Wind-generated waves

Athanassios A. Dimas

Department of Civil Engineering, University of Patras

Outline

1. Environmental loads on OWTs
2. Wind
3. Wind-generated waves

1. Environmental loads on OWTs

The most important environmental loads on OWTs are due to:

- Wind
- Waves
- Currents
- Tides

Other environmental phenomena, which may play a role in the design of OWTs, are:

Ice, Earthquake, Soil conditions, Temperature, Fouling, Visibility.

2. Wind – Introduction

Role of wind in the design of OWTs:

- Direct load
- Wave generation

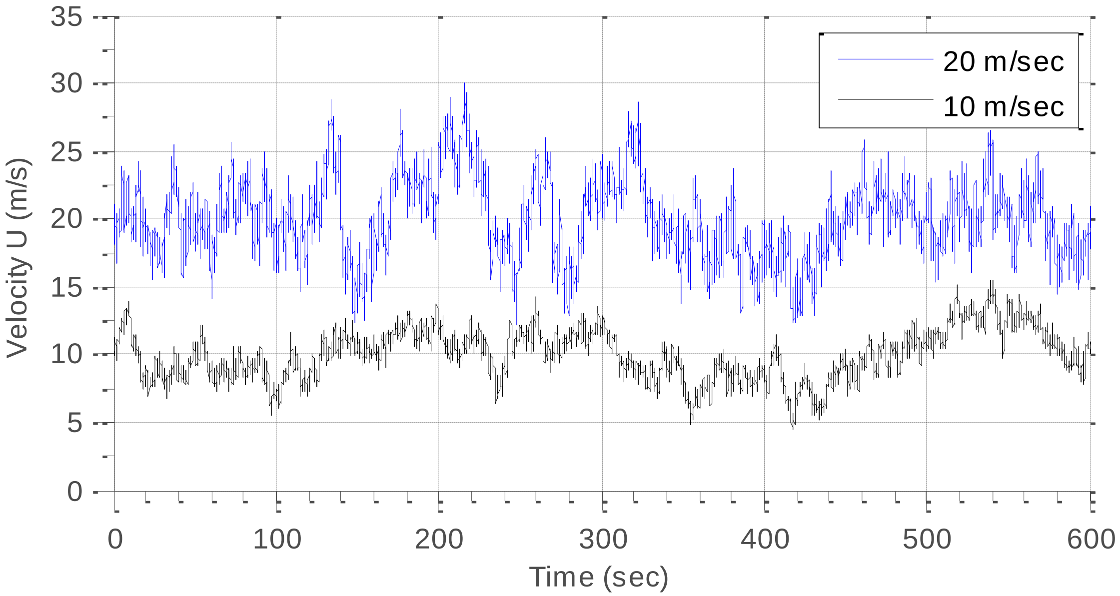
References:

- DNV 2010. Environmental Conditions and Environmental Loads. DNV-RP-C205 (RECOMMENDED PRACTICE), DET NORSKE VERITAS AS.
- DNV 2011. Design of Offshore Wind Turbine Structures. DNV-OS-J101 (OFFSHORE STANDARD), DET NORSKE VERITAS AS.
- IEC 2009. Wind Turbines - Part 3: Design Requirements for Offshore Wind Turbines. IEC (International Electrotechnical Commission) & BS (British Standards) EN 61400-3.

2. Wind – Wind speed parametrization

Analysis of measurements in order to identify appropriate values of wind speed (normal, severe and extreme) to be used in the design.

Wind speed, related to the computation of environmental loads, varies with time and height.



Commonly, wind speed is presented after averaging over 1 minute, 10 minutes or 1 hour. Commonly, a reference height of 10 m is used.

The 10-minute mean wind speed at 10 m height: U_{10}

The 10-minute standard deviation of wind speed at 10 m height: σ_U

Turbulence intensity: σ_U / U_{10}

2. Wind – Wind speed statistics

- **For reliable design, the wind climate database for computing U_{10} and σ_U should preferably cover a 10-year period or more of continuous data with a sufficient time resolution.**

- Wind speed measured at heights different than 10 m should be converted using either local data if they exist or by means of fitting profiles (presented next).

- If wind data are scarce and uncertain, and wind velocity measurements can not be carried out, then a reliable hindcast wind model should be validated and used.

- The assumption of stationary wind conditions during the 10 minutes is not always valid. For example, wind gusts induced by front passages and unstable weather conditions, tropical storms, etc.

2. Wind – Height profiles

The most common wind speed profiles with respect to height above the mean sea level (MSL) are the logarithmic, the power law and the Froya profiles.

The logarithmic profile is:

$$U(z) = \frac{u^*}{\kappa} \ln \frac{z}{z_0}$$

$$u^* = \sqrt{\frac{\tau}{\rho_a}} = f_a \cdot U_{10} \Rightarrow f_a = \frac{\kappa}{\ln(Z/z_0)}$$

where u^* is the friction velocity, κ=0.40 is the von Karman constant, z_0 is the sea roughness height (0.0001m for open still sea and 0.01m for open sea or coastal area with waves), Z=10 m is the reference height, τ is the shear stress on the sea surface, ρ_α=1.225 kg/m^3 is the air density and f_a is the friction coefficient.

2. Wind – Height profiles

The power law profile is: $U(z) = U(Z)\left(\frac{z}{Z}\right)^{a}$

where the exponent *a*=0.12-0.14 for open seas with waves and *Z*=10 m.

The Froya wind profile (DNV 2010) is: $U(T,z) = U_0\left(1 + C \cdot \ln\left(\frac{z}{Z}\right)\right)\left(1 - 0.41 \cdot I_U(z) \cdot \ln\left(\frac{T}{T_0}\right)\right)$

where U_0 is the 1-hour mean wind speed, T_0=1 hour, $T<T_0$,

$$C = 5.73 \cdot 10^{-2}\sqrt{1 + 0.148 \cdot U_0} \qquad I_U(z) = 0.06 \cdot (1 + 0.043 \cdot U_0) \cdot \left(\frac{z}{H}\right)^{-0.23}$$

The Froya profile is recommended by DNV for extreme mean winds with return periods of 50 years.

2. Wind – Mean wind speed

Mean wind speed is usually obtained by one of the following sampling schemes:

- A U_{10} value is computed for every 10-minute period consecutively, i.e. six (6) values are obtained every hour or 144 values per day.
- A U_{10} value is computed for one 10-minute period every hour, i.e. 24 values per day.
- A U_{10} value is computed for one 10-minute period every three or six hours, i.e. 8 or 4 values per day.

Regardless of the sampling method, the collected U_{10} values, obtained over a time span of several years, are used to compute the cumulative probability function $F_{U10}(U)$.

2. Wind – Mean wind speed

Unless the dataset of U_{10} values indicates otherwise, a Weibull distribution can be assumed for the cumulative probability (πιθανότητα εμφάνισης) function at a given height:

$$F_{U10}\left(U_{10} \leq U\right) = 1 - \exp\left(-\left(\frac{U}{A}\right)^{k}\right)$$

where A is the scale parameter and k is the shape parameter, which are site and height dependent, and they are obtained by fitting of the data set values.

Note that $F_{U10}\left(U_{10} \leq A\right) = 0.632$

2. Wind – Mean wind speed

Example:

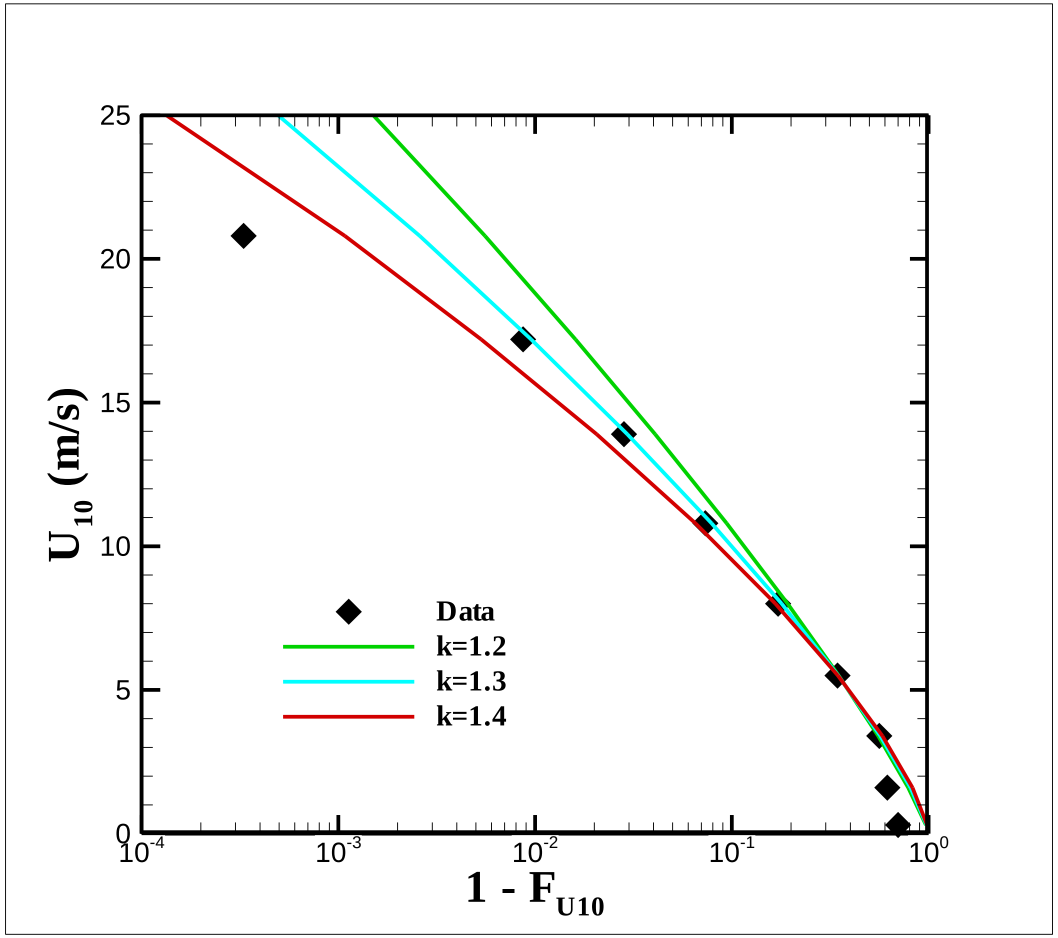
For the dataset shown in the figure,

k=1.4 gives the best fitting.

$$F_{U10}\left(U_{10} \le A\right) = 0.632 \Rightarrow$$

$$1 - F_{U10} = 0.368 \Rightarrow$$

$$A = 5.272 \text{ m/s}$$



2. Wind – Extreme mean wind speed

The cumulative probability distribution of extreme wind speeds can be approximated by the Gumbel distribution:

$$F_{U10,\max,1-yr}(U) = \exp\left(-\exp\left(-a\left(U-b\right)\right)\right)$$

The 10-minute mean wind speed with a return period of T_R in years is the one whose probability of exceedance is $1/T_R$, hence:

$$F_{U10,\max,1-yr}\left(U_{10,T_R}\right) = 1-\frac{1}{T_R} \quad \Rightarrow \quad (\text{Gumbel})$$

$$F_{U10,\max,1-yr}\left(U_{10,T_R}\right) = \exp\left(-\exp\left(-a\left(U_{10,T_R}-b\right)\right)\right) \Rightarrow U_{10,T_R} = b-\frac{1}{a}\ln\left(-\ln\left(1-\frac{1}{T_R}\right)\right)$$

2. Wind – Extreme operating gust

Especially for the design of wind turbines, the Extreme Operating Gust is defined as:

$$V = \min\left\{1.35\left(U_{EWM,hub,1-yr} - U_{10,hub,1-yr}\right), \frac{3.3\sigma_{U,1-yr}}{1+0.1\dfrac{D}{\Lambda_L}}\right\}$$

where *D* is the rotor diameter and $\Lambda_L = L_U / 8.1$ is the longitudinal scale of turbulence and is related to the integral length scale of the wind speed spectrum:

$$L_U = \begin{cases} 3.33z & \text{for} \quad z < 60\text{m} \\ 200\text{m} & \text{for} \quad z \geq 60\text{m} \end{cases}$$

3. Waves – Introduction

Wind-generated waves are not regular (monochromatic) but irregular (spectral):

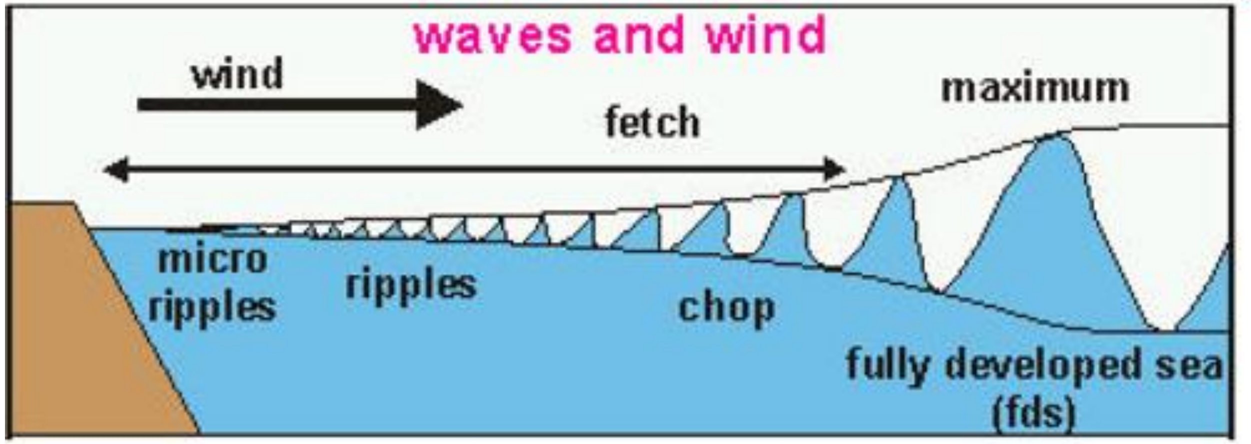
- Correlation between wind and wave parameters.
- Statistical and spectral analysis of irregular waves.
- Hindcast of proper values (normal, severe and extreme) of wave parameters to be used in the design of Floating Offshore Wind Turbines (FOWT).

References:

- DNV 2010. Environmental Conditions and Environmental Loads. DNV-RP-C205 (RECOMMENDED PRACTICE), DET NORSKE VERITAS AS.
- U.S.A.C.E., 2002. *Coastal Engineering Manual*, Engineer Manual 1110–2-1100, U.S. Army Corps of Engineers, Part II, Chapters 1 & 2, Washington, DC.

3. Waves – Generation and growth

Transfer of energy from winds in the lower atmosphere to the water in the upper sea (Phillips 1957, Miles 1960):



Turbulent boundary layer of air flow over initially calm sea induces pressure fluctuations on the sea surface and the generation if ripples.

The developing rough sea surface induces generation of eddies in the air flow, which in turn induce stronger pressure fluctuations on the sea surface and the generation of sea surface undulations that gradually grow and become waves.

During this process, air flow separation occurs at the crests of the undulations while the pressure on the sea surface is out of phase with the sea surface elevation.

The process reaches an equilibrium state where wave growth ceases.

3. Waves – Wind and wave parameters

Wind parameters:

- Speed, U_{10}.
- Fetch, F.
- Duration, t_d.

Wave parameters:

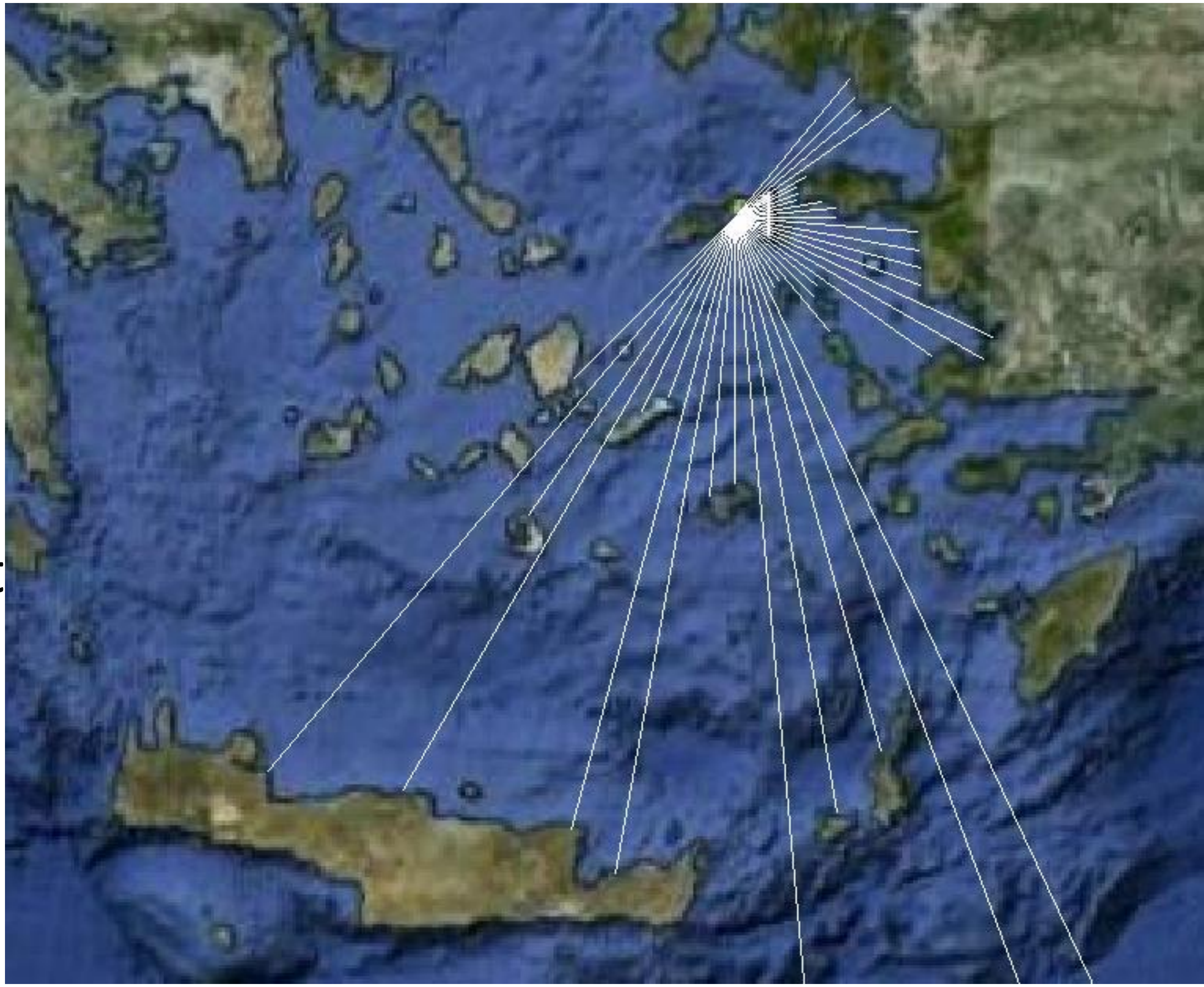
- Significant wave height, H_s.
- Significant wave period, T_s.

3. Waves – Fetch

Fetch = characteristic length of a straight and free of obstructions (e.g. islands) path of wind blow over the sea surface.

The end point of an individual fetch, F_i, is the location where the wave parameters are required, for example at the location of a FOWT, while the start point is at the nearest coastline in the direction of the wind under consideration.

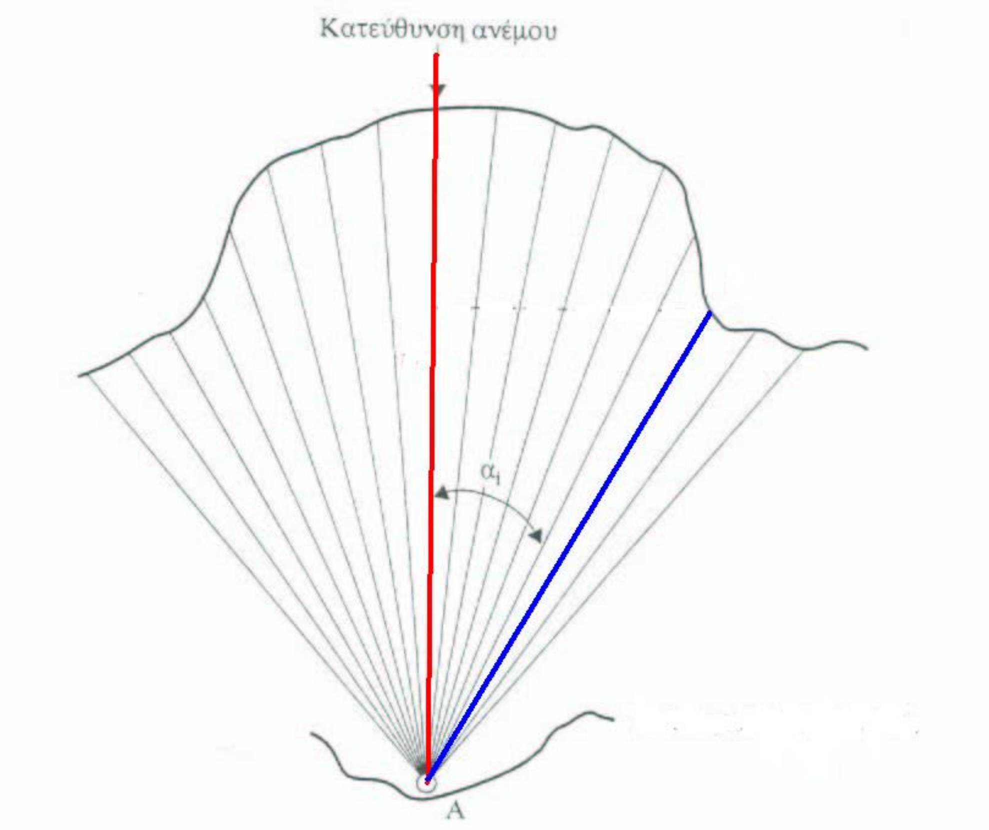
Example: several F_i lines for a location offshore of Agios Kirykos at Ikaria Island, Greece.



3. Waves – Fetch

Fetch is computed as a weighted average of the individual lengths, F_i, around the wind direction under consideration:

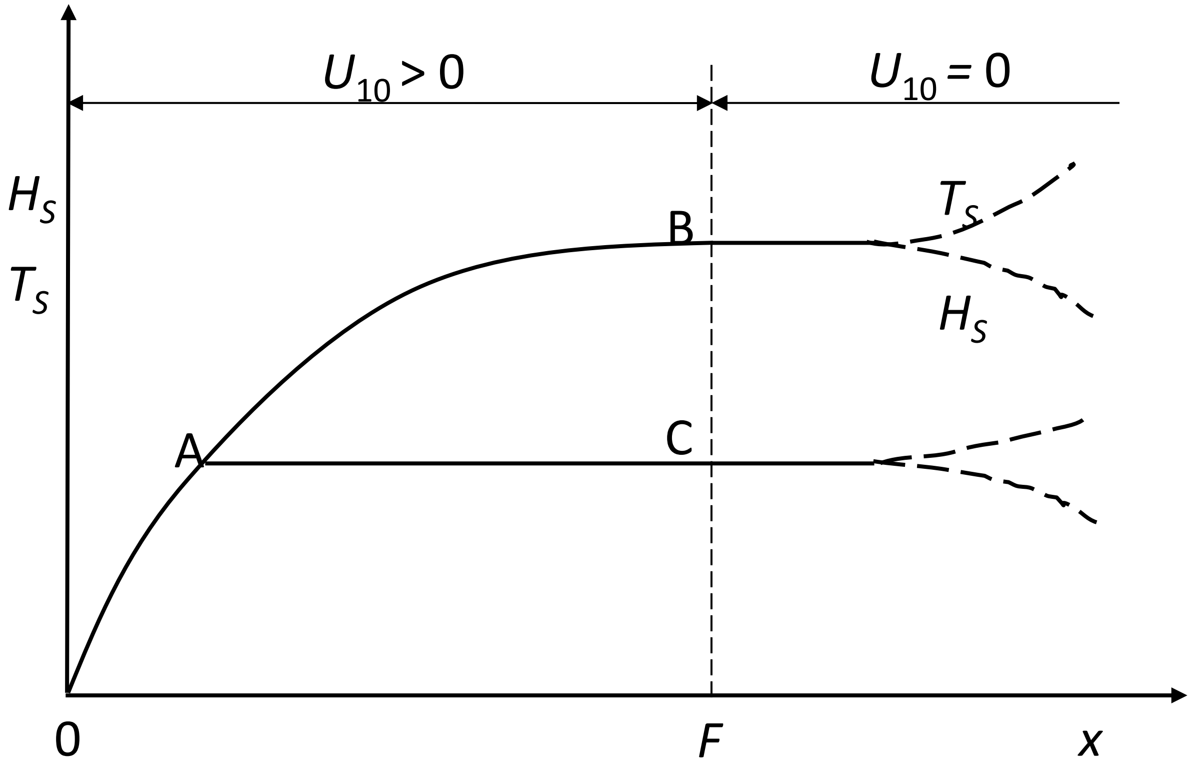
$$F = \frac{\sum_{i=-N}^{N} F_i \cos^2 a_i}{\sum_{i=-N}^{N} \cos a_i}$$



where $\alpha_i = i \cdot \Delta\alpha$, and usually $\Delta\alpha = 5^o$ and $N = 9$,
i.e. circular sector of ±45° with respect to the wind direction

3. Waves – Wind-wave parameter correlation

Sketch of the spatial development of wave parameters (significant wave height and period) induced by wind of constant speed U_{10} blowing over a given fetch F.

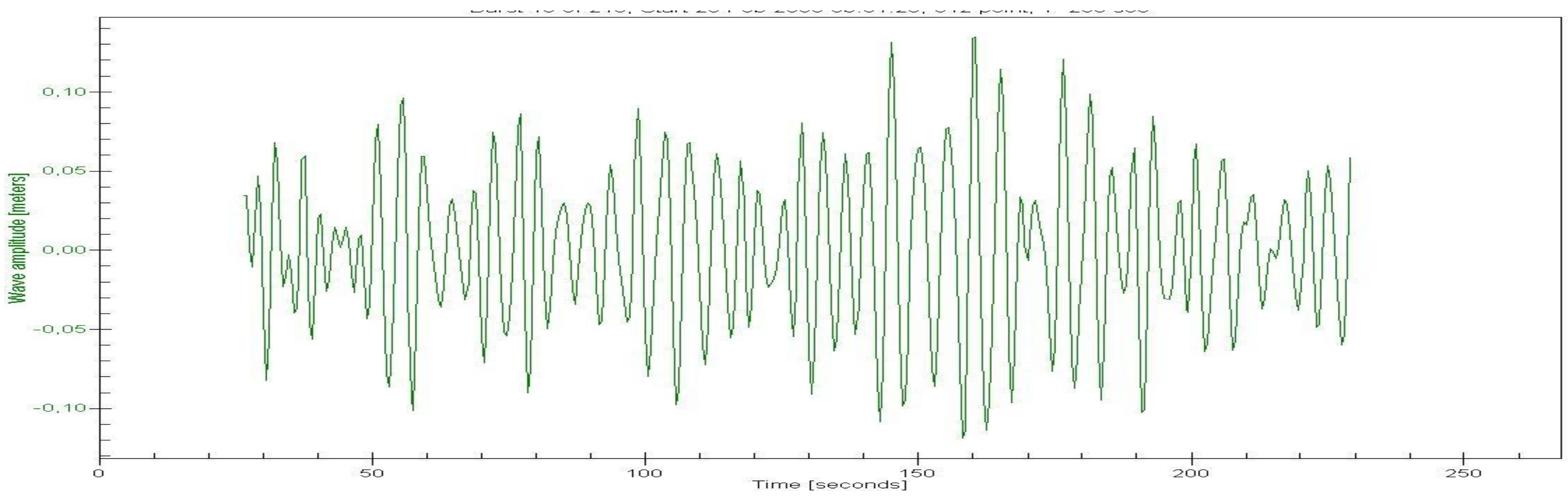


3. Waves – Wind-wave parameter correlation

- Fully developed waves: wave parameters depend only on the wind speed ($F \rightarrow \infty$, $t_d \rightarrow \infty$)
- Fetch-limited waves: 0 → (developing) → B → (developed)
- Duration-limited waves: 0 → (developing) → A → (developed) → C
- The sea state beyond B or C, where waves are developed but there is no wind blow, is called swell.
- After some distance or equivalently after some time with no wind blow, friction prevails over gravity and the waves are dissipating.

3. Waves – Irregular waves

The random-looking, sea-surface elevation resulting by the propagation of irregular waves can be analyzed by: **(i) statistical analysis in the time domain**, or **(ii) spectral analysis in the frequency domain**.

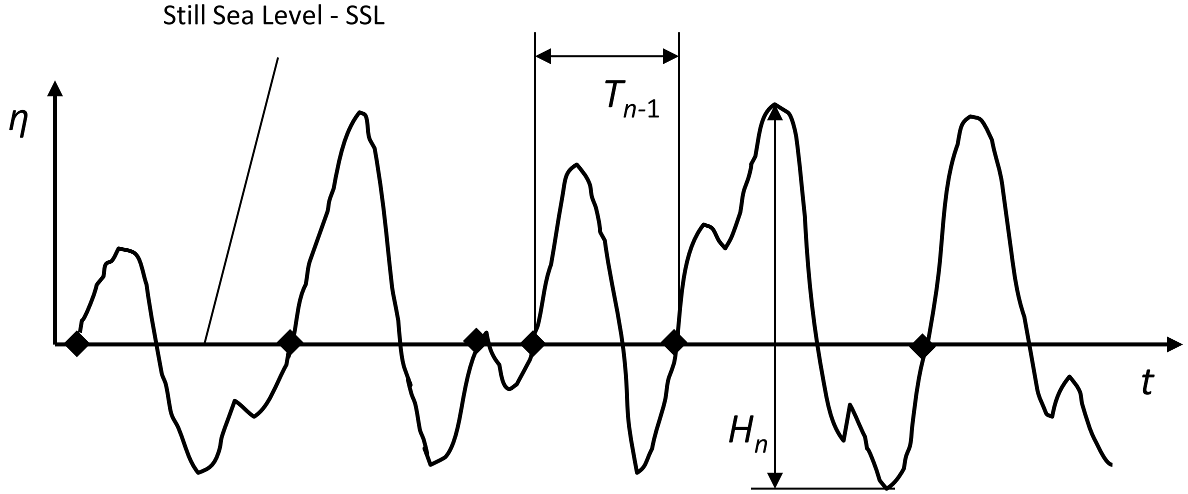


Typical temporal evolution of the sea-surface elevation at a fixed location in space during propagation of irregular waves

3. Waves – Statistical analysis of irregular waves

The analysis is based on temporal records of the sea-surface elevation, η.

In a record, a "wave" corresponds to the part of the record between two successive zero upcrossing points, and its height H_n and period T_n are defined as shown in the following sketch:

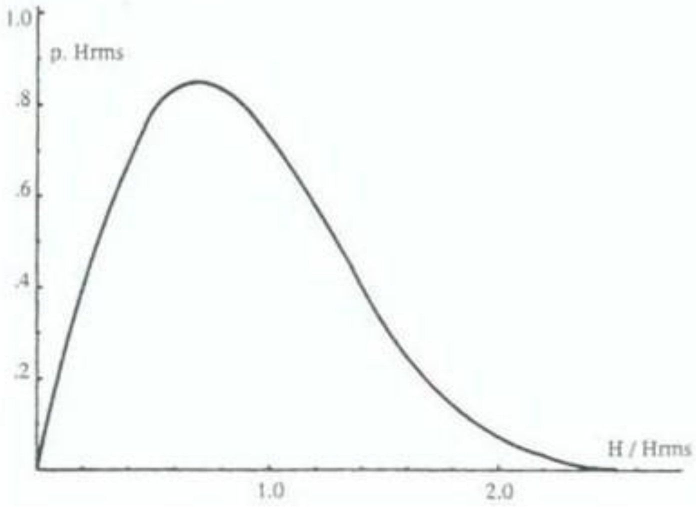


The N "waves" of each record are ranked in descending order according to their heights: H_1 is the maximum and H_N is the minimum.

3. Waves – Statistical analysis of irregular waves

The cumulative probability of the wave height in records of irregular waves follows the Rayleigh distribution:

$$P(H)=\int_0^H p(x)\,dx = 1-\exp\left(-\left(\frac{H}{H_{rms}}\right)^2\right)$$



where the probability density is: $p(x)=\frac{2x}{H_{rms}^2}\exp\left(-\left(\frac{x}{H_{rms}}\right)^2\right)$

The most representative wave heights of the waves in a record or a storm are:

RMS (root mean square) wave height

$$H_{rms}=\sqrt{\frac{1}{N}\sum_{n=1}^{N}H_n^2}$$

Significant wave height

$$H_s=H_{33}=\frac{3}{N}\sum_{n=1}^{N/3}H_n$$

3. Waves – Statistical analysis of irregular waves

The exceedance probability of any wave height value H_n in the record or the storm is:

$$1-P\left(H \le H_n\right)=\frac{n}{N}=\exp\left(-\left(\frac{H_n}{H_{rms}}\right)^2\right) \Rightarrow H_n = H_{rms}\sqrt{-\ln\frac{n}{N}}$$

The lower wave height in the definition of the significant wave height is:

$$H_{n=N/3} = H_{rms}\sqrt{-\ln\frac{1}{3}} = 1.048 \cdot H_{rms}$$

Therefore, the definition of the significant wave height, using the Rayleigh probability density, results into:

$$H_s = H_{33} = \frac{\int_{H_{N/3}}^{\infty} Hp\left(H\right)dH}{\int_{H_{N/3}}^{\infty} p\left(H\right)dH} = \sqrt{2} \cdot H_{rms}$$

3. Waves – Statistical analysis of irregular waves

The exceedance probability of the significant wave height H_s is:

$$1-P(H_s)=\exp\left(-\left(\frac{H_s}{H_{rms}}\right)^2\right)=\exp(-2)=0.135$$

For other representative wave heights of a record or a storm:

$$\bar{H}=H_{100}=\frac{1}{N}\sum_{n=1}^{N}H_n=\frac{\sqrt{\pi}}{2}H_{rms}=0.63\cdot H_s \qquad H_{10}=\frac{10}{N}\sum_{n=1}^{N/10}H_n=1.27\cdot H_s$$

$$H_{\max}=H_{n=1}=H_{rms}\sqrt{\ln N}=H_s\sqrt{\frac{1}{2}\ln N}=\begin{cases}1.86\cdot H_s & & N=1000\\ 1.95\cdot H_s & \text{for} & N=2000\\ 2.04\cdot H_s & & N=4000\end{cases}$$

3. Waves – Statistical analysis of irregular waves

The significant wave period of the waves in a record or a storm is: $T_s = T_{33} = \frac{3}{N}\sum_{n=1}^{N/3} T_n$

The wave period of irregular waves does not follow any probabilistic distribution.

According to Det Norske Veritas (DNV-RP-C205 2010), the wave period that corresponds to the maximum wave height in a storm is:

$$T_{\max} = 2.94\sqrt{H_{\max}}$$

where the period is in sec and the wave height is in m.

3. Waves – Spectral analysis of irregular waves

The sea-surface elevation is decomposed into a Fourier series: $\eta(t)=\sum_{i=1}^{M}\frac{H_i}{2}\cos(2\pi f_i t+\varphi_i)$

The dispersion of the sea-surface elevation in the time-domain is: $\overline{\eta^2}=\frac{1}{t_K}\int_0^{t_K}\eta^2 dt=\sum_{i=1}^{M}\frac{H_i^2}{8}=\frac{1}{8}H_{rms}^2$

The corresponding definition in the frequency domain is: $\overline{\eta^2}=\sum_{i=1}^{M}S(f_i)\Delta f$ where *S*(f) is the spectrum.

The moments of the spectrum are defined as: $m_n=\int_0^{\infty}f^n S(f)\,df$

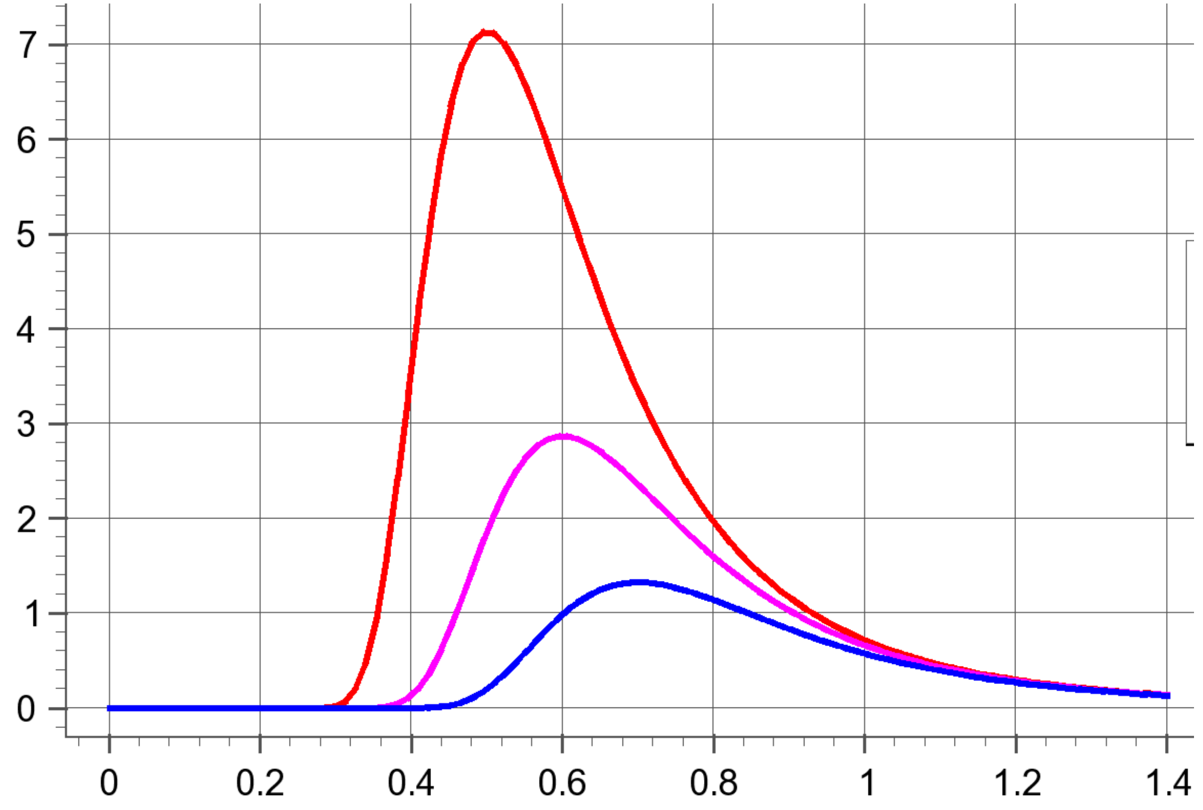
The 0th moment is related to the energy density and the characteristic heights of the irregular waves: $\int_0^{\infty}S(f)\,df=m_0=\frac{\bar{E}}{\rho g}=\frac{H_{rms}^2}{8}$

$$H_S=H_{m0}=4\sqrt{m_0}=\sqrt{2}\cdot H_{rms}$$

3. Waves – Spectral analysis of irregular waves

General form of the spectrum of wind-generated irregular waves:

$$S(f) = \frac{Ag^2}{f^5} \exp\left(-1.25 \frac{f_P^4}{f^4} \right)$$



3. Waves – Spectral analysis of irregular waves

PM (Pierson-Moskowitch) spectrum:

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left(-0.74 \left(\frac{g}{2\pi W f} \right)^4 \right) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left(-1.25 \left(\frac{f_P}{f} \right)^4 \right)$$

where $\alpha = 0.0081$, f_P is the spectral peak frequency (at the local maximum value of the spectrum), and $W \approx 1.075 \cdot U_{10}$ is the wind speed at $Z = 19.5$m above the still sea level.

$$\left. \frac{dS}{df} \right|_{f=f_P} = 0 \Rightarrow f_P = \frac{1}{T_P} = 0.877 \frac{g}{2\pi W} \qquad \alpha = 5\pi^4 \frac{H_s^2 f_p^4}{g^2} \Rightarrow H_s = 0.21 \frac{W^2}{g}$$

The PM spectrum represents fully-developed waves.

3. Waves – Spectral analysis of irregular waves

JONSWAP (JOint North Sea WAve Project):) spectrum:

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left(-1.25\left(\frac{f_P}{f}\right)^4\right) \gamma^{\exp\left(-\frac{1}{2}\left(\frac{f-f_p}{\sigma f_p}\right)^2\right)}$$

where

$$a = 0.0081(1 - 0.287 \ln \gamma)$$

$$1 < \gamma < 7$$

$$\gamma = 3.3$$

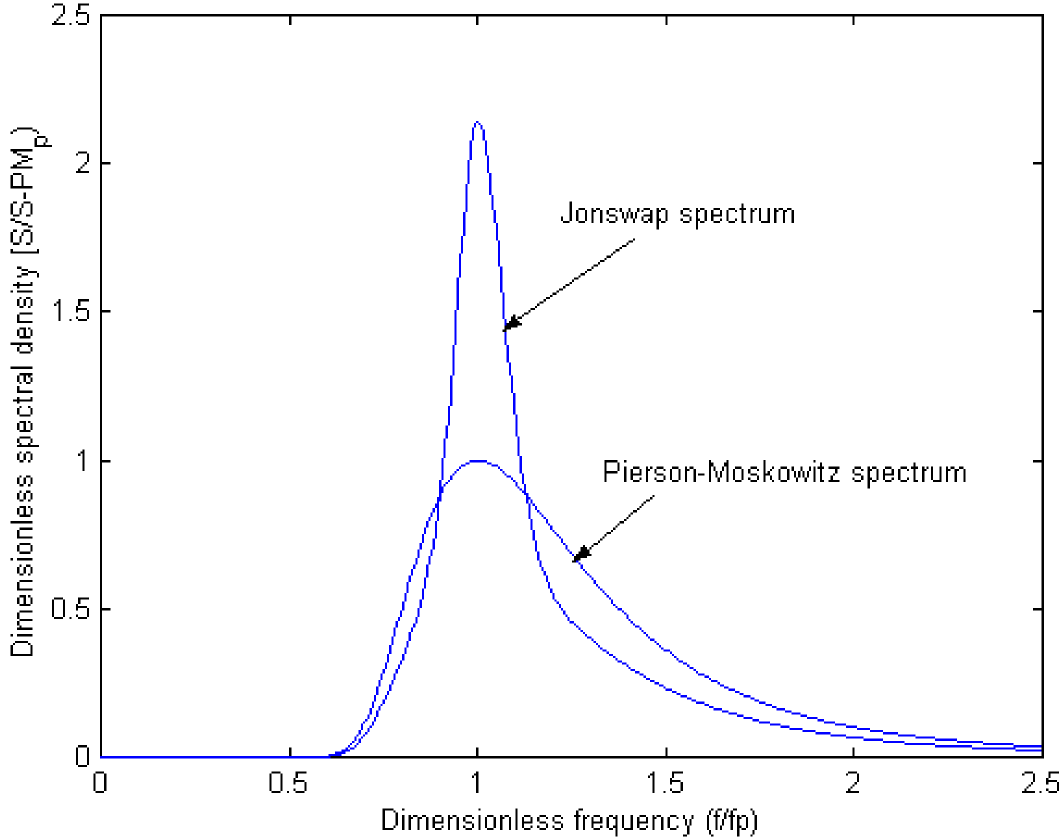
$$\sigma = \begin{cases} 0.07 & f \le f_P \\ 0.09 & f > f_P \end{cases}$$

The JONSWAP spectrum represents fetch-limited, developed waves.

For $\gamma = 1$, the JONSWAP spectrum is identical to the PM one.
For $\gamma \to \infty$, the wave becomes regular with wave period equal to T_P.

3. Waves – Spectral analysis of irregular waves

Comparison between PM and JONSWAP spectrums:



3. Waves – Spectral analysis of irregular waves

The significant wave period is computed according to the Shore Protection Manual (USACE 1984) as:

$$T_s \approx 0.95 \cdot T_P$$

The mean wave period, which corresponds to the mean interval of zero upcrossings, is defined as:

$$\bar{T} = T_{m02} = \sqrt{\frac{m_0}{m_2}}$$

For the JONSWAP spectrum, the mean wave period is related to the significant one:

$$\bar{T} = T_P \sqrt{\frac{5+\gamma}{11+\gamma}} \Rightarrow T_s = \begin{cases} 1.34 \cdot \bar{T} & \gamma = 1 \\ 1.25 \cdot \bar{T} \quad \text{for} & \gamma = 3.3 \\ 1.20 \cdot \bar{T} & \gamma = 5 \end{cases}$$

3. Waves – Hindcast of irregular waves

Numerical simulations

Wave generation and growth based on the numerical solution of the wave action density, *N*, equation (3rd generation of spectral wave models):

$$\frac{\partial N}{\partial t}+\frac{\partial\left(\left(C_{gx}+U\right)N\right)}{\partial x}+\frac{\partial\left(\left(C_{gy}+U\right)N\right)}{\partial y}+\frac{\partial\left(C_{r}N\right)}{\partial \omega_r}+\frac{\partial\left(C_{\theta}N\right)}{\partial \theta}=\frac{S_E}{\omega_r}$$

where $N=E/\omega_r$, $E=\rho g S$ is the energy density, S is the spectrum, ω_r is the relative wave radial frequency, U is the current velocity, t is time, (x,y,z) are the spatial coordinates, (C_{gx},C_{gy}) are the wave celerity components, C_θ is the wave direction rate of change, $C_r=d\omega_r/dt$, and S_E is the summation of source and sink term.

The most important source term introduces the effect of wind shear stress, while the most important sink term is the effect of wave breaking.

3. Waves – Hindcast of irregular waves

Empirical methods: CEM-JONSWAP method

Friction velocity:

$$\frac{u_*}{U_{10}} = \sqrt{C_D} = \sqrt{0.001\left(1.1 + 0.035 \cdot U_{10}\right)}$$

For fetch-limited waves, the significant wave height and period are:

$$\frac{gH_s}{u_*^2} = 0.0413\left(\frac{gF}{u_*^2}\right)^{0,5} \qquad \frac{gT_s}{u_*} = 0.71345\left(\frac{gF}{u_*^2}\right)^{0,33}$$

For duration-limited waves, first the equivalent fetch is computed:

$$\frac{gF_{eq}}{u_*^2} = 0.00523\left(\frac{gt_d}{u_*}\right)^{1,5}$$

If $F > F_{eq}$, the wave is duration-limited and the wave parameters are computed by setting $F = F_{eq}$.

3. Waves – Hindcast of irregular waves

Example for JONSWAP method

Wind parameters: $U_{10} = 30 \text{ m/s} \qquad F = 20 \text{ km} \qquad t_d = 3 \text{ hr}$

Friction velocity: $u_* = U_{10}\sqrt{0.001(1.1+0.035 \cdot U_{10})} = 30 \cdot 0.04637 = 1.391 \text{ m/s}$

Equivalent fetch: $F_{eq} = \frac{u_*^2}{g} 0.00523 \left(\frac{g t_d}{u_*}\right)^{1,5} = \frac{1.391^2}{9.807} 0.00523 \left(\frac{9.807 \cdot 3 \cdot 3600}{1.391}\right)^{1,5} = 21680 \text{ m}$

So $F < F_{eq}$ → the wave is fetch-limited.

3. Waves – Hindcast of irregular waves

Example: CEM-JONSWAP method

Significant wave height:

$$H_s = \frac{u_*^{\,2}}{g} 0.0413 \left(\frac{gF}{u_*^{\,2}} \right)^{0,5} = \frac{1.391^2}{9.807} 0.0413 \left(\frac{9.807 \cdot 20000}{1.391^2} \right)^{0,5} = 2.59 \text{ m}$$

Significant wave period:

$$T_s == \frac{u_*}{g} 0.71345 \left(\frac{gF}{u_*^{\,2}} \right)^{0,33} = \frac{1.391}{9.807} 0.71345 \left(\frac{9.807 \cdot 20000}{1.391^2} \right)^{0,33} = 4.54 \text{ s}$$

Maximum values in a storm:

$$H_{max} \approx 2H_s = 5.18 \text{ m} \qquad T_{max} = 2.94\sqrt{H_{max}} = 6.69 \text{ s}$$

3. Waves – Extreme irregular wave parameters

Severe values of the significant wave height, H_s , corresponding to long return periods (50 or 100 years), T_r, are computed using proper data extrapolation.

The cumulative probability distribution of severe waves extreme with very long period of return does not follow the Rayleigh distribution but the Weibull or the Gumbel ones. The Gumbel is:

$$\text{Gumbel}: \quad P(H) = e^{-e^{-\left(\frac{H-B}{A}\right)}}$$

$$A = 0.779 \cdot \sigma$$

$$B = \bar{H} - 0.45 \cdot \sigma$$

$$\bar{H} = \frac{1}{N}\sum_{i=1}^{N} H_i$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(H_i - \bar{H}\right)^2}$$

3. Waves – Extreme irregular wave parameters

The exceedance probability, **1 - *P***, is related to the return period, ***T_r***, and the interval, ***r***, between measurements :

$$\frac{T_r}{r} = \frac{1}{1-P(H)} \Leftrightarrow P(H) = 1 - \frac{r}{T_r}$$

Using Gumbel, results into:

$$\frac{H_{T_r}}{H_r} = \frac{A\left(-\ln\left(-\ln\left(1-\frac{r}{T_r}\right)\right)\right)+B}{H_r}$$

3. Waves – Extreme irregular wave parameters

The encounter probability of an extreme value of the significant wave height at a time ***t << T_r*** is given by the expression:

$$E(t) = 1 - e^{-\frac{t}{T_r}}$$

Hence, the encounter probability of the wave with return period T_r = 100 yr during the 1st year (t = 1 yr) is 1%, while during the first 10 years (t = 10 έτη) increases to 9.5%.

For the period of the extreme wave, Goda (2003) suggests: $T_s = 3.3(H_s)^{0.63}$

where H_s is in m.

3. Waves – Extreme irregular wave parameters

Example: Extreme wave parameters

For mean annual data (r = 1 yr) with $H_{s,1yr}$ = 2.59 m and σ = 0.2·$H_{s,1yr}$, the Gumbel distribution gives:

$$H_{s,100-yr} = 1.63 \cdot H_{s,1-yr} = 1.63 \cdot 2.59 = 4.22 \text{ m}$$

while according to Goda:

$$T_{s,100-yr} = 3.3\left(H_{S,100-yr}\right)^{0.63} = 8.18 \text{ s}$$

The corresponding max values of extreme waves during a storm are:

$$H_{\max,100-yr} \approx 2H_{S,100-yr} = 8.44 \text{ m} \qquad T_{\max,100-yr} = 2.94\sqrt{H_{\max,100-yr}} = 8.54 \text{ s}$$