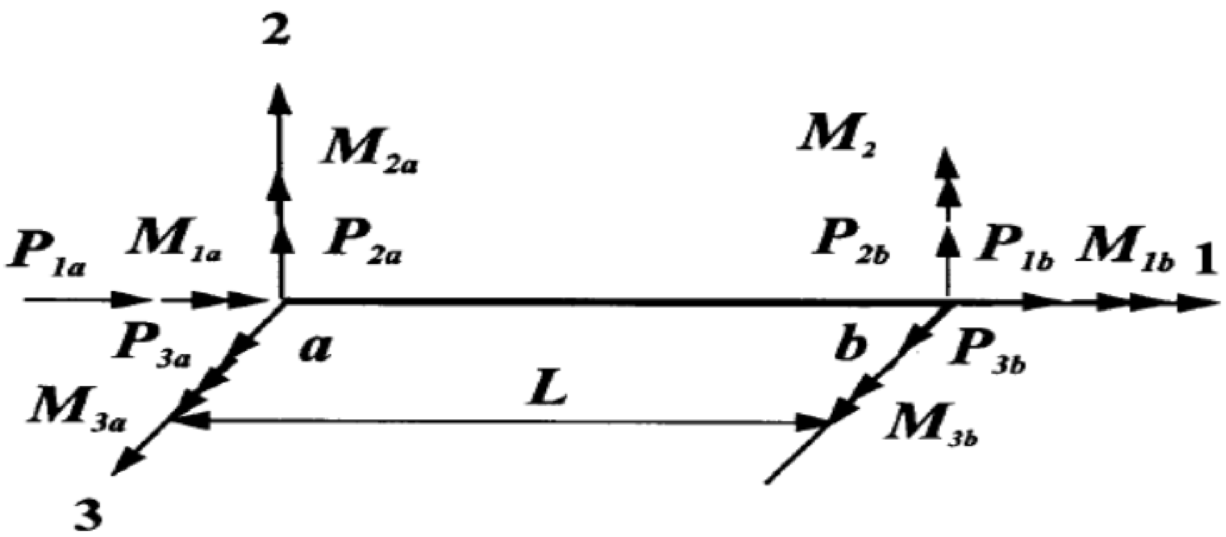
$$\{F\} = [K]\{D\}$$

3-D

$$\{F\} = \{P_{1a}, F_{2a}, F_{3a}, M_{1a}, M_{2a}, M_{3a}, P_{1b}, F_{2b}, F_{3b}, M_{1b}, M_{2b}, M_{3b}\}^T$$

$$\{D\} = \{u_{1a}, u_{2a}, u_{3a}, \theta_{1a}, \theta_{2a}, \theta_{3a}, u_{1b}, u_{2b}, u_{3b}, \theta_{1b}, \theta_{2b}, \theta_{3b}\}^T$$





$$
[\boldsymbol{K}] = \begin{bmatrix}
\phi_{s1} & 0 & 0 & 0 & 0 & 0 & -\phi_{s1} & 0 & 0 & 0 & 0 & 0 \\
 & \phi_{s7} & 0 & 0 & 0 & \phi_{s6} & 0 & -\phi_{s7} & 0 & 0 & 0 & \phi_{s6} \\
 & & \phi_{s9} & 0 & -\phi_{s8} & 0 & 0 & 0 & -\phi_{s9} & 0 & -\phi_{s8} & 0 \\
 & & & \dfrac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & -\dfrac{GJ}{L} & 0 & 0 \\
 & & & & \phi_{s4} & 0 & 0 & 0 & \phi_{s8} & 0 & \phi_{s5} & 0 \\
 & & & & & \phi_{s2} & 0 & -\phi_{s6} & 0 & 0 & 0 & \phi_{s3} \\
 & & & & & & \phi_{s1} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \phi_{s7} & 0 & 0 & 0 & -\phi_{s6} \\
 & & \text{Sym.} & & & & & & \phi_{s9} & 0 & \phi_{s8} & 0 \\
 & & & & & & & & & \dfrac{GJ}{L} & 0 & 0 \\
 & & & & & & & & & & \phi_{s4} & 0 \\
 & & & & & & & & & & & \phi_{s2}
\end{bmatrix}
$$

3-D Stiffness matrix

ϕ_{si}	Stability Functions S_i	Compression	Tension
$\phi_{s1} = S_1 \dfrac{EA}{L}$	S_1	$\dfrac{1}{1-\dfrac{EA}{4P^3L^2}\left[H_y + H_z\right]}$	$\dfrac{1}{1-\dfrac{EA}{4P^3L^2}\left[H'_y + H'_z\right]}$
$\phi_{s2} = S_2 \dfrac{(4+\phi_y)EI_Z}{(1+\phi_y)L}$	S_2	$\dfrac{(\alpha L)(\sin\alpha L - \alpha L\cos\alpha L)}{4\phi_\alpha}$	$\dfrac{(\alpha L)(\alpha L\cosh\alpha L - \sinh\alpha L)}{4\phi_\alpha}$
$\phi_{s3} = S_2 \dfrac{(2-\phi_y)EI_Z}{(1+\phi_y)L}$	S_3	$\dfrac{(\alpha L)(\alpha L - \sin\alpha L)}{2\phi_\alpha}$	$\dfrac{(\alpha L)(\sinh\alpha L - \alpha L)}{2\phi_\alpha}$
$\phi_{s4} = S_4 \dfrac{(4+\phi_z)EI_y}{(1+\phi_z)L}$	S_4	$\dfrac{(\beta L)(\sin\beta L - \beta L\cos\beta L)}{4\phi_\beta}$	$\dfrac{(\beta L)(\beta L\cosh\beta L - \sinh\beta L)}{4\phi_\beta}$
$\phi_{s5} = S_2 \dfrac{(2-\phi_z)EI_y}{(1+\phi_z)L}$	S_5	$\dfrac{(\beta L)(\beta L - \sin\beta L)}{2\phi_\beta}$	$\dfrac{(\beta L)(\sinh\beta L - \beta L)}{2\phi_\beta}$
$\phi_{s6} = S_6 \dfrac{6EI_Z}{(1+\phi_y)L^2}$	S_6	$\dfrac{(\alpha L)^2(1-\cos\alpha L)}{6\phi_\alpha}$	$\dfrac{(\alpha L)^2(\cosh\alpha L - 1)}{6\phi_\alpha}$
$\phi_{s7} = S_7 \dfrac{12EI_Z}{(1+\phi_y)L^3}$	S_7	$\dfrac{(\alpha L)^3\sin\alpha L}{12\phi_\alpha}$	$\dfrac{(\alpha L)^3\sinh\alpha L}{12\phi_\alpha}$
$\phi_{s8} = S_8 \dfrac{6EI_y}{(1+\phi_z)L^2}$	S_8	$\dfrac{(\beta L)^2(1-\cos\beta L)}{6\phi_\beta}$	$\dfrac{(\beta L)^2(\cosh\beta L - 1)}{6\phi_\beta}$
$\phi_{s9} = S_9 \dfrac{12EI_y}{(1+\phi_z)L^3}$	S_9	$\dfrac{(\beta L)^3\sin\beta L}{12\phi_\beta}$	$\dfrac{(\beta L)^3\sinh\beta L}{12\phi_\beta}$
$\alpha = \sqrt{P/EI_Z}$	ϕ_α	$2-2\cos\alpha L - \alpha L\sin\alpha L$	$2-2\cosh\alpha L + \alpha L\sinh\alpha L$
$\beta = \sqrt{P/EI_y}$	ϕ_β	$2-2\cos\beta L - \beta L\sin\beta L$	$2-2\cosh\beta L + \beta L\sinh\beta L$

$$H_y = \beta L(M_{ya}^2 + M_{yb}^2)(\cot\beta L + \beta L\,\mathrm{cosec}^2\beta L) - 2(M_{ya}+M_{yb})^2 + 2\beta L M_{ya}M_{yb}(\mathrm{cosec}\,\beta L)(1+\beta L\cot\beta L)$$

$$H_z = \alpha L(M_{za}^2 + M_{zb}^2)(\cot\alpha L + \alpha L\,\mathrm{cosec}^2\alpha L) - 2(M_{za}+M_{zb})^2 + 2\alpha L M_{za}M_{zb}(\mathrm{cosec}\,\alpha L)(1+\alpha L\cot\alpha L)$$

$$H'_y = \beta L(M_{ya}^2 + M_{yb}^2)(\coth\beta L + \beta L\,\mathrm{cosech}^2\beta L) - 2(M_{ya}+M_{yb})^2 + 2\beta L M_{ya}M_{yb}(\mathrm{cosech}\,\beta L)(1+\beta L\coth\beta L)$$

$$H'_z = \alpha L(M_{za}^2 + M_{zb}^2)(\coth\alpha L + \alpha L\,\mathrm{cosech}^2\alpha L) - 2(M_{za}+M_{zb})^2 + 2\alpha L M_{za}M_{zbb}(\mathrm{cosech}\,\alpha L)(1+\alpha L\coth\alpha L)$$

Linear Elastic Matrix

$$\phi_{e1} = \frac{AE}{L}\ ; \quad \phi_{e2} = \frac{4EI_Z}{L}$$

$$\phi_{e3} = \frac{2EI_z}{L}\ ; \quad \phi_{e4} = \frac{4EI_y}{L}$$

$$\phi_{e5} = \frac{2EI_y}{L}\ ; \quad \phi_{e6} = \frac{6EI_Z}{L^2}$$

$$\phi_{e7} = \frac{12EI_Z}{L^3}\ ; \quad \phi_{e8} = \frac{6EI_y}{L^2}$$

$$\phi_{e9} = \frac{12EI_y}{L^3}$$

Geometric Nonlinear Matrix

$$\phi_{g1} = 0\ ; \quad \phi_{g2} = \phi_{g4} = \frac{2F_{xb}L}{15}\ ; \quad \phi_{g3} = \phi_{g5} = \frac{F_{xb}L}{30}$$

$$\phi_{g7} = \phi_{g9} = \frac{6F_{xb}}{5L}\ ; \quad \phi_{g6} = \phi_{g8} = \frac{F_{xb}}{10}\ ; \quad \phi_{g10} = \frac{M_{za}+M_{zb}}{L^2}$$

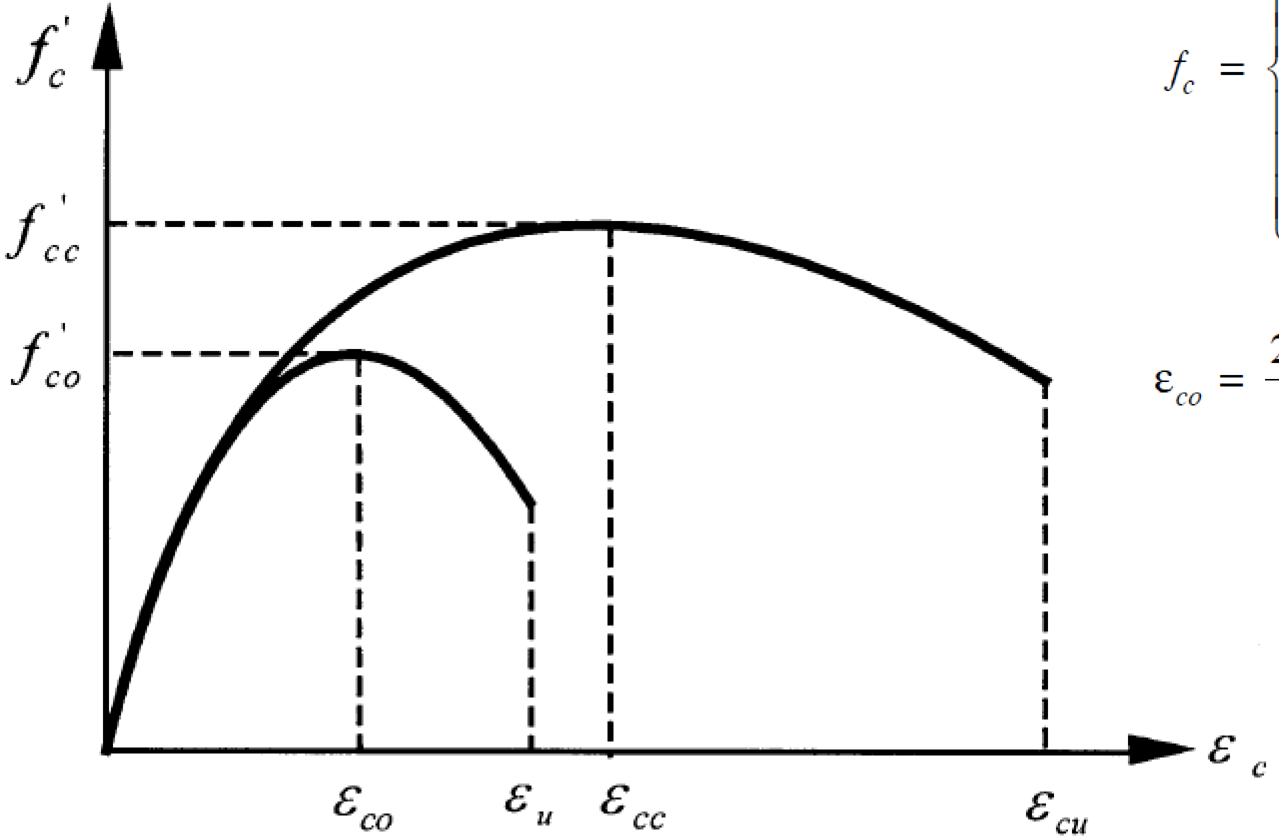
$$\phi_{g11} = \frac{M_{ya}+M_{yb}}{L^2}\ ; \quad \phi_{g12} = \frac{M_{ya}}{L}\ ; \quad \phi_{g13} = \frac{M_{xb}}{L}$$

$$\phi_{g14} = \frac{M_{yb}}{L}\ ; \quad \phi_{g15} = \frac{M_{za}}{L}\ ; \quad \phi_{g16} = \frac{M_{zb}}{L}$$

$$\phi_{g17} = \frac{F_{xb}I_p}{AL}\ ; \quad \phi_{g18} = \frac{M_{zb}}{6} - \frac{M_{za}}{3}\ ; \quad \phi_{g19} = \frac{M_{ya}}{3} - \frac{M_{yb}}{6}$$

$$\phi_{g20} = \frac{M_{za}+M_{zyb}}{6}\ ; \quad \phi_{g21} = \frac{M_{ya}+M_{yb}}{6}$$

Material Nonlinearity Formulations



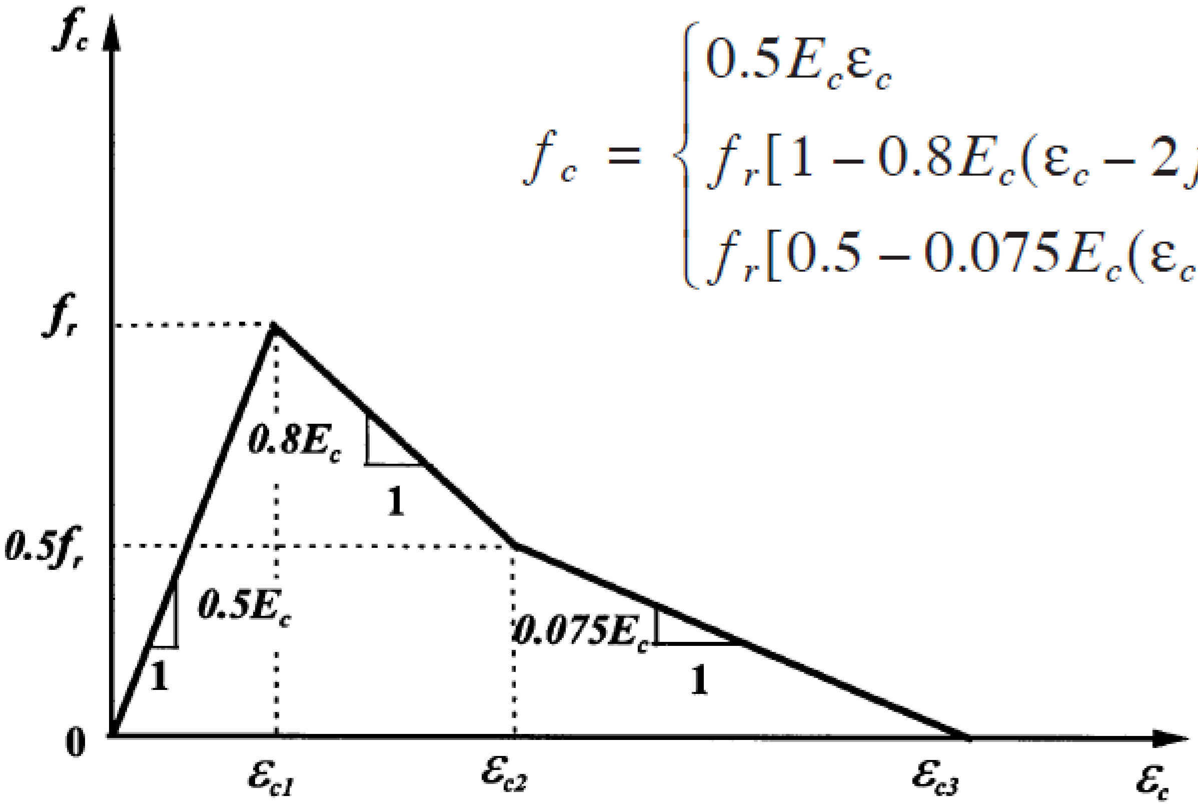
Idealized stress–strain curves for concrete in uniaxial <u>compression</u>

$$f_c = \begin{cases} f'_{co}\left[\dfrac{2\varepsilon_c}{\varepsilon_{co}} - \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] & \varepsilon_c \leq \varepsilon_{co} \\ f'_{co}\left[1 - \beta\left(\dfrac{\varepsilon_c - \varepsilon_o}{\varepsilon_u - \varepsilon_{co}}\right)\right] & \varepsilon_{co} < \varepsilon_c \leq \varepsilon_u \end{cases}$$

$$\varepsilon_{co} = \frac{2 f'_{co}}{E_c}$$

Concrete: towers / foundation / piles

Material Nonlinearity Formulations

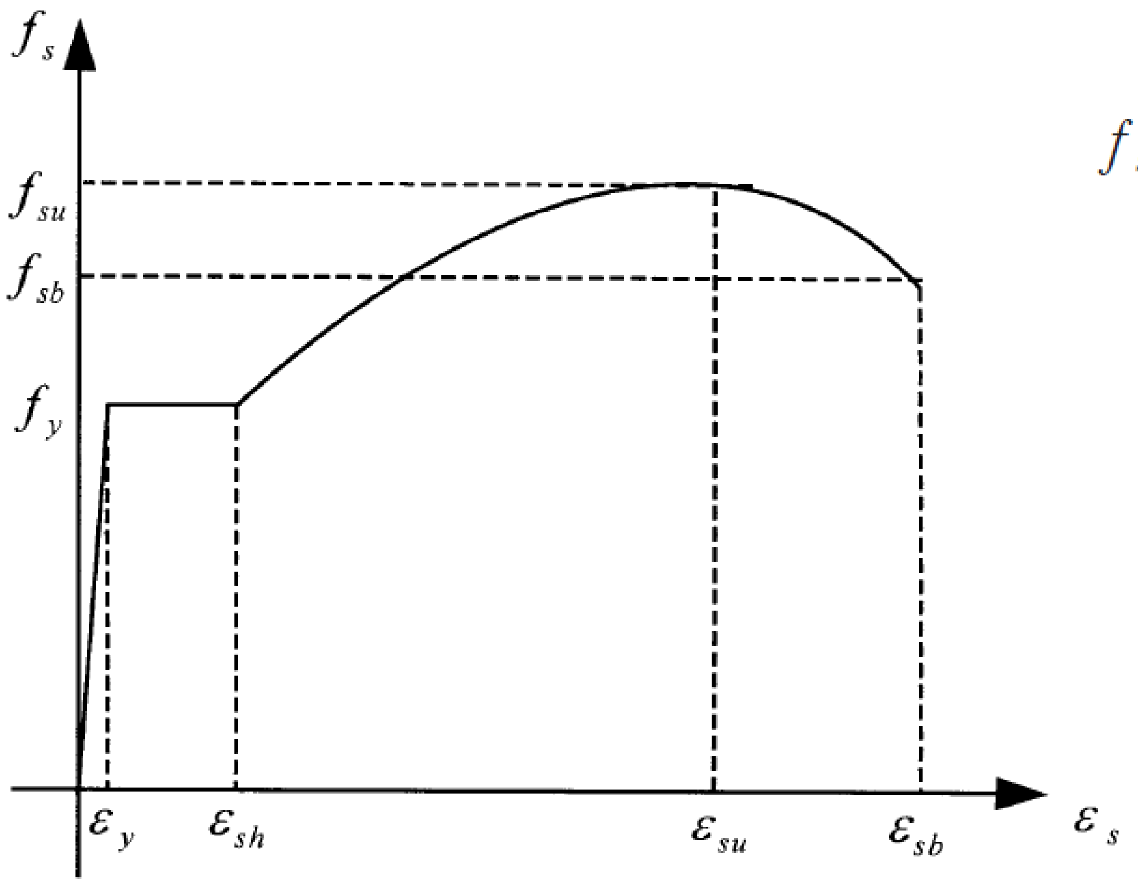


$$f_c = \begin{cases} 0.5E_c\varepsilon_c & \varepsilon_c \le \varepsilon_{c1} = 2f_r/E_c \\ f_r[1 - 0.8E_c(\varepsilon_c - 2f_r/E_c)] & \varepsilon_{c1} < \varepsilon_c \le \varepsilon_{c2} = 2.625f_r/E_c \\ f_r[0.5 - 0.075E_c(\varepsilon_c - 2.625f_r/4E_c)] & \varepsilon_c < \varepsilon_{c3} = 9.292f_r/E_c \end{cases}$$

Idealized stress–strain curves for concrete in uniaxial **tension**

Concrete: towers / foundation / piles

Material Nonlinearity Formulations

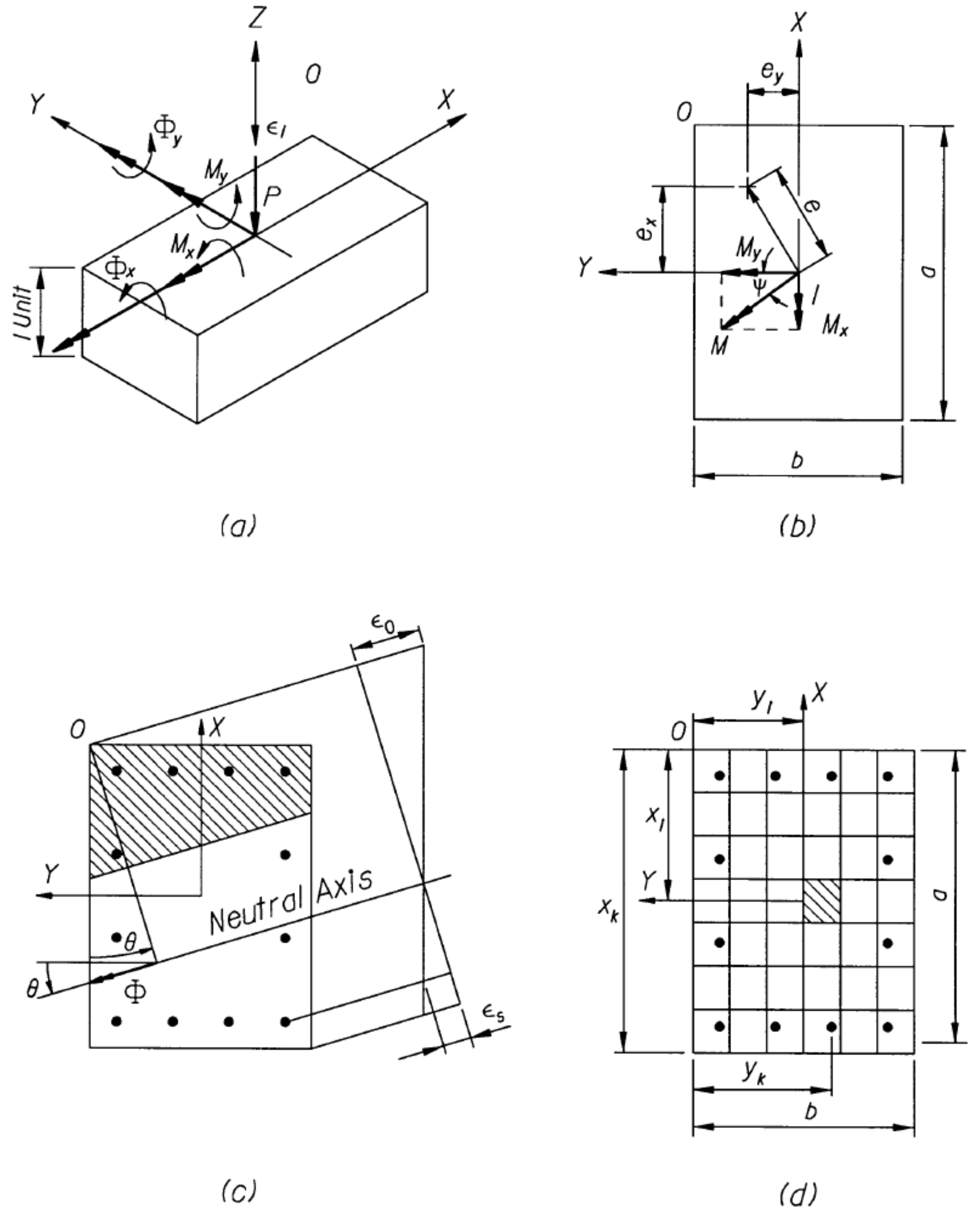


$$f_s = \begin{cases} E_s \varepsilon_s & 0 \le \varepsilon_s \le \varepsilon_y \\ f_y & \varepsilon_{sy} < \varepsilon_s \le \varepsilon_{sh} \\ f_y + \dfrac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_{su} - \varepsilon_{sh}}(f_{su} - f_y) & \varepsilon_{sh} < \varepsilon_s \le \varepsilon_{su} \\ f_u - \dfrac{\varepsilon_s - \varepsilon_{su}}{\varepsilon_{sb} - \varepsilon_{su}}(f_{su} - f_{sb}) & \varepsilon_{cu} < \varepsilon_s \le \varepsilon_{sb} \end{cases}$$

Idealized stress–strain curves for steel in uniaxial tension & compression

Steel: towers / reinforcement / shaft / hub etc.

Nonlinear Section Analysis



Compatibility equations

$$\phi_x = \varepsilon / y$$

$$\phi_y = \varepsilon / x$$

$$P = \int_A \sigma \, dA = \sum_{i=1}^{n} \sigma_i A_i$$

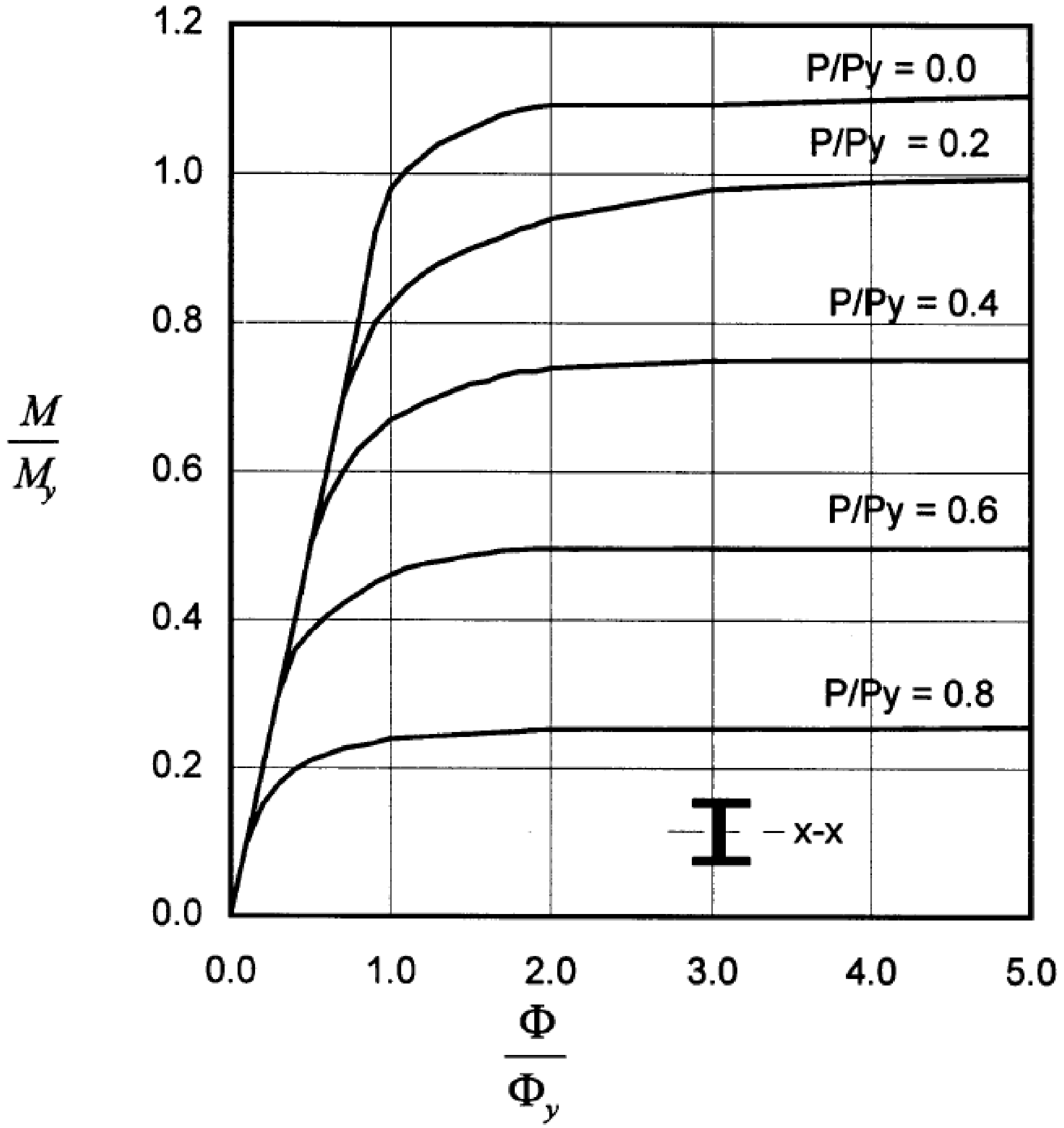
Equilibrium equations

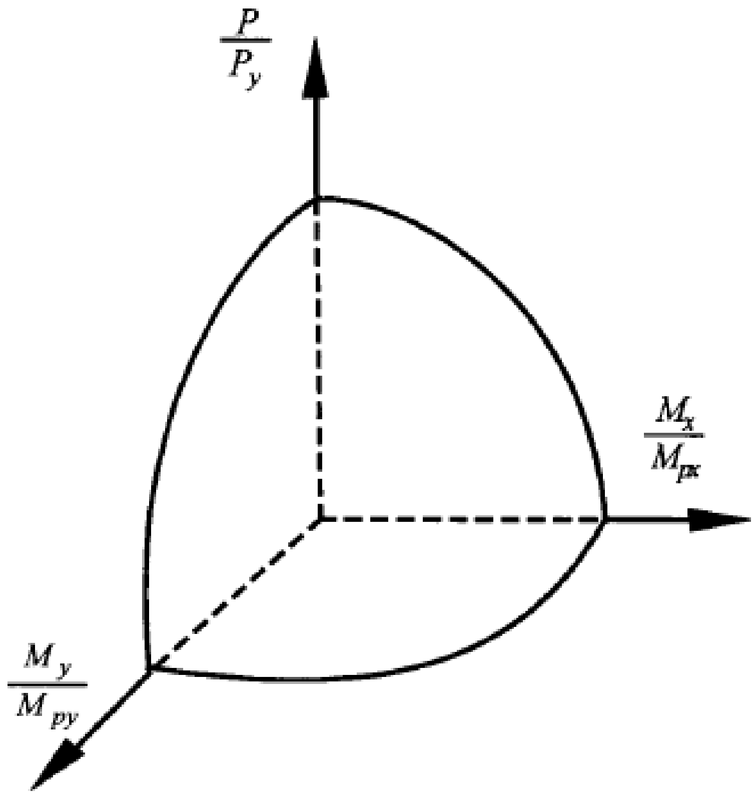
$$M_x = \int_A \sigma \, y \, dA = \sum_{i=1}^{n} \sigma_i y_i A_i$$

$$M_y = \int_A \sigma \, x \, dA = \sum_{i=1}^{n} \sigma_i x_i A_i$$

Nonlinear Section Analysis

Moment–thrust–curvature curve for a steel I-section (**2-D** loads)

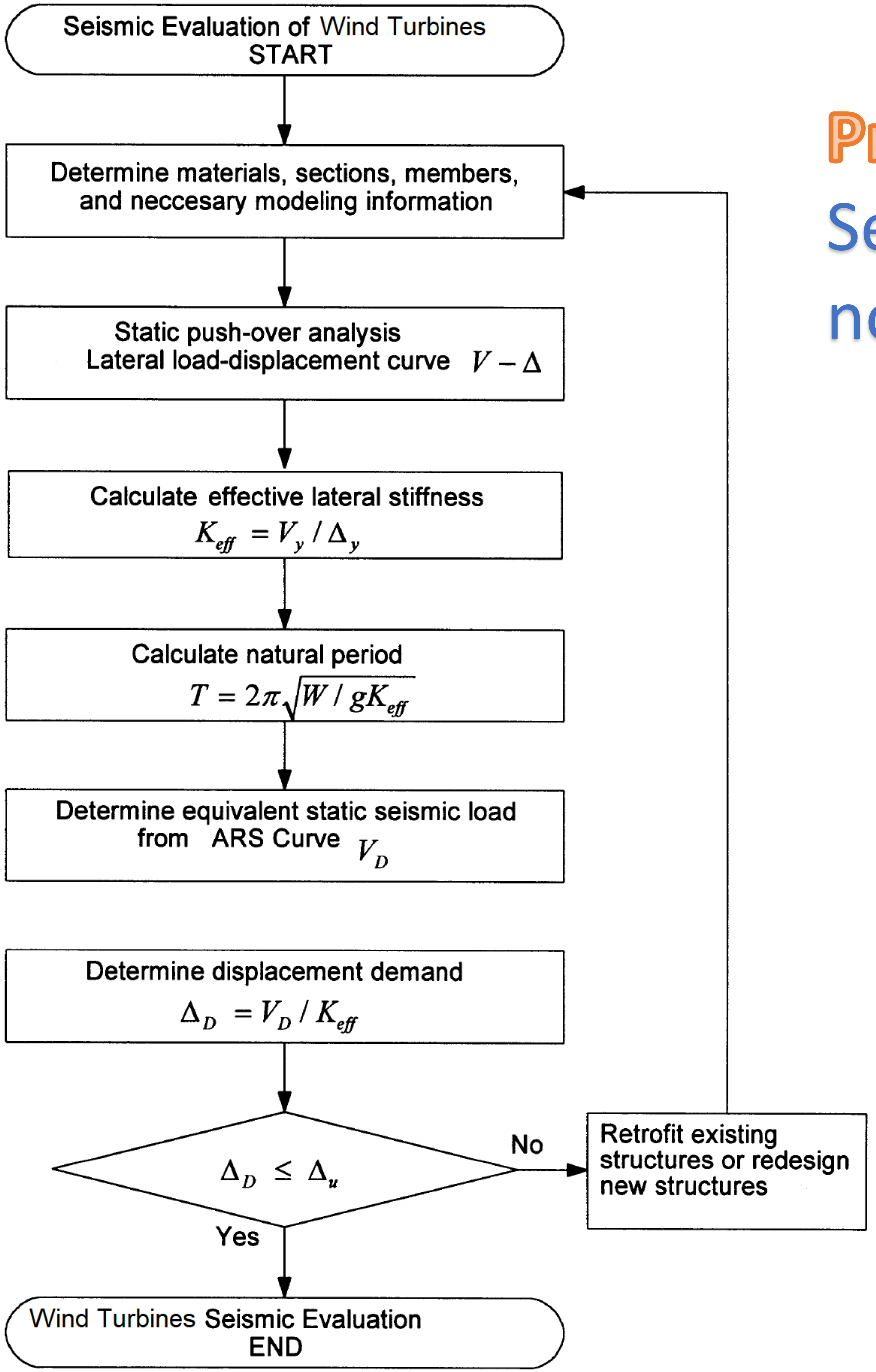


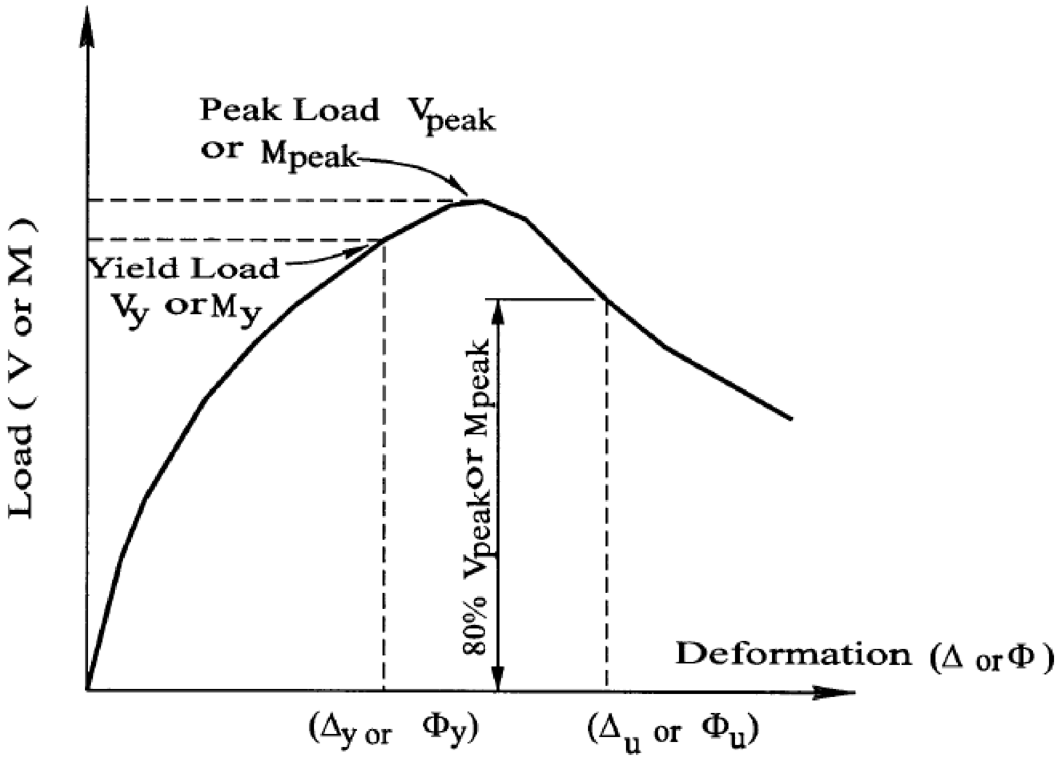


General yield surface for a steel I-section (**3-D** loads)

Practical Applications

Seismic evaluation of wind turbines with nonlinear behavior





Practical Applications

Linear and Nonlinear Dynamic Analysis

$$[M]\cdot\ddot{u}+[C]\cdot\dot{u}+[K]\cdot u=R(t)$$

$[M]$	Mass matrix
$[C]$	Damping matrix
$[K]$	Stiffness matrix (constant)
$R(t)$	Vector of external loads
$u,\dot{u},\ddot{u}$	Displacement, velocity and acceleration vectors

$[M]$, $[C]$, $[K]$ symmetrical

Practical Applications

Linear and Nonlinear Dynamic Analysis

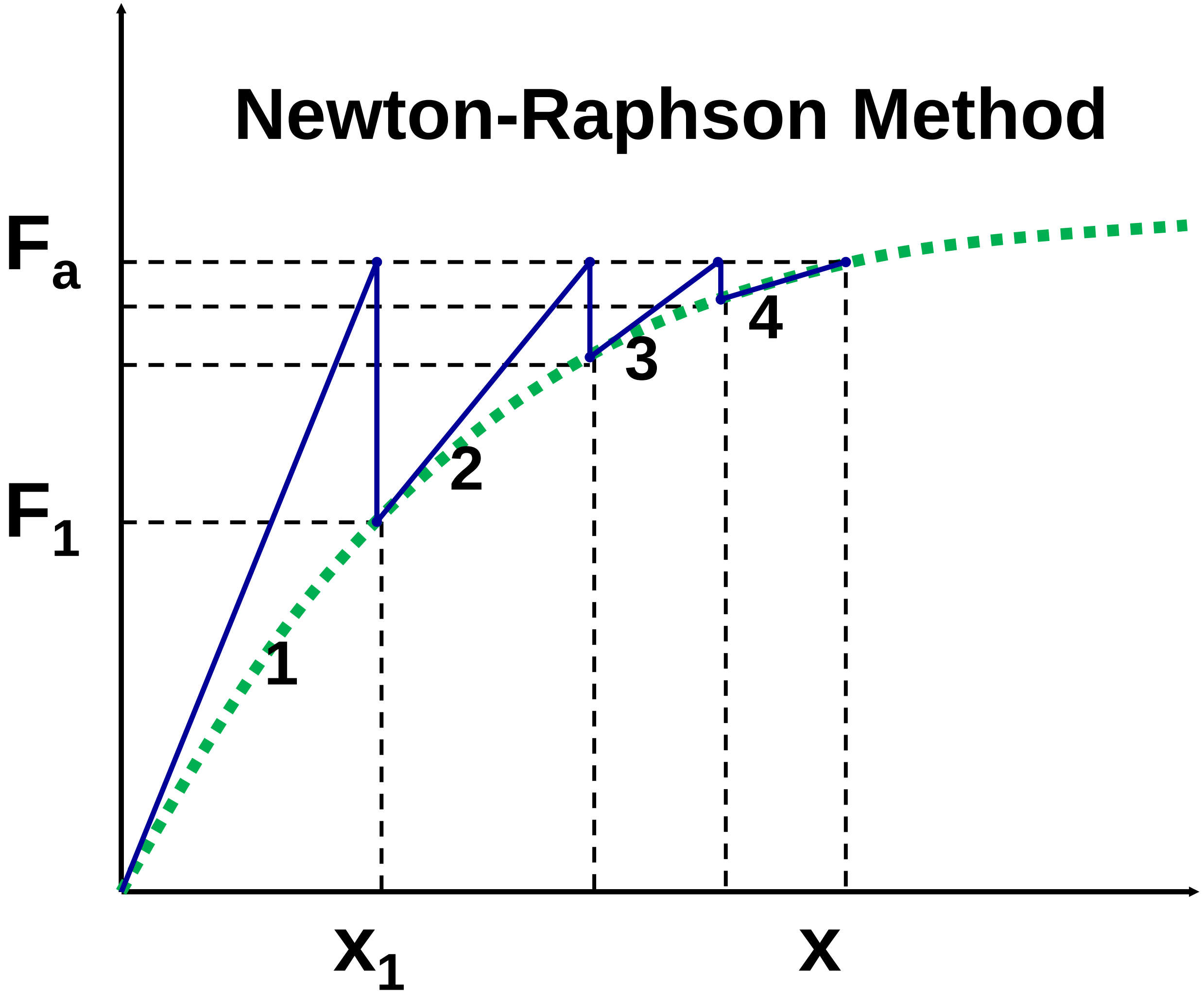
$$[M]\cdot\ddot{u}+[C]\cdot\dot{u}+[K(t)]\cdot u=R(t)$$

$[M]$	Mass matrix
$[C]$	Damping matrix
$[K(t)]$	Stiffness matrix (time depended)
$R(t)$	Vector of external loads
$u, \dot{u}, \ddot{u}$	Displacement, velocity and acceleration vectors

$[M]$, $[C]$, $[K(t)]$ symmetrical

Practical Applications

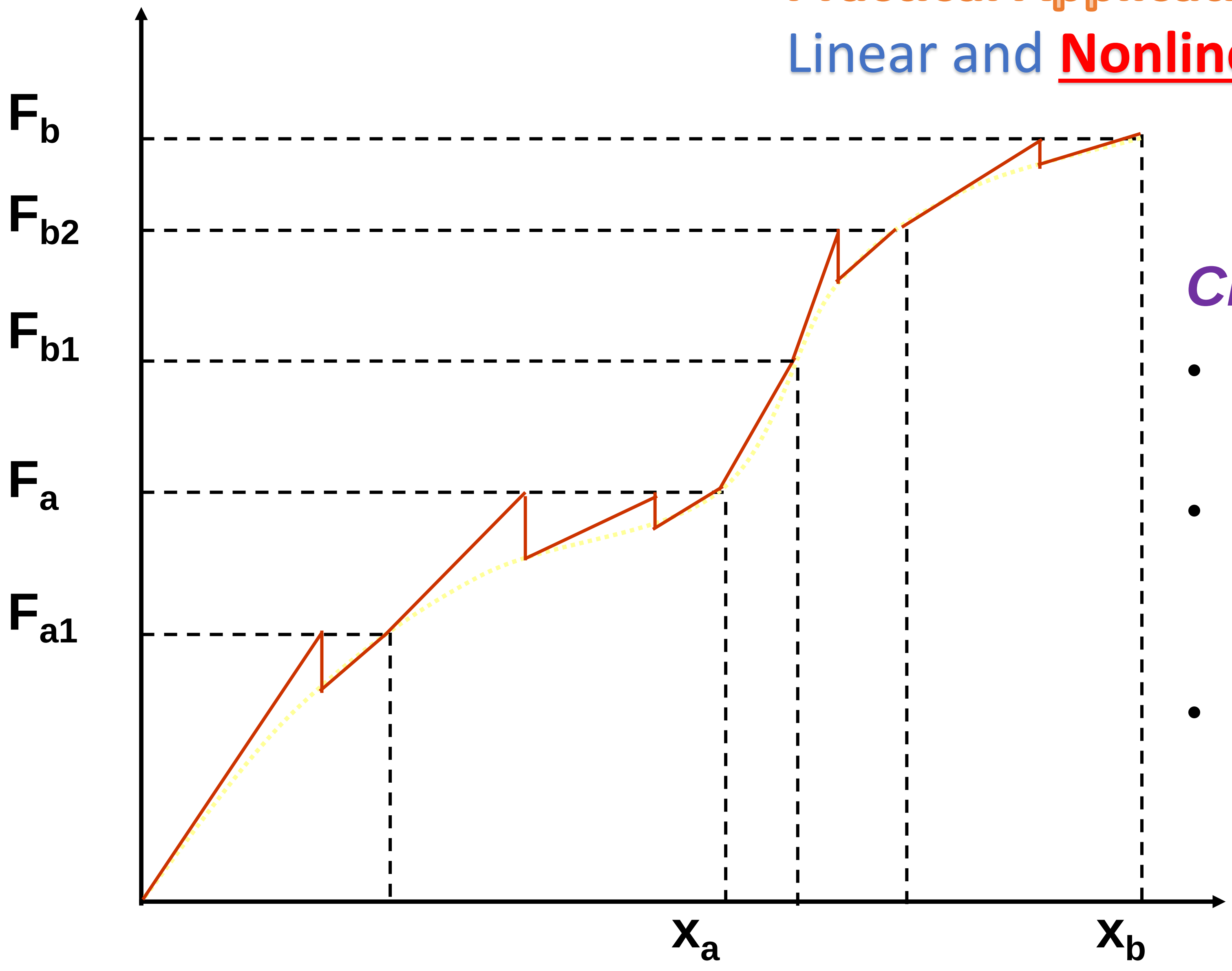
Linear and Nonlinear Dynamic Analysis



- The actual relationship between load and displacement is not known beforehand.
- Consequently, a series of linear approximations with corrections is performed. This is a simplified explanation of the Newton-Raphson method.

Practical Applications

Linear and **Nonlinear** Dynamic Analysis

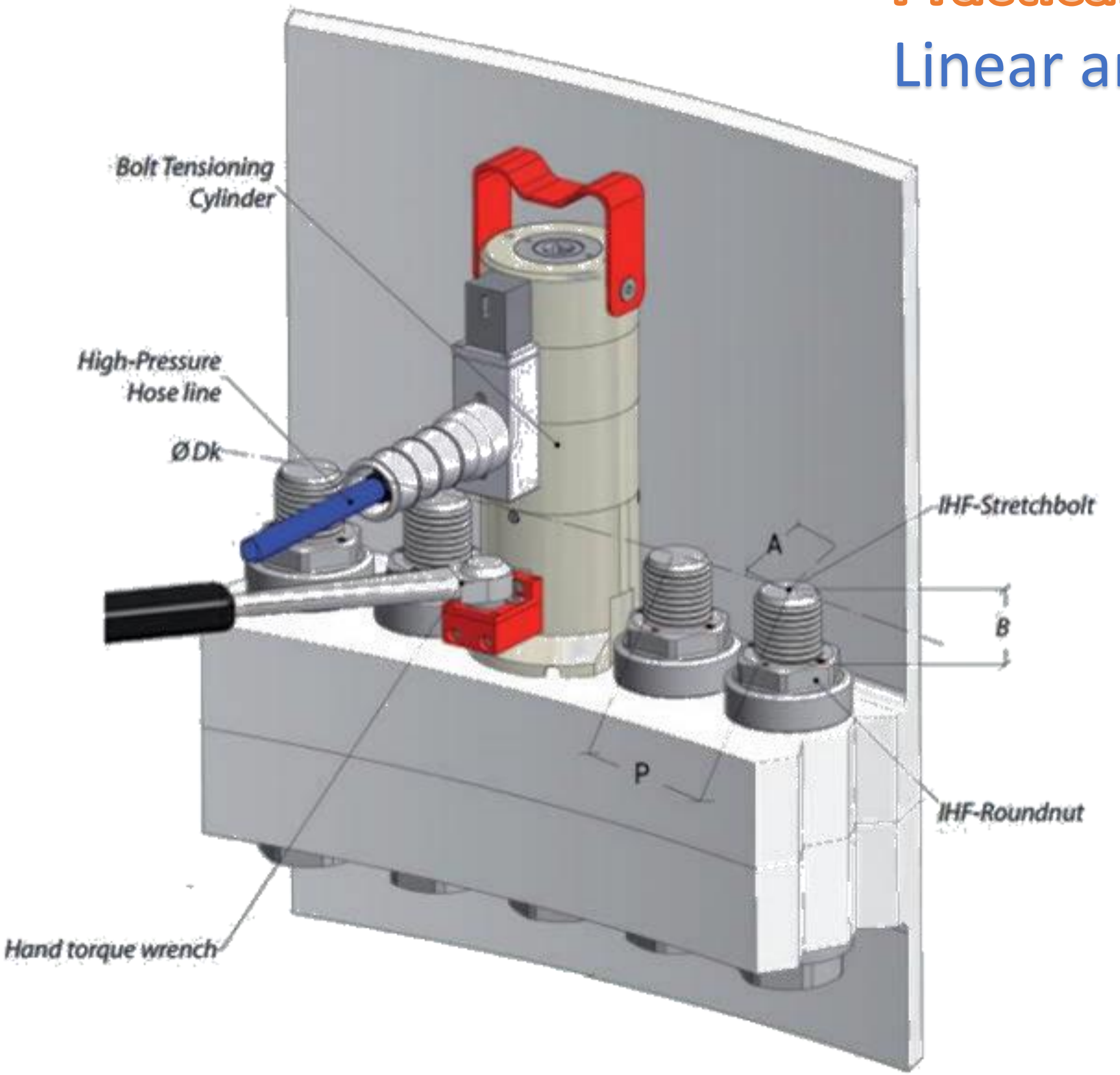


Critical facts

- ***Substeps*** **apply the loads in an incremental fashion**
- ***Equilibrium iterations*** **are the corrective solutions to obtain a converged substep**
- ***Load steps*** **are changes in general loading.**

Practical Applications

Linear and Nonlinear Dynamic Analysis



Critical facts

- ***Substeps*** **apply the loads in an incremental fashion**
- ***Equilibrium iterations*** **are the corrective solutions to obtain a converged substep**
- ***Load steps*** **are changes in general loading.**

Practical Applications

Linear and Nonlinear Dynamic Analysis

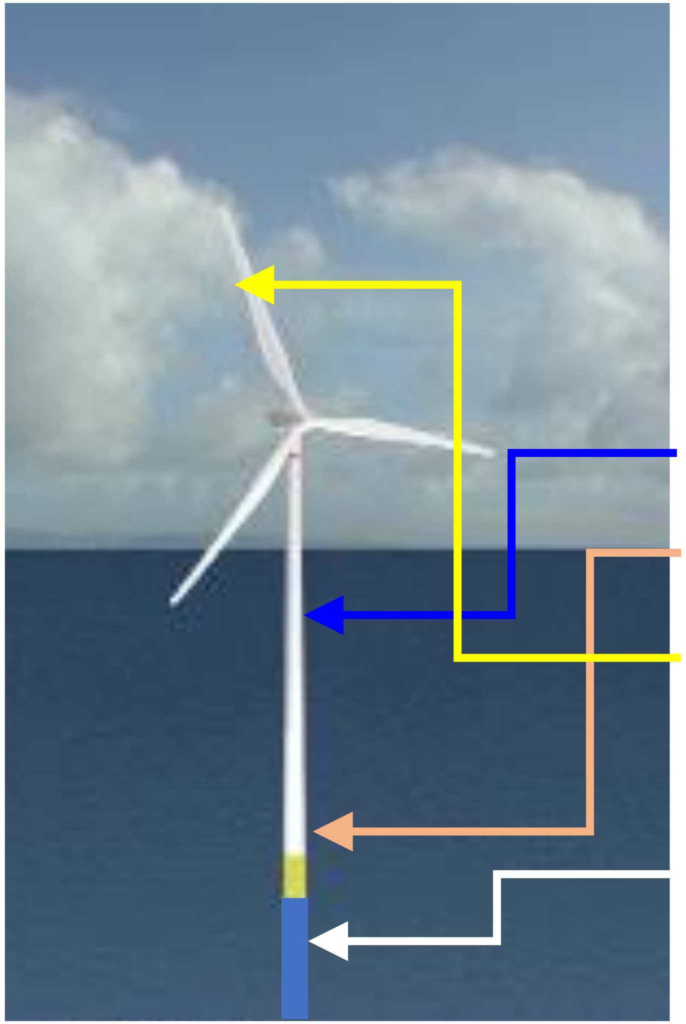
The complete nonlinear dynamic aeroelastic (or aeroinelastic) problem

$$[M]\cdot\ddot{u}+[C+G]\cdot\dot{u}+[K(t)+Z(t)+H(t)]\cdot u=R(t)$$

$[M]$	Mass matrix
$[C]$	Damping matrix
$[G]$	Coriolis matrix
$[K(t)]$	Stiffness matrix
$[Z(t)]$	Centrifugal stiffness matrix
$[H(t)]$	Acceleration stiffness matrix
$R(t)$	Vector of external loads
$u,\dot{u},\ddot{u}$	Displacement-, velocity- and acceleration vectors

$[M]$, $[C]$, $[K]$ and $[Z]$ symmetrical

$[G]$ and $[H]$ anti-symmetrical



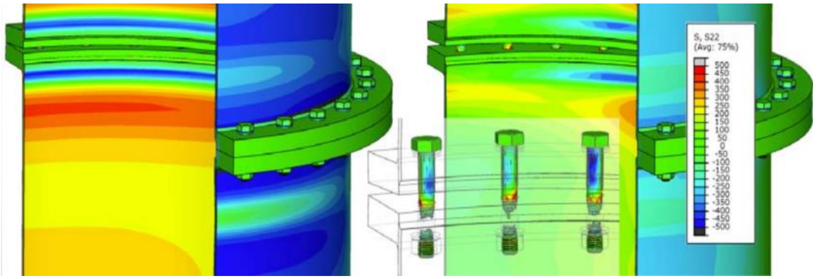
NONLINEAR STRUCTURAL BEHAVIOR OF WIND TURBINE COMPONENTS

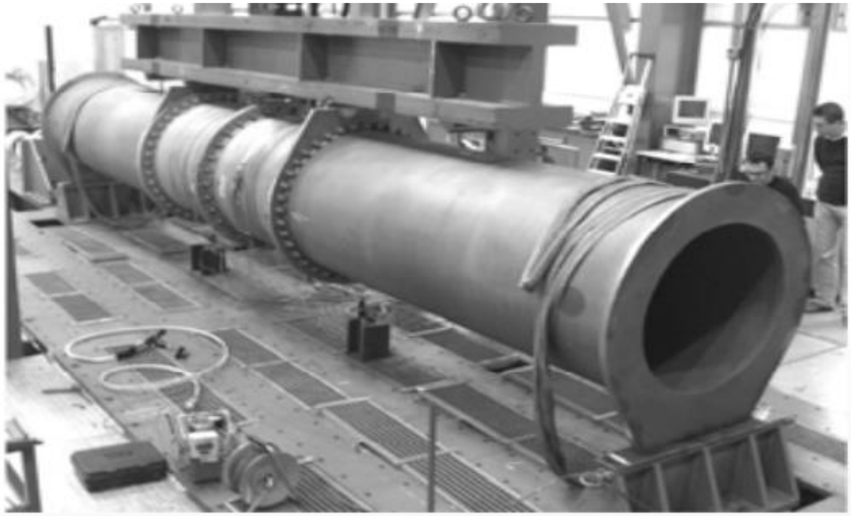
CASE STUDIES

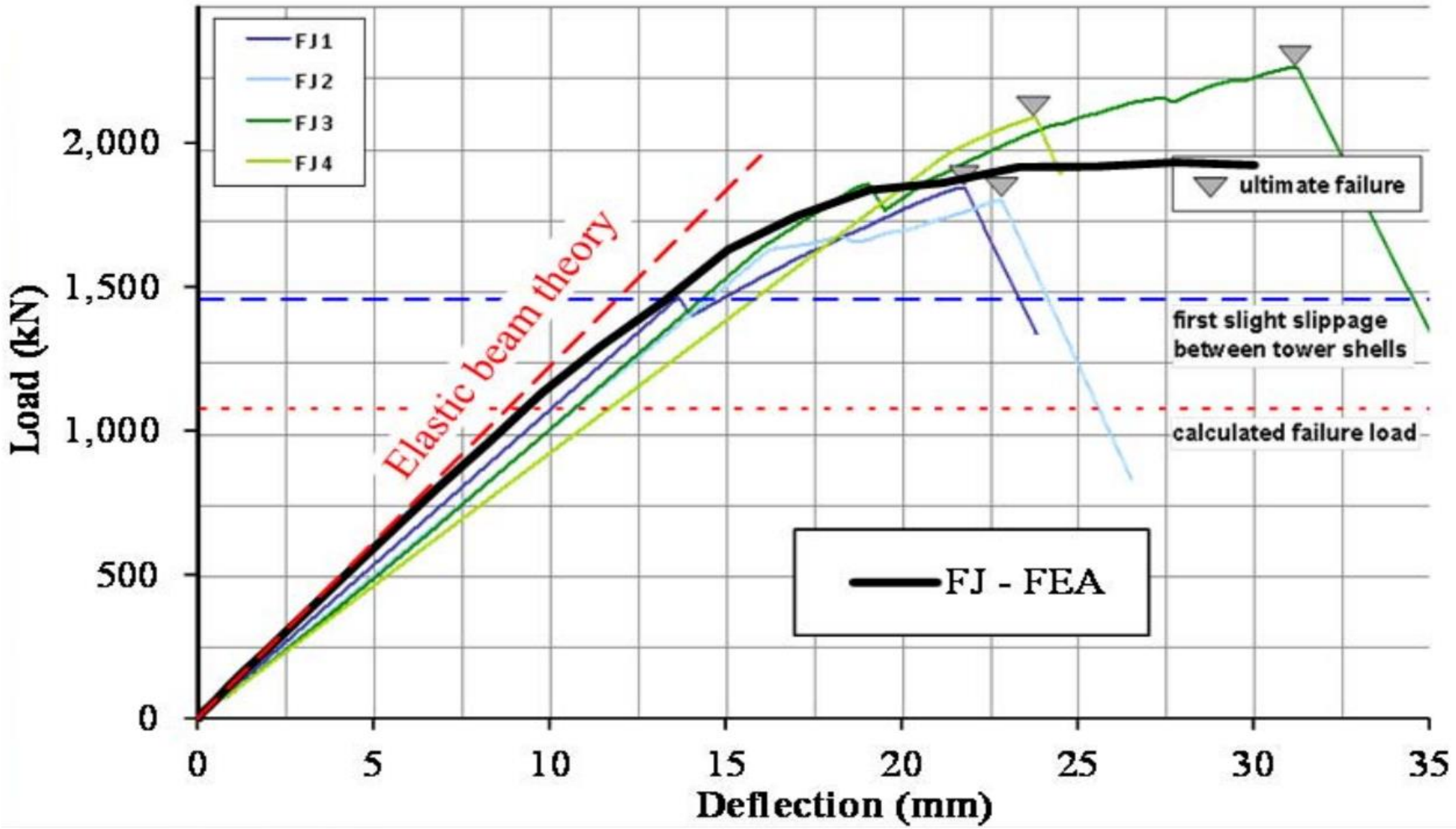
1) Ring flange connections
2) Stability of tower (opening/stiffening)
3) Delamination & local buckling in blades
4) Nonlinear behavior of monopile

Nonlinear structural engineering and wind turbines

Case study 1: Ring flange connection



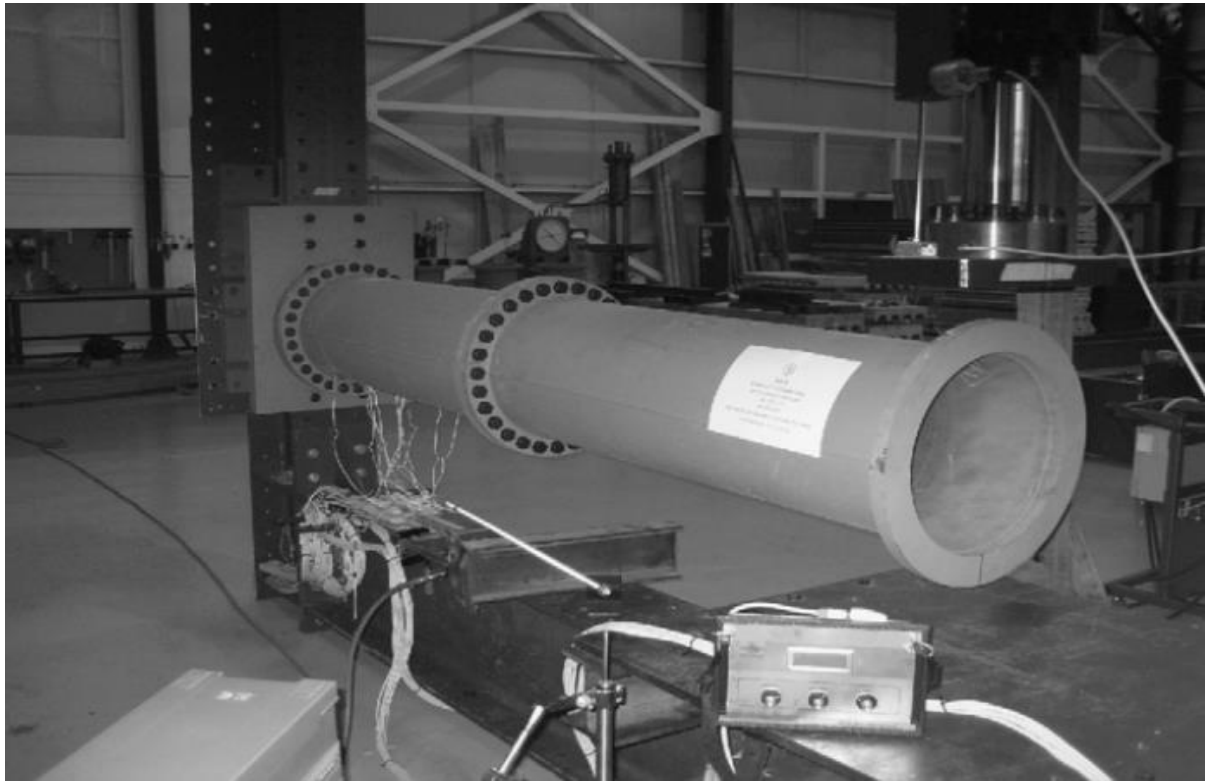




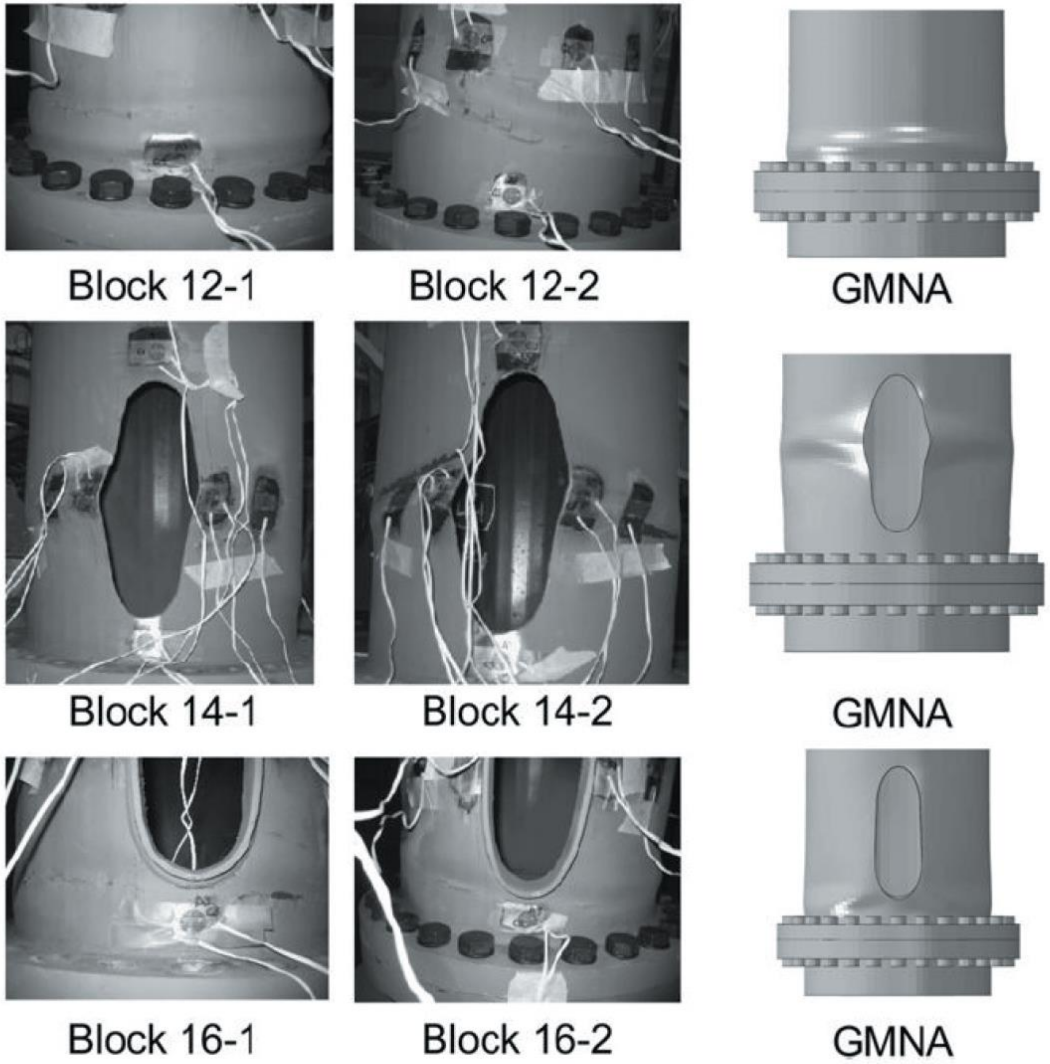
Veljkovic, M. et al (2010), 'High-strength steel tower for wind turbine, HISTWIN' Final Report RFSR-CT-2006-00031, Brussels: RFCS Publications, European Commission

Nonlinear structural engineering and wind turbines

Case study 2: Nonlinear behavior of wind turbine towers

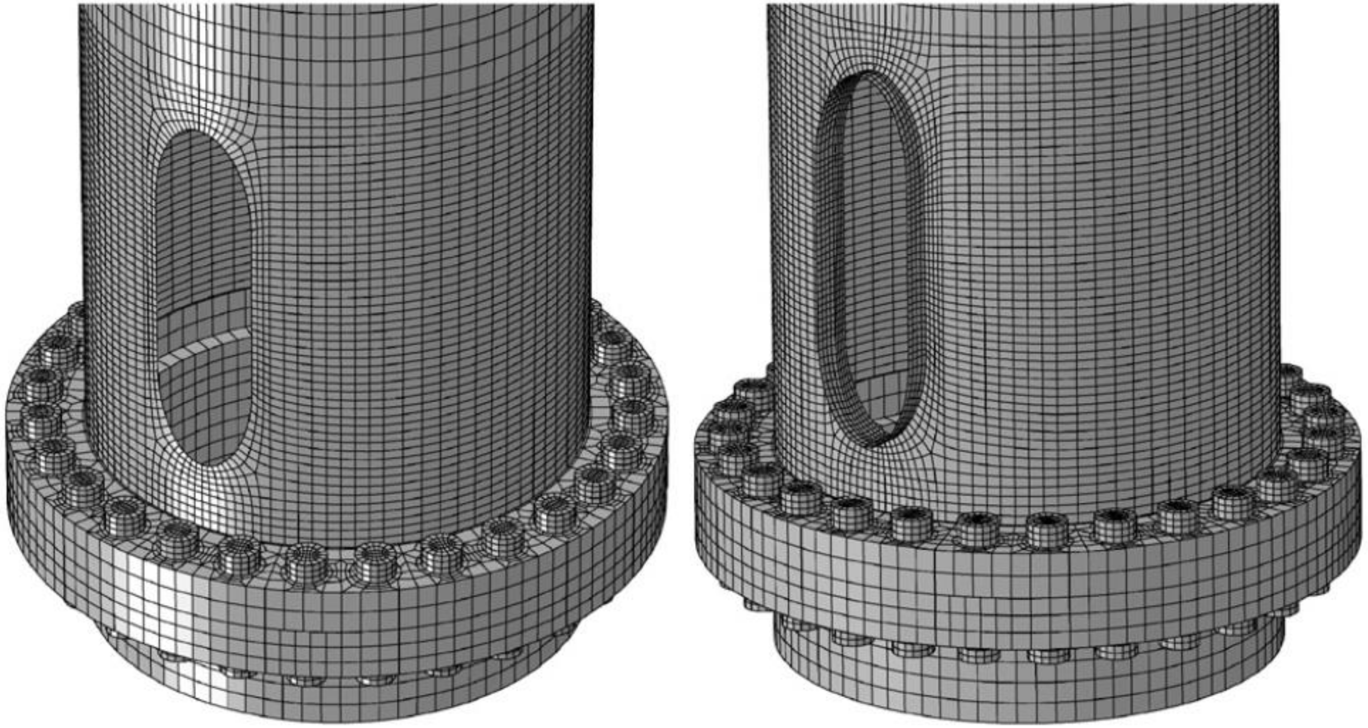


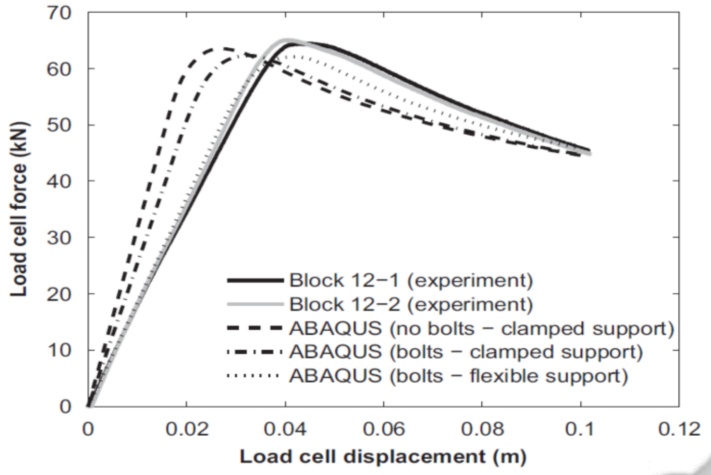
C.A. Dimopoulos, C.J. Gantes, Experimental investigation of buckling of wind turbine tower cylindrical shells with opening and stiffening under bending, Thin-Walled Structures, vol. 54, pp. 140-155, 2012.

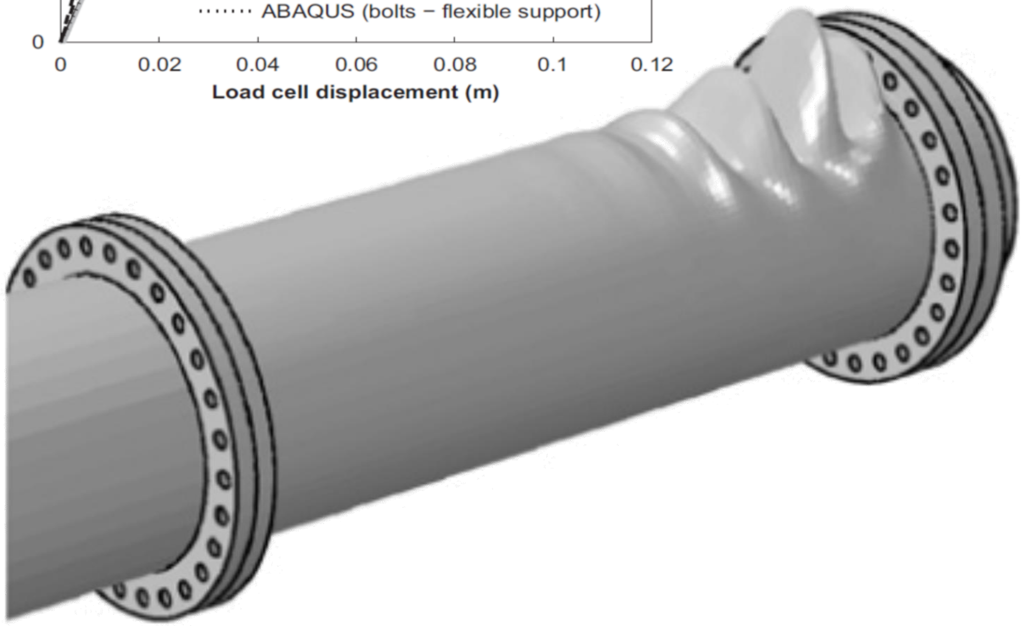


Nonlinear structural engineering and wind turbines

Case study 2: Nonlinear behavior of wind turbine towers







C.A. Dimopoulos, C.J. Gantes, Experimental investigation of buckling of wind turbine tower cylindrical shells with opening and stiffening under bending, Thin-Walled Structures, vol. 54, pp. 140-155, 2012.

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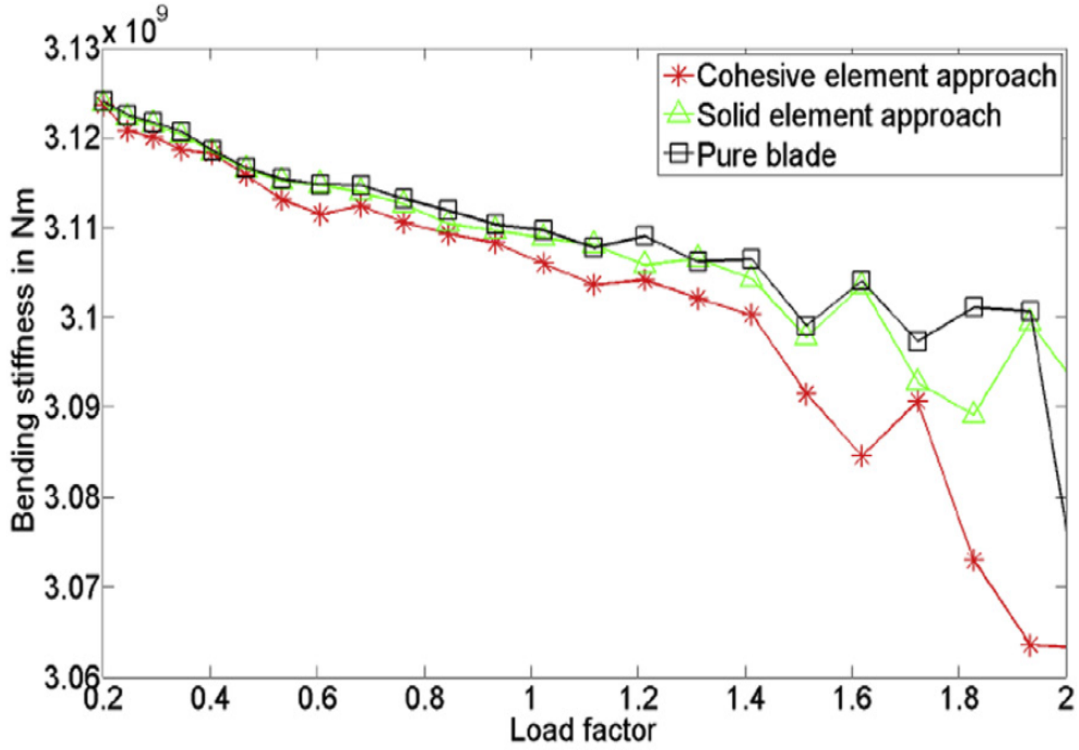
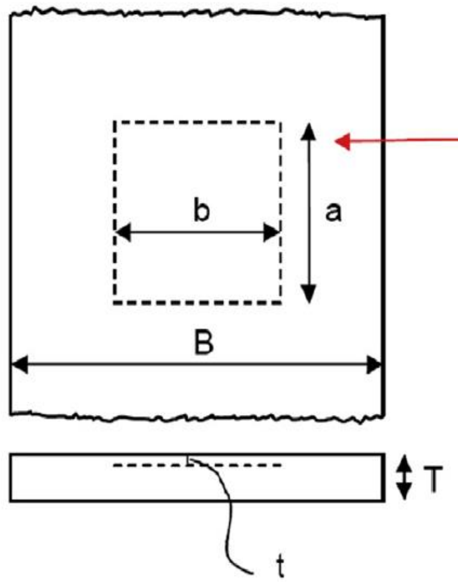
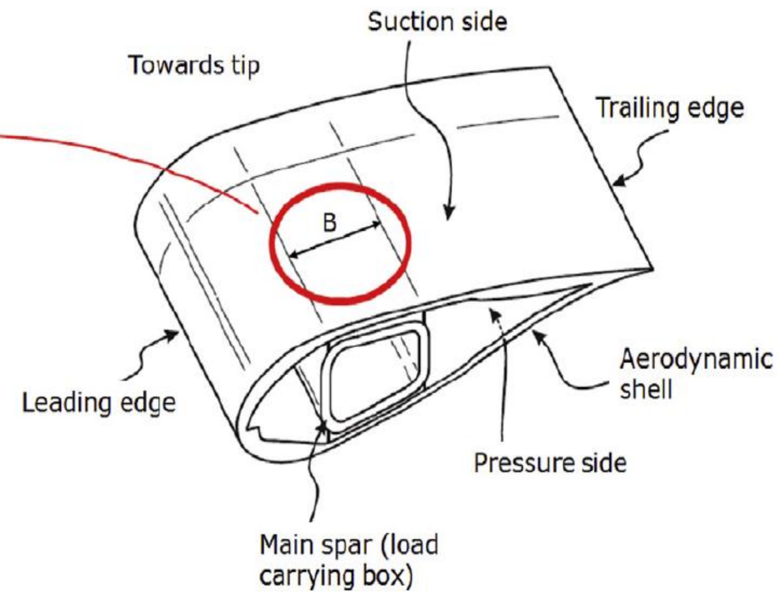
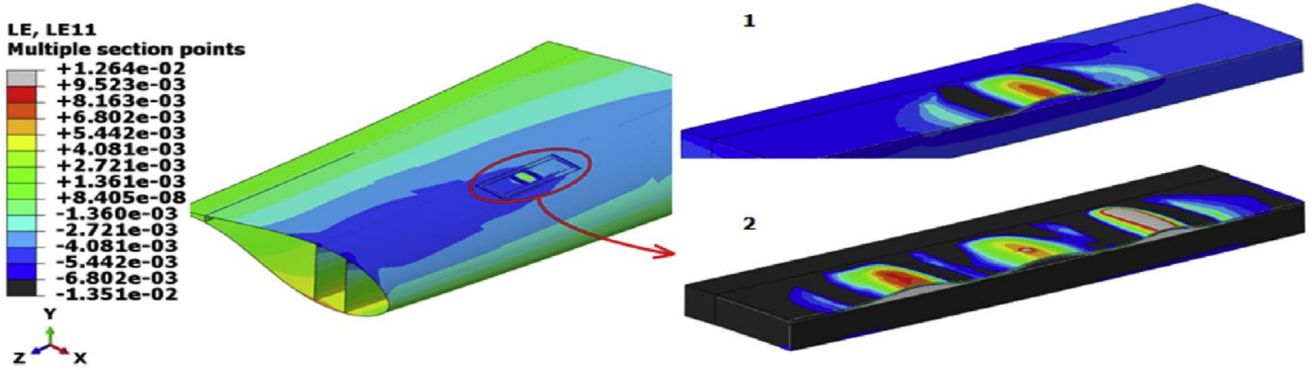
Case study 3: Nonlinear behavior of wind turbine blades









Haselbach, P. U., Bitsche, R. D., & Branner, K. (2016). The effect of delaminations on local buckling in wind turbine blades. Renewable Energy, 85, 295-305.

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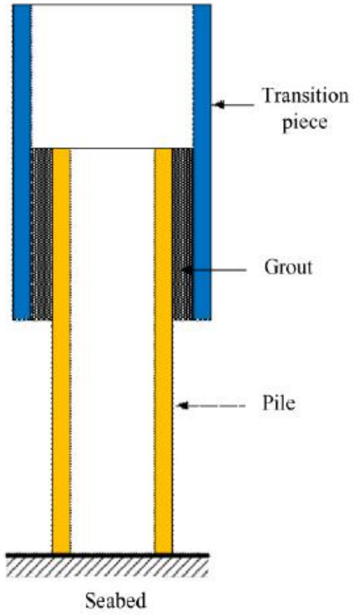
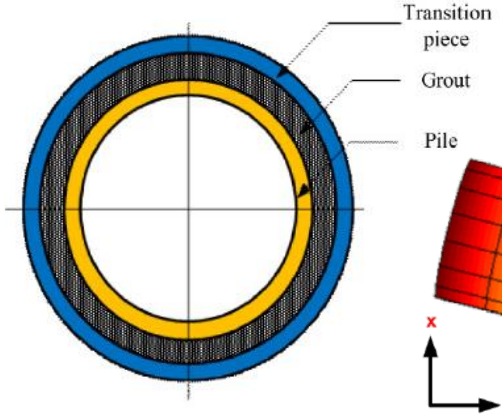
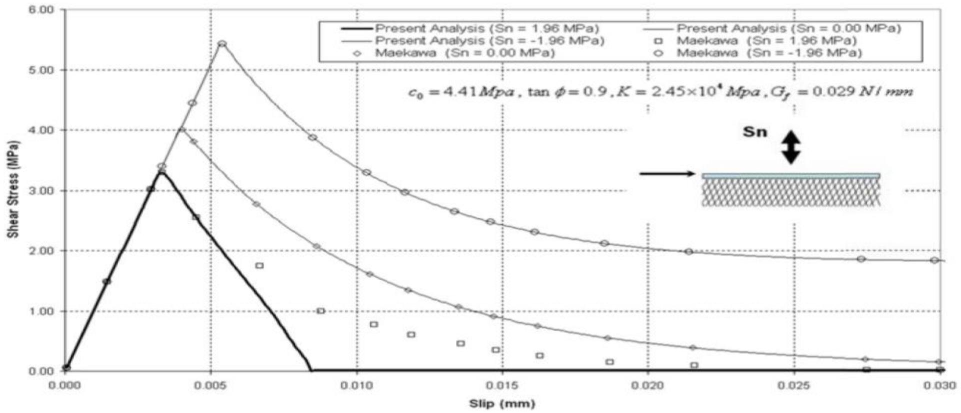
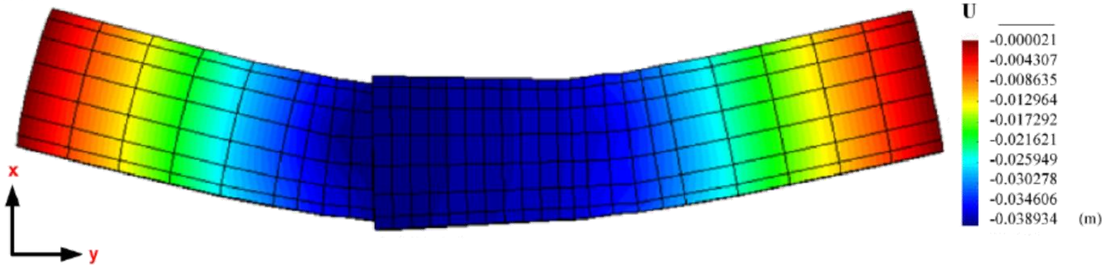
Case study 4: Nonlinear behavior of monopile grouted connection

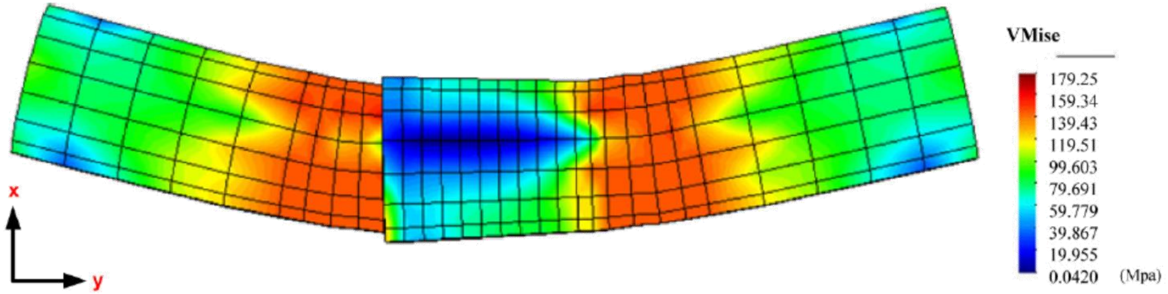








Kim, K. D., Plodpradit, P., Kim, B. J., Sinsabvarodom, C., & Kim, S. (2014). Interface behavior of grouted connection on monopile wind turbine offshore structure. International Journal of Steel Structures, 14(3), 439-446.



Thank you for your attention